Multi-Agent Dialogue Games and Dialogue Sequents
Workshop on Logical Dialogue Games 2015

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Lorenzen-Dialogues as flexible reasoning procedure for \textbf{Intuitionistic} and \textbf{Modal Logic}

- strategies

\textbf{Parallelized} Dialogues

- two \textbf{parties}, more players
- distributed problem-solving
- round-based scheduling & access restrictions
- \textbf{normalization} simplifies proof-search

\textbf{Dialogue Sequents}

- clear rules, no ambiguity
- easier to show soundness/completeness
- also suitable for expressing parallelism
P must decide whether to defend with $\varphi$ or $\psi$.

In classical logic, he can defend more than once.
<table>
<thead>
<tr>
<th></th>
<th>O</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( (A \land B) \supset (A \supset B) )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( ? \supset (1) )</td>
<td>( A \land B \supset (2) ) )</td>
</tr>
<tr>
<td>3</td>
<td>( ? \supset (2) )</td>
<td>( A \supset (3) B )</td>
</tr>
<tr>
<td>4</td>
<td>( B? (3) )</td>
<td>( \triangle ?L (2) )</td>
</tr>
<tr>
<td>5</td>
<td>( ! (4) )</td>
<td>( A \supset ?R (2) )</td>
</tr>
<tr>
<td>6</td>
<td>( ! (5) )</td>
<td>( B \supset ! (4) )</td>
</tr>
</tbody>
</table>

- O may **attack atoms**, P may not.
- P may defend atom-attacks if O stated atom herself (*ipse dixisti!*).
- In **intuitionistic dialogues**, P may only defend against the last open attack.
Dialogue Sequents (Barth and Krabbe, 1982)

$\Pi, [\Delta]/T/_{\mathcal{N}}Z, [\Gamma]$

- $N$
  Party (O or P) whose turn it is

- $T$
  local thesis: last statement attacked by O

- $\Pi$
  concessions stated by O

- $[\Delta]$
  possible defences for O

- $[\Gamma]$
  possible defences for P

- $Z$
  sentence stated by P, that can be attacked by O in next move
<table>
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<tr>
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<tr>
<td>1</td>
<td>H</td>
<td>((A \land B) \supset (A \supset B))</td>
</tr>
<tr>
<td>2</td>
<td>(\mathcal{O} \supset (1))</td>
<td>(A \land B)</td>
</tr>
<tr>
<td>3</td>
<td>(\mathcal{O} \supset (2))</td>
<td>(A)</td>
</tr>
<tr>
<td>4</td>
<td>(B? (3))</td>
<td>(\mathcal{O})</td>
</tr>
<tr>
<td>5</td>
<td>(\mathcal{O} \supset (4))</td>
<td>(A)</td>
</tr>
<tr>
<td>6</td>
<td>(\mathcal{O} \supset (5))</td>
<td>(B)</td>
</tr>
</tbody>
</table>

\[
\begin{array}{ll}
\varnothing & / \quad /_O (A \land B) \supset (A \supset B) \\
A \land B & / \quad (A \land B) \supset (A \supset B) /_P [(A \supset B)] \\
A \land B & / \quad (A \land B) \supset (A \supset B) /_O A \supset B \\
A, A \land B & / \quad A \supset B /_P [B] \\
A, A \land B & / \quad A \supset B /_O B \\
A, A \land B & / \quad B /_P \varnothing \\
A, A \land B, [A] & / \quad B /_O \varnothing \\
A, A \land B, A & / \quad B /_P \varnothing \\
A, A \land B, [B] & / \quad B /_O \varnothing \\
A, A \land B, A, B & / \quad B /_P \varnothing \quad \text{ipse dixisti!}
\end{array}
\]
- When P has a choice, another agent is introduced.
- O argues with them separately.
<table>
<thead>
<tr>
<th></th>
<th>O</th>
<th>P0</th>
<th>P1</th>
<th>P2</th>
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<td>H</td>
<td>(A \land B) \supset (A \supset B)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>?_{P_0} \supset A \land B</td>
<td>!</td>
<td>A \supset B</td>
<td>?L</td>
</tr>
<tr>
<td></td>
<td>\quad \uparrow P_1</td>
<td>A</td>
<td></td>
<td>?R</td>
</tr>
<tr>
<td>3</td>
<td>?_{P_0} \supset A</td>
<td>!</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>\quad \uparrow P_1</td>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>\quad \uparrow P_2</td>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>B \supset P_0</td>
<td>!</td>
<td>\triangle</td>
<td></td>
</tr>
</tbody>
</table>
How to enforce intuitionism?

$$\mathcal{O} \iff \mathcal{P}_i$$

$$\Pi \vdash \Sigma$$

- **Separated Contexts**
  - Hypersequents
    - $$\Pi_1 \vdash C_1 | \Pi_2 \vdash C_2 | \ldots$$
  - Single-conclusion
    - classical: $$\Pi_i, \Pi_k \vdash C_i$$
    - intermediate:$$\Pi_i \vdash C_i | \Pi_k \vdash \bot$$
  - **merging**
    - intuitionistic: $$\mathcal{O}$$
    - sub-classical: $$\mathcal{P}_i$$

- **Shared Context**
  - single sequents
    - classical: $$\Pi \vdash C_1, C_2, \ldots$$
  - Multi-conclusion
    - classical: $$\Pi \vdash C_i, C_k$$
    - sub-classical: $$\mathcal{O}$$
  - **isolation**
    - intuitionistic: $$\Pi \vdash C_i$$


- **Structural Rules** (shared context, classical logic)

  - Start with initial P-Agent P0. States **hypothesis**.
  - In every **round**/row every P-agent may perform ONE move. O reacts on all of the P-agents’ moves of previous round.
  - All players MUST perform moves when possible.
  - Only O may attack atoms. P-agents can defend against atom-attacks only if atom has been stated by O (*ipse dixisti!*).
  - Last moving party wins.
Shared context and intuitionistic logic

Distinguish between critical and non-critical attacks:
- critical: \( ? \supset ? \)  
- non-critical: \( ?L \quad ?R \quad ?V \quad ?\bot \quad a? \)

- corresponds to \( \supset r \) and \( \neg r \) rules of multi-conclusion sequent calculus.
- O’s concessions in a critical attack only committed if the corresp. P-agent defends (⇒ promise).

Structural rules (cont.)

- If several agents are attacked in one round by O then
  - all non-critically attacked agents may react or
  - one critically attacked agent may react.
  Then the other agents are deactivated. (⇒ isolation)
- Critical defences may be delayed.
Multi-Agent Dialogue Sequents

\[ \Gamma \vdash_{\alpha} \Delta \]

- \( \Gamma \)
  - **signed formulas** stated by O
- \( \Delta \)
  - **signed formulas** stated by P
- \( \alpha \)
  - **phase**: \( \alpha \in \{O, PN, PD\} \)

- signed formula: announcer label + formula
  - \( o_p : \varphi \) player/agent \( o \) stated \( \varphi \) — \( p \) is the **addressee**
  - \( \overline{o}_p : \varphi \) assertion is **attacked** (by \( p \))
  - \( \overline{\overline{o}}_p : \varphi \) assertion is **blocked** (optimization)
Round Cycle (Phases)

- **O**
  O performs her moves
  - attacks P-agents’ assertions
  - defends against attacks

- **PD**
  P-agents **decide** whether
  - one of them reacts on a **critical** attack
  - or not

- **PN**
  P-agents performs their moves
  - attack O’s assertions
  - defends **non-critically** against attacks

- Last party loses.
Sequent Rules for $O$

\[
\begin{array}{c}
\Gamma \vdash_0 \Delta, \bar{p} : A \supset B \\
\hline
\Gamma \vdash_0 \Delta, p : A \supset B
\end{array}
\]

\[
\begin{array}{c}
\bar{o}_p : A \supset B, \Gamma \vdash_0 \Delta, p : A \\
\hline
\bar{o}_p : A \supset B, \Gamma \vdash_0 \Delta
\end{array}
\]

\[
\begin{array}{c}
\Gamma \vdash_0 \Delta, \bar{p}^L : A \land B \\
\hline
\Gamma \vdash_0 \Delta, p : A \land B
\end{array}
\]

\[
\begin{array}{c}
\bar{o}_p : A, \Gamma \vdash_0 \Delta \\
\hline
\bar{o}_p^L : A \land B, \Gamma \vdash_0 \Delta
\end{array}
\]

\[
\begin{array}{c}
o_p : A, \Gamma \vdash_0 \Delta \\
\hline
\bar{o}_p^R : A \land B, \Gamma \vdash_0 \Delta
\end{array}
\]

\[
\begin{array}{c}
\Gamma \vdash_0 \Delta, \bar{p} : A \lor B \\
\hline
\Gamma \vdash_0 \Delta, p : A \lor B
\end{array}
\]

\[
\begin{array}{c}
o_p : A, \Gamma \vdash_0 \Delta \\
\hline
\bar{o}_p : A \lor B, \Gamma \vdash_0 \Delta
\end{array}
\]

\[
\begin{array}{c}
o_p : B, \Gamma \vdash_0 \Delta \\
\hline
\bar{o}_p^L : A \land B, \Gamma \vdash_0 \Delta
\end{array}
\]

\[
\begin{array}{c}
\Gamma \vdash_0 \Delta, \bar{p} : \neg A \\
\hline
\Gamma \vdash_0 \Delta, p : \neg A
\end{array}
\]

\[
\begin{array}{c}
\bar{o}_p : \neg A, \Gamma \vdash_0 \Delta \\
\hline
\bar{o}_p : \neg A, \Gamma \vdash_0 \Delta
\end{array}
\]

\[
\begin{array}{c}
\Gamma \vdash_0 \Delta, \bar{p} : A \\
\hline
\Gamma \vdash_0 \Delta, p : \bot
\end{array}
\]

\[
\begin{array}{c}
\bar{o}_p : \neg A, \Gamma \vdash_0 \Delta \\
\hline
\bar{o}_p : \neg A, \Gamma \vdash_0 \Delta
\end{array}
\]

\[
\begin{array}{c}
\Gamma \vdash PD \Delta \\
\hline
\Gamma \vdash_0 \Delta
\end{array}
\]

only applicable if no other rule application is possible.
Sequent Rules for P-agents

**p-rules – decide phase**

\[
\frac{o_q : A, \Gamma \vdash_{PN} p : B}{\Gamma \vdash_{PD} \Delta, \bar{p} : A \supset B} \quad P!\supset
\]

\[
\frac{o_p : A, \Gamma \vdash_{PN} \emptyset}{\Gamma \vdash_{PD} \Delta, \bar{p} : \neg A} \quad P*\neg
\]

\[
\Gamma^\delta =_{df} (\Gamma \setminus \{\tilde{o}_p : f \mid p \in \text{Agents}, f \in \text{Form}\}) \cup \{o_p : f \mid \tilde{o}_p : f \in \Gamma, p \in \text{Agents}, f \in \text{Form}\}
\]

\(q\) is a new P-agent.

**p-rules – normal phase**

\[
\frac{o_p : A \supset B, \Gamma \vdash_{PN} \Delta}{o_p : A \supset B, \Gamma \vdash_{PN} \Delta} \quad P?\supset
\]

\[
\frac{o_p : \neg A, \Gamma \vdash_{PN} \Delta}{o_p : \neg A, \Gamma \vdash_{PN} \Delta} \quad P?\neg
\]

\[
\frac{\bar{o}_p^L : A \land B, \bar{o}_q^R : A \land B, \Gamma \vdash_{PN} \Delta}{o_p : A \land B, \Gamma \vdash_{PN} \Delta} \quad P?\land
\]

\[
\frac{\Gamma \vdash_{PN} \Delta}{\Gamma \vdash_{PN} \Delta, \bar{p} : A \land B} \quad P!L
\]

\[
\frac{\bar{o}_p^R : A \land B, \bar{o}_q^L : A \land B, \Gamma \vdash_{PN} \Delta}{o_p : A \land B, \Gamma \vdash_{PN} \Delta} \quad P?\lor
\]

\[
\frac{\Gamma \vdash_{PN} \Delta, p : A}{\Gamma \vdash_{PN} \Delta, \bar{p} : A \lor B} \quad P!R
\]

\[
\frac{o_p : A \lor B, \Gamma \vdash_{PN} \Delta}{o_p : A \lor B, \Gamma \vdash_{PN} \Delta} \quad P?\lor
\]

\[
\frac{\Gamma \vdash_{PN} \Delta, p : A, q : B}{\Gamma \vdash_{PN} \Delta, \bar{p} : A \lor B} \quad P!V
\]

\[
\frac{o_p : A, \Gamma \vdash_{PN} \Delta, \bar{s} : A}{P!!}
\]

\[
\frac{o_p : \bot, \Gamma \vdash_{PN} \Delta, s : A}{P?\bot}
\]

\[
\frac{\Gamma \vdash_{O} \Delta}{\Gamma \vdash_{PN} \Delta} \quad c_P
\]

only applicable if no other rule application is possible

\(q\) is a new P-agent in each case.
Sequent system does not implement structural rules literally.
  ▶ Rules $O ? \supset$ and $O ? \neg$ do not add concessions directly.
  ▶ "Trigger Rules" $P * \neg$ and $O * \supset$

Moves in phases O and PN can be performed in any order.
  ⇒ always same result wrt. P-winning-strategy

Decisions
  ▶ by O cause branching in sequent tree
  ▶ by Ps happen only in decide phase

Similar to focus sequents for single-conclusion calculi
  (Liang, Miller, 2009; Simmons 2014)
  ⇒ proof normalization
  ⇒ strategy analysis/optimization for Ps
Several agents on O-side?

- Frank Van Dun (1972)  
  ⇒ Modal Logic
- O-agents correspond to Kripke worlds related due to coalition relation (usually reflexive)
All of my friends can show you that $\varphi \land \psi$ is true.

That guy shall show it!

Show them!

$\varphi \land \psi$ is true.

Show me that $\varphi$ is true.

Uhm...

And show me that $\psi$ is true.
Pairwise communication

O-agent states a modal expression

\( \Box \varphi \) All of my coalition partners can show you that \( \varphi \) is true.
You choose!

\( \Diamond \varphi \) At least one of my coalition partners can show you that \( \varphi \) is true.
I choose!

P-agent states a modal expression

\( \Box \varphi \) I can show that \( \varphi \) is true to all of your coalition partners.
You choose!

\( \Diamond \varphi \) I can show that \( \varphi \) is true to at least one of your coalition partners.
I choose!

Different coalition partners are bound to different commitments.

⇒ intuitionistic connectives \( \supset \) and \( \neg \) implemented with implicit \( \Box \) in S4-frame.

More under construction...

S4, S5, IK...
Open questions / Future Work:

- parallelism in Barth & Krabbe’s sequent system?
- dialogical sequents for modal logic (sound/complete)
- optimization due to strategies
- comparison to focus calculi
Contact

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