Giles Games, Vague Quantifiers, and Contexts

Workshop on Logical Dialogue Games

Matthias Hofer Christoph Roschger
joint work with Chris Fermüller

Institute of Computer Languages
Theory and Logic Group

September 29, 2015

Outline

Giles's Game

Precisifications and Context Sensitivity

Vague Quantification - 'Many' and Dialogue Games

Conclusion and Outlook

Giles's game

Overview

Giles's game for Łukasiewicz logic $Ł_\infty$

- evaluation game: determine truth in a model
- each player asserts a multiset of formulas
- initially, I assert the formula in question
- dialogue part and betting part

- can be motivated as a generalization of Hintikka's evaluation game for classical logic

Giles's game

Dialogue part

- Decompose formulas according to dialogue rules

General Game Rule

One player chooses one of their opponent's compound propositions F and attacks it. After the according defense move F is deleted from the game.

- no further regulations

Giles's game

Dialogue part

- Decompose formulas according to dialogue rules

General Game Rule

One player chooses one of their opponent's compound propositions F and attacks it. After the according defense move F is deleted from the game.

- no further regulations

(LLA): I can declare not to attack an assertion.

(LLD): I can just assert $\perp$ in reply to your attack.

- rules for your assertions are dual

Giles's game

Dialogue rules

- Decompose formulas according to dialogue rules

($R_{\rightarrow}$): If I assert $F \rightarrow G$ then you attack by asserting F, and I defend by asserting G.

($R_{\vee}$): If I assert $F \vee G$, then my defense consists in asserting F or G at my choice.

($R_{\wedge}$): If I assert $F \wedge G$, then my defense consists in asserting F or G at your choice.

($R_{\forall}$): If I assert $\forall x.F(x)$, then my defense consists in asserting $F(c)$ where c has been chosen by you.

($R_{\exists}$): If I assert $\exists x.F(x)$, then my defense consists in asserting $F(c)$ for some c at my choice.

Giles's game

Dialogue rules

- Decompose formulas according to dialogue rules

($R_{\rightarrow}$): If I assert $F \rightarrow G$ then you attack by asserting F, and I defend by asserting G.

($R_{\vee}$): If I assert $F \vee G$, then my defense consists in asserting F or G at my choice.

($R_{\wedge}$): If I assert $F \wedge G$, then my defense consists in asserting F or G at your choice.

($R_{\forall}$): If I assert $\forall x.F(x)$, then my defense consists in asserting $F(c)$ where c has been chosen by you.

($R_{\exists}$): If I assert $\exists x.F(x)$, then my defense consists in asserting $F(c)$ for some c at my choice.

($R_{\&}$): If I assert $F \& G$, then my defense consist in asserting both F and G.

Giles's game

Betting part and correspondence to $Ł_\infty$

- dispersive binary experiments
- success probabilities correspond to v_M
- experiment fails: the asserting player pays 1€
- Risk: the expected amount of money I have to pay

Theorem (Giles)

The least final risk r I can enforce directly corresponds to $v_M(F)$:

- I have a strategy, that my risk is not greater than r
- you have a strategy, that my risk is not smaller than r

Many-Valued Logics

Łukasiewicz Logic Ł

- one of the three fundamental fuzzy logics
- domain of truth values: unit interval $[0, 1]$
- based on the Łukasiewicz t-norm $\max(0, x + y - 1)$

Connectives of Łukasiewicz Logic

Connectives: $\rightarrow$, &, $\wedge$, $\vee$, $\neg$ with truth functions:

- $f_{\&}(x, y) = \max(0, x + y - 1)$,
- $f_{\rightarrow}(x, y) = \min(1, 1 - x + y)$,
- $f_{\wedge}(x, y) = \min(x, y)$,
- $f_{\vee}(x, y) = \max(x, y)$,
- $f_{\neg}(x) = 1 - x$.

A formula is called *true* in Ł under a given interpretation iff it evaluates to 1.

Fuzzy Quantification

Motivation

- model vague natural language quantifiers like *about half*, *nearly a third*
- focus on semi-fuzzy quantifiers

Random witness selection

If I assert $\Pi x.\hat{F}(x)$ then I have to assert $\hat{F}(c)$ for a randomly picked c.

- uniform distribution over a finite domain $\mathcal{D}$
- blind choice and deliberate choice

Precisifications and Context Sensitivity

Example

Example:

- ♦ two objects, say Eve and Joe (a_1, a_2)
- ♦ two distinct predicates, say 'child' and 'poor' (P_1, P_2)
- ♦ one statement: "All children are poor"

Classically equivalent:

$\forall_x(A \to B) \leftrightarrow^c \forall_x(\neg A \vee B)$,

Not in mathematical fuzzy logic:

$v_I(\forall_x(A \to B)) \neq v_I(\forall_x(\neg A \vee B))$

Precisifications and Context Sensitivity

Example

Premise: $v_I(P_1(a_1)) = v_I(P_1(a_2)) = v_I(P_2(a_1)) = v_I(P_2(a_2)) = \frac{1}{2}$

Then:

$v_I(\forall_x(child(x) \rightarrow child(x))) = 1$, and
$v_I(\forall_x(child(x) \rightarrow poor(x))) = 1$.

Also,

$v_I(\forall_x(\neg child(x) \vee child(x))) = \frac{1}{2}$, and
$v_I(\forall_x(\neg child(x) \vee poor(x))) = \frac{1}{2}$.

So, we introduce a notion of context..

Precisifications and Context Sensitivity

Context

Definition

A context C is a non-empty finite set of classical interpretations over the same signature that share the same universe U_C and assign the same domain elements to the constant symbols.

remarks:

- ♦ each element $I \in C$ is called precisification and signifies an admissible manner to classify the elements of U_C as either satisfying or not satisfying a given predicate.
- ♦ the assumption of finiteness is not crucial.

Precisifications and Context Sensitivity

Ring Operator

One new game rule:

($R_\circ$) If I assert $\circ F$ then, in reply to your attack, some precisification $I \in C$ is chosen randomly and $\circ F$ is replaced with $F \uparrow I$ in my tenet.

Context sensitive truth values $v_{I^C}(\circ F) = 1 - \langle \circ F \rangle_C$

$$v_{I^C}(\circ F) = \frac{1}{|C|} \sum_{I \in C} 1_{\{v_I(F)=1\}}$$

$$\langle \circ F \rangle_C = \frac{1}{|C|} \sum_{I \in C} 1_{\{v_I(F)=0\}}$$

Precisifications and Context Sensitivity

Ring Operator

Two results on the ring-operator

- ⧫ $v_{I^C}(\circ\forall_x F) \leq v_{I^C}(\forall_x \circ F)$
- ⧫ $v_{I^C}(\forall_x(\neg P(x) \vee G(x)) \leq v_{I^C}(\forall_x \circ (\neg P(x) \vee G(x)) = v_{I^C}(\forall_x \circ (P(x) \rightarrow G(x)) \leq v_{I^C}(\forall_x(P(x) \rightarrow G(x))$

The latter also holds with:

- ⧫ existential quantifier
- ⧫ no quantifier

Vague Quantification - 'Many' and Dialogue Games

Semantics of 'Many'

S. Lappin (et al.): extensional and intensional readings of 'many'.

Today: focus on extensional side

Example: (LEGO-bricks)

Figure: a precisification

Vague Quantification - 'Many' and Dialogue Games

Semantics of 'Many'

NL statement: **"Many LEGO-bricks are brown."**

Different colors are comparison classes c_i, with $i \in \{1, .., n\}$, and we define for each such i:

Definition

$p_i := |\|LEGOs\| \cap \|c_i\||$, and $p_{br} := |\|LEGOs\| \cap \|brown\||$.

Also, we define:

$I_a := \{i : i \in \{1, \ldots, n\}, p_i > p_{br}\}$, and $a := |I_a|$.
$I_b := \{i : i \in \{1, \ldots, n\}, p_i = p_{br}\}$, and $b := |I_b|$.
$I_c := \{i : i \in \{1, \ldots, n\}, p_i < p_{br}\}$, and $c := |I_c|$.

Vague Quantification - 'Many' and Dialogue Games

Semantics of 'Many'

Truth value of the NL statement

$$\text{Many}(LEGOs, brown) = \frac{\sum_{i \in I_c} p_i}{\sum_{i \in I_c} p_i + \sum_{i \in I_a} p_i} =: z$$

Compatible with Dialogue Games through:

(R^{ext}_{many}) If I assert Many(A, B) then, in reply to your attack, I have to replace Many(A, B) with one distinguished atom of truth value z in my tenet.

Vague Quantification - 'Many' and Dialogue Games

Ring Operator and 'Many'

Combination of the Two: 'Ring Operator' and 'Many'

- ♦ Here, **precisifications correspond to** possible manifestations of our **actual situation** (the picture)
- ♦ Having the **ring in front of a 'many'-formula** amounts to calculating the truth value of Many(A,B) w.r.t. any precisification available and then to average the results.
- ♦ Having the **ring inside a 'many'-formula** amounts to averaging all precisifications into one, and then calculating the respective truth value for this merged information host.

Not true: $v_{I^C}(\circ Many(A, B)) \leq v_{I^C}(Many(A, \circ B))$

Conclusion and Outlook

- ◊ Dialogue Games can express NL quantifiers
- ◊ Context can be combined with Dialogue Games
- ◊ Vagueness comes from quantifiers and predicates

- ◊ Impose structure on contexts
- ◊ Extend the list of quantifiers to augment applicability
- ◊ Treat more complex NL statements
- ◊ Handle fuzzy predicates

Thank you for attending!!