Multi-Agent Dialogue Games and Dialogue Sequents

Workshop on Logical Dialogue Games 2015

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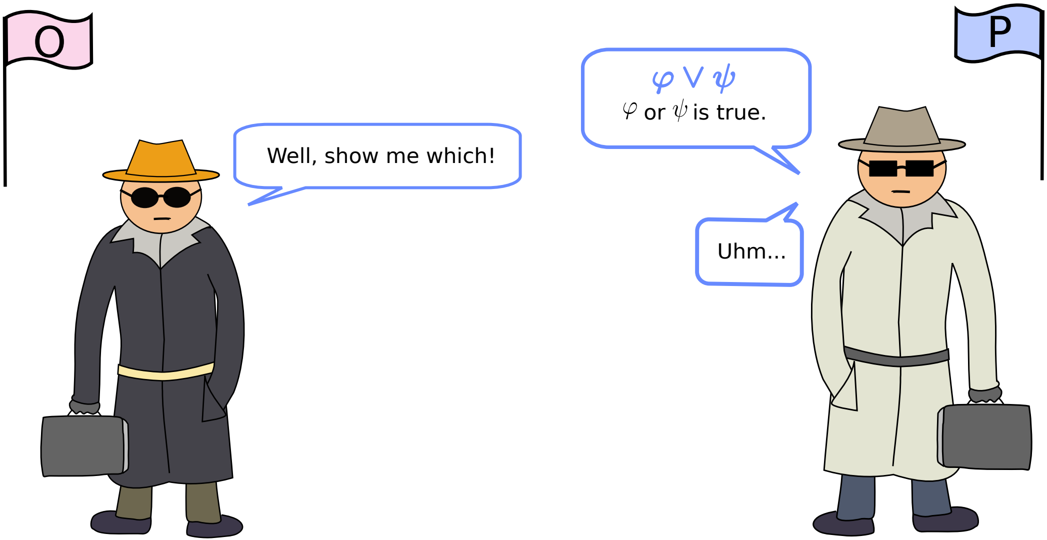
- Lorenzen-Dialogues as flexible reasoning procedure for **Intuitionistic** and **Modal Logic**
  - strategies
- **Parallelized** Dialogues
  - two **parties**, more players
  - distributed problem-solving
  - round-based scheduling & access restrictions
  - **normalization** simplifies proof-search
- Dialogue **Sequents**
  - clear rules, no ambiguity
  - easier to show soundness/completeness
  - also suitable for expressing parallelism



- P must decide whether to defend with φ or ψ.
- In classical logic, he can defend more than once.

	O		P	
1			**H**	$(A \wedge B) \supset (A \supset B)$
2	**?**$\supset$ (1)	$A \wedge B$	**!** (2)	$A \supset B$
3	**?**$\supset$ (2)	A	**!** (3)	B
4	B**?** (3)	$\lhd$	**?**L (2)	$\lhd$
5	**!** (4)	A	**?**R (2)	$\lhd$
6	**!** (5)	B	**!** (4)	$\lhd$

- O may **attack atoms**, P may not.
- P may defend atom-attacks if O stated atom herself (***ipse dixisti!***).
- In **intuitionistic dialogues**, P may only defend against the **last open attack**.

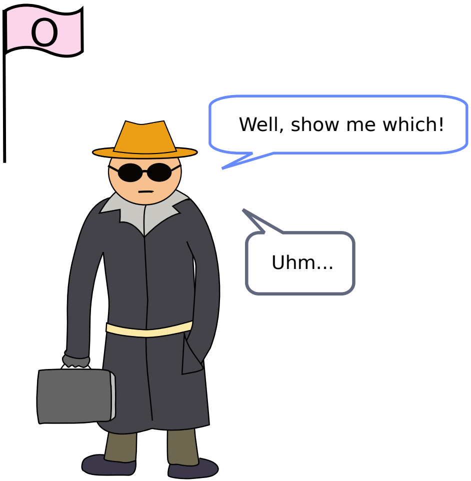
- **Dialogue Sequents** (Barth and Krabbe, 1982)

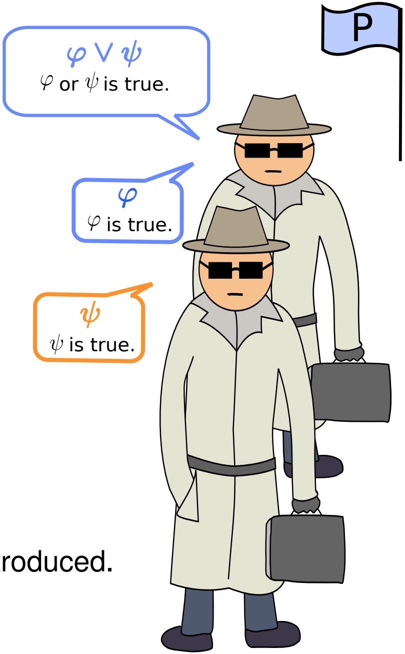
$$\Pi, [\Delta] / T /_{N} Z, [\Gamma]$$

  - N
    Party (O or P) whose **turn** it is
  - T
    local thesis: last statement attacked by O
  - Π
    concessions stated by O
  - $[\Delta]$
    possible **defences** for O
  - $[\Gamma]$
    possible **defences** for P
  - Z
    sentence stated by P, that can be **attacked** by O in **next move**

	O		P	
1			**H**	$(A \wedge B) \supset (A \supset B)$
2	**?**$\supset$ (1)	$A \wedge B$	**!** (2)	$A \supset B$
3	**?**$\supset$ (2)	A	**!** (3)	B
4	B**?** (3)	$\lhd$	**?**L (2)	$\lhd$
5	**!** (4)	A	**?**R (2)	$\lhd$
6	**!** (5)	B	**!** (4)	$\lhd$

$$\emptyset \; / \qquad\qquad /_{\mathrm{O}}\; (A \wedge B) \supset (A \supset B)$$
$$A \wedge B \; / \; (A \wedge B) \supset (A \supset B) \; /_{\mathrm{P}}\; [(A \supset B)]$$
$$A \wedge B \; / \; (A \wedge B) \supset (A \supset B) \; /_{\mathrm{O}}\; A \supset B$$
$$A, A \wedge B \; / \; A \supset B \; /_{\mathrm{P}}\; [B]$$
$$A, A \wedge B \; / \; A \supset B \; /_{\mathrm{O}}\; B$$
$$A, A \wedge B \; / \; B \; /_{\mathrm{P}}\; \emptyset$$
$$A, A \wedge B, [A] \; / \; B \; /_{\mathrm{O}}\; \emptyset$$
$$A, A \wedge B, A \; / \; B \; /_{\mathrm{P}}\; \emptyset$$
$$A, A \wedge B, A, [B] \; / \; B \; /_{\mathrm{O}}\; \emptyset$$
$$A, A \wedge B, A, B \; / \; B \; /_{\mathrm{P}}\; \emptyset \quad \textit{ipse dixisti!}$$





- When P has a choice, another agent is introduced.
- O argues with them separately.

	O		P0		P1		P2	
1			**H**	$(A \wedge B) \supset (A \supset B)$				
2	$?_{P0}\supset$	$A \wedge B$	**!**	$A \supset B$	$?L$	$\lhd$	$?R$	$\lhd$
3	$?_{P0}\supset$	A	**!**	B				
	$!_{P1}$	A			—	—		
	$!_{P2}$	B					—	—
4	$B?_{P0}$	$\lhd$	**!**	$\lhd$	—	—	—	—

- How to enforce **intuitionism**?

O $\lightning$ Pi

$i \in \mathbb{N}$

$\Pi \vdash \Sigma$

Π	formulas stated by O
Σ	formulas stated by Pi

- **Separated Contexts**
- Hypersequents
  $\Pi_1 \vdash C_1 \mid \Pi_2 \vdash C_2 \mid \dots$
- Single-conclusion

classical $\Pi_i, \Pi_k \vdash C_i$

↑ *merging* intermediate

intuitionistic $\Pi_i \vdash C_i \mid \Pi_k \vdash \bot$

- **Shared Context**
- single sequents
  $\Pi \vdash C_1, C_2, \dots$
- Multi-conclusion

classical $\Pi \vdash C_i, C_k$

↓ *isolation* sub-classical

intuitionistic $\Pi \vdash C_i$

- **Structural Rules** (shared context, classical logic)
  - Start with initial P-Agent P0.
    States **hypothesis**.
  - In every **round**/row every P-agent may perform ONE move.
    O reacts on all of the P-agents' moves of previous round.
  - All players MUST perform moves when possible.
  - Only O may attack atoms.
    P-agents can defend against atom-attacks only if atom has been stated by O (*ipse dixisti!*).
  - Last moving party wins.

- Shared context and **intuitionistic** logic

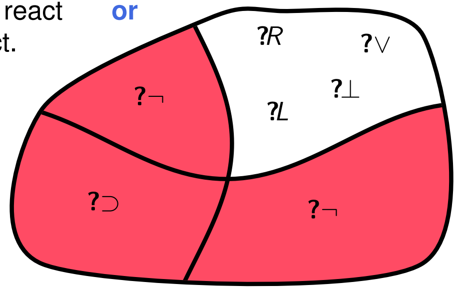
Distinguish between **critical** and **non-critical attacks**:

- critical: $?\supset$ $?\neg$
- non-critical: $?L$ $?R$ $?\vee$ $?\bot$ $a?$

  - corresponds to $\supset r$ and $\neg r$ rules of multi-conclusion sequent calculus.
  - O's concessions in a critical attack only **committed** if the corresp. P-agent defends ($\Rightarrow$ **promise**).

- **Structural rules** (cont.)

  - If **several agents** are attacked in one round by O then
    - **all non-critically attacked** agents may react **or**
    - **one critically attacked** agent may react. Then the other agents are **deactivated**. $\Rightarrow$ *isolation*
  - Critical defences may be delayed.



- Multi-Agent Dialogue Sequents

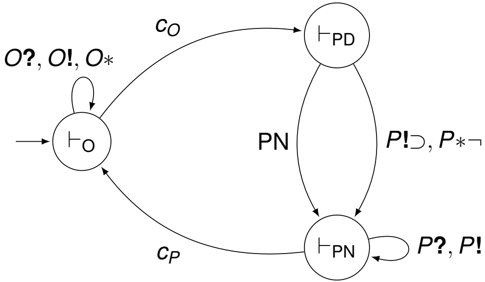
$$\Gamma \vdash_\alpha \Delta$$

  - Γ
    signed formulas stated by O
  - Δ
    signed formulas stated by P
  - α
    phase: $\alpha \in \{\mathsf{O}, \mathsf{PN}, \mathsf{PD}\}$

- signed formula: announcer label + formula
  - $o_p : \varphi$ player/agent o stated φ — p is the **addressee**
  - $\overline{o_p} : \varphi$ assertion is **attacked** (by p)
  - $\widetilde{o_p} : \varphi$ assertion is **blocked** (optimization)

- Round Cycle (Phases)

- **O**
  O performs her moves
  - attacks P-agents' assertions
  - defends against attacks
- **PD**
  P-agents **decide** whether
  - one of them reacts on a **critical** attack
  - or not
- **PN**
  P-agents performs their moves
  - attack O's assertions
  - defends **non-critically** against attacks
- Last party loses.



- Sequent Rules for O

$$\frac{\Gamma \vdash_O \Delta, \overline{p} : A \supset B}{\Gamma \vdash_O \Delta, p : A \supset B}\ O\mathbf{?}{\supset} \qquad \frac{\widetilde{o_p} : A \supset B, \Gamma \vdash_O \Delta, p : A \qquad o_p : B, \Gamma \vdash_O \Delta}{\overline{o_p} : A \supset B, \Gamma \vdash_O \Delta}\ O{*}{\supset}$$

$$\frac{\Gamma \vdash_O \Delta, \overline{p}^L : A \wedge B \qquad \Gamma \vdash_O \Delta, \overline{p}^R : A \wedge B}{\Gamma \vdash_O \Delta, p : A \wedge B}\ O\mathbf{?}{\wedge} \qquad \frac{\Gamma \vdash_O \Delta, \overline{p} : A \vee B}{\Gamma \vdash_O \Delta, p : A \vee B}\ O\mathbf{?}{\vee}$$

$$\frac{o_p : A, \Gamma \vdash_O \Delta}{\overline{o_p}^L : A \wedge B, \Gamma \vdash_O \Delta}\ O\mathbf{!}L \qquad \frac{o_p : A, \Gamma \vdash_O \Delta \qquad o_p : B, \Gamma \vdash_O \Delta}{\overline{o_p} : A \vee B, \Gamma \vdash_O \Delta}\ O\mathbf{!}{\vee}$$

$$\frac{o_p : B, \Gamma \vdash_O \Delta}{\overline{o_p}^R : A \wedge B, \Gamma \vdash_O \Delta}\ O\mathbf{!}R \qquad \frac{\Gamma \vdash_O \Delta, \overline{p} : \neg A}{\Gamma \vdash_O \Delta, p : \neg A}\ O\mathbf{?}{\neg} \qquad \frac{\widetilde{o_p} : \neg A, \Gamma \vdash_O \Delta, p : A}{\overline{o_p} : \neg A, \Gamma \vdash_O \Delta}\ O{*}{\neg}$$

$$\frac{\Gamma \vdash_O \Delta, \overline{p} : A}{\Gamma \vdash_O \Delta, p : A}\ O\mathbf{?}a \qquad \frac{\Gamma \vdash_O \Delta}{\Gamma \vdash_O \Delta, p : \bot}\ O\mathbf{?}{\bot} \qquad \frac{\Gamma \vdash_{PD} \Delta}{\Gamma \vdash_O \Delta}\ c_O$$

only applicable if no other rule application is possible

Sequent Rules for P-agents

p-rules – decide phase

$$\frac{o_q : A,\ \Gamma^{\delta} \vdash_{\mathrm{PN}}\ p : B}{\Gamma \vdash_{\mathrm{PD}}\ \color{red}{\Delta}\color{black}{,\ \overline{p} : A \supset B}}\ \color{red}{P!\supset}$$

$$\frac{o_p : A,\ \Gamma^{\delta} \vdash_{\mathrm{PN}}\ \emptyset}{\Gamma \vdash_{\mathrm{PD}}\ \color{red}{\Delta}\color{black}{,\ \overline{p} : \neg A}}\ \color{red}{P{*}\neg}$$

$$\frac{\Gamma \vdash_{\mathrm{PN}}\ \Delta}{\Gamma \vdash_{\mathrm{PD}}\ \Delta}\ \mathrm{PN}$$

$$\Gamma^{\delta} =_{df} (\Gamma \setminus \{\widetilde{o}_p : f \mid p \in \mathit{Agents}, f \in \mathit{Form}\}) \cup \{o_p : f \mid \widetilde{o}_p : f \in \Gamma, p \in \mathit{Agents}, f \in \mathit{Form}\}$$

q is a new P-agent.

p-rules – normal phase

$$\frac{\overline{o_p} : A \supset B,\ \Gamma\ \vdash_{\mathrm{PN}}\ \Delta}{o_p : A \supset B,\ \Gamma\ \vdash_{\mathrm{PN}}\ \Delta}\ P?\supset$$

$$\frac{\overline{o_p} : \neg A,\ \Gamma\ \vdash_{\mathrm{PN}}\ \Delta}{o_p : \neg A,\ \Gamma\ \vdash_{\mathrm{PN}}\ \Delta}\ P?\neg$$

$$\frac{\Gamma\ \vdash_{\mathrm{PN}}\ \Delta,\ p : A}{\Gamma\ \vdash_{\mathrm{PN}}\ \Delta,\ \overline{p}^{L} : A \wedge B}\ P!L$$

$$\frac{\overline{o_p}^{L} : A \wedge B,\ \overline{o_q}^{R} : A \wedge B,\ \Gamma\ \vdash_{\mathrm{PN}}\ \Delta}{o_p : A \wedge B,\ \Gamma\ \vdash_{\mathrm{PN}}\ \Delta}\ P?\wedge$$

$$\frac{\Gamma\ \vdash_{\mathrm{PN}}\ \Delta,\ p : B}{\Gamma\ \vdash_{\mathrm{PN}}\ \Delta,\ \overline{p}^{R} : A \wedge B}\ P!R$$

$$\frac{\overline{o_p} : A \vee B,\ \Gamma\ \vdash_{\mathrm{PN}}\ \Delta}{o_p : A \vee B,\ \Gamma\ \vdash_{\mathrm{PN}}\ \Delta}\ P?\vee$$

$$\frac{\Gamma\ \vdash_{\mathrm{PN}}\ \Delta,\ p : A,\ q : B}{\Gamma\ \vdash_{\mathrm{PN}}\ \Delta,\ \overline{p} : A \vee B}\ P!\vee$$

$$\frac{}{o_p : A,\ \Gamma\ \vdash_{\mathrm{PN}}\ \Delta,\ \overline{s} : A}\ P!!$$

$$\frac{}{o_p : \bot,\ \Gamma\ \vdash_{\mathrm{PN}}\ \Delta,\ s : A}\ P?\bot$$

$$\frac{\Gamma\ \vdash_{\mathrm{O}}\ \Delta}{\Gamma\ \vdash_{\mathrm{PN}}\ \Delta}\ c_P$$

only applicable if no other rule application is possible

q is a new P-agents in each case.

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{o_{p1} : A \vdash_{\mathsf{PN}} \emptyset}{\emptyset \vdash_{\mathsf{PD}} \overline{p0} : A,\ \overline{p1} : \neg A}\ P{*}\neg}{\emptyset \vdash_{\mathsf{O}} \overline{p0} : A,\ \overline{p1} : \neg A}\ c_O}{\emptyset \vdash_{\mathsf{O}} \overline{p0} : A,\ p1 : \neg A}\ O\mathbf{?}\neg}{\emptyset \vdash_{\mathsf{O}} p0 : A,\ p1 : \neg A}\ O\mathbf{?}a}{\emptyset \vdash_{\mathsf{PN}} p0 : A,\ p1 : \neg A}\ c_P}{\emptyset \vdash_{\mathsf{PN}} \overline{p0} : A \vee \neg A}\ P\mathbf{!}\vee}{\emptyset \vdash_{\mathsf{PD}} \overline{p0} : A \vee \neg A}\ \mathsf{PN}}{\emptyset \vdash_{\mathsf{O}} \overline{p0} : A \vee \neg A}\ c_O}{\emptyset \vdash_{\mathsf{O}} p0 : A \vee \neg A}\ O\mathbf{?}\vee
$$

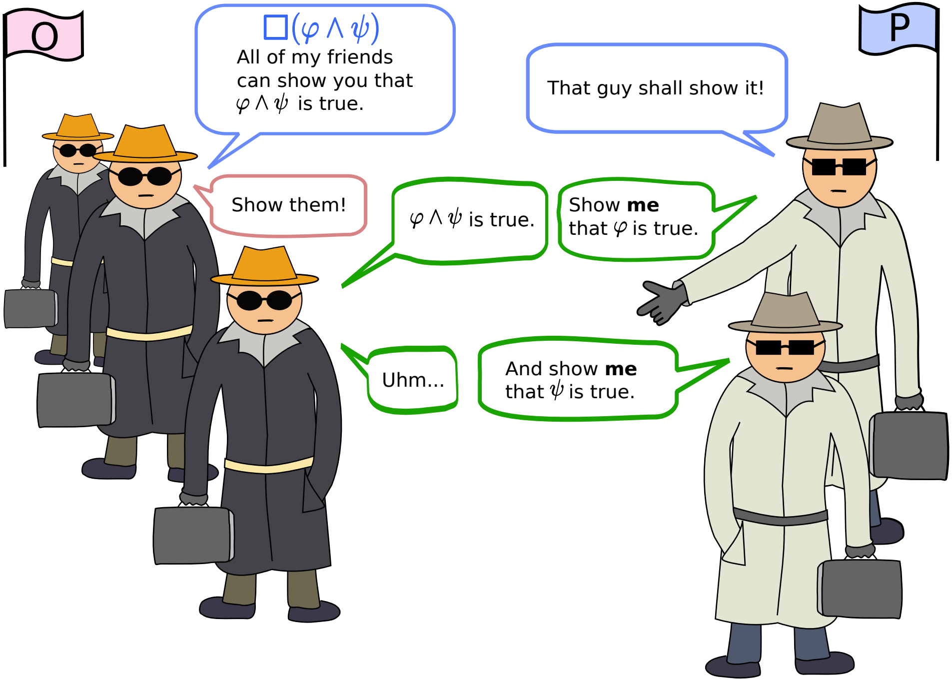
- Sequent system does not implement structural rules **literally**.
  - Rules $O\mathbf{?}\supset$ and $O\mathbf{?}\neg$ do not add concessions directly.
  - "Trigger Rules" $P{*}\neg$ and $O{*}\supset$
- Moves in phases O and PN can be performed in **any order**.
  $\Rightarrow$ always same result wrt. P-winning-strategy
- Decisions
  - by O cause **branching** in sequent tree
  - by Ps happen only in **decide phase**
- Similar to **focus sequents** for single-conclusion calculi (Liang, Miller, 2009; Simmons 2014)

$\Rightarrow$ proof normalization

$\Rightarrow$ strategy analysis/optimization for Ps

Several agents on **O**-side?

- Frank Van Dun (1972)
  $\Rightarrow$ **Modal Logic**
- O-agents correspond to Kripke worlds
  related due to **coalition relation** (usually reflexive)


O
P
$\Box(\varphi \wedge \psi)$
All of my friends
can show you that
$\varphi \wedge \psi$ is true.
That guy shall show it!
Show them!
$\varphi \wedge \psi$ is true.
Show **me**
that φ is true.
Uhm...
And show **me**
that ψ is true.

- Pairwise communication
- O-agent states a **modal expression**
  - $\Box\varphi$ ***All*** *of my coalition partners can show you that* φ *is true.* ***You choose!***
  - $\Diamond\varphi$ *At least* ***one*** *of my coalition partners can show you that* φ *is true.* ***I choose!***
- P-agent states a **modal expression**
  - $\Box\varphi$ *I can show that* φ *is true to* ***all*** *of your coalition partners.* ***You choose!***
  - $\Diamond\varphi$ *I can show that* φ *is true to at least* ***one*** *of your coalition partners.* ***I choose!***
- Different coalition partners are bound to different **commitments**.
  $\Rightarrow$ intuitionistic connectives $\supset$ and $\neg$ implemented with **implicit** $\Box$ in **S4**-frame.
- More under construction...
  S4, S5, IK ...

Open questions / Future Work:

- parallelism in Barth&Krabbe's sequent system?
- dialogical sequents for modal logic (sound/complete)
- optimization due to strategies
- comparison to focus calculi

- Contact
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