Evaluation Games for Shapiro's Logic of Vagueness in Context

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Shapiro's Logic

Main principles

- Shapiro's Goal: Model reasoning with *vague* predicates.
- Assertions are stated in course of a conversation.

Main Thesis

The extension and antiextension of a vague predicate uttered in a conversation depend on the *conversational record*.

Judgement dependence

A predicate P is *judgement dependent* if for some objects o a competent speaker of the english language may decide that $P(o)$ or $\neg P(o)$ without losing her competency.

- Vagueness due to judgement dependence.

Shapiro's Logic

The model

$F = \langle W, M \rangle$ a Kripke-like tree structure called *frame*

W a set of partial interpretations called *sharpenings* defining an extension and an antiextension for each predicate

$M \in W$ the base

- Propositions true at the base are externally determined.
- Moving alongside a branch away from the base corresponds to precifying asserted statements — require monotonicity of partial interpretations.
- Similar to supervaluation, but without the completability requirement.

- Main setting for a conversation

Sorites Paradox

Imagine 2000 men lined up where man #1 has full hair and man #2000 has no hair at all. The men are ordered by their amount of hair.
A group of conversationalists is repeatedly asked if they judge man #i as bald, starting with man #1, continuing until man #2000 is reached.

Principle of tolerance

A conversationalist judging man m to be (not) bald cannot judge another man m' in any other way at the same time, if m and m' differ only marginally.

Forcing

A formula ϕ is forced at a sharpening N, if for each sharpening N' of N there is a further sharpening N'' of N' such that ϕ holds at N''.

- Intuitively: There is always a way that ϕ will eventually get true in course of the conversation.

- Corresponds to truth at a partial interpretation
- Statements being forced at the base are called *determinately true*.
- *Weak* forcing: ϕ is weakly forced at N if there is no sharpening N' of N such that ϕ is false at N'.
- Note that (weak) forcing is not represented as a connective.

An example

The father promisses his children “If the weather is good tomorrow, we will go swimming and if the weather is not good tomorrow we will go to the cinema". The children are happy because they believe, they are going to do something.

- The next day the weather is cloudy but it's not raining.
- The father decides not to do anything with his children.

Shapiro’s Logic

Forcing vs. Weak forcing

An example

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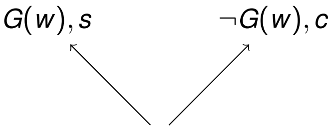

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Shapiro's Logic

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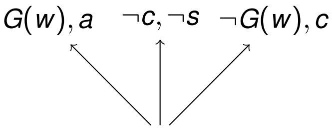

- The next day the weather is cloudy but it's not raining.
- The father decides not to do anything with his children.

$(G(w) \rightarrow s) \wedge (\neg G(w) \rightarrow c)$ is not forced at the base, but *weakly* forced.

Shapiro's Logic

Connectives and quantifiers

- $\wedge, \vee, \neg, \rightarrow, \forall, \exists$ are evaluated locally at a sharpening
- Observation: $\exists x.\phi(x)$ can be forced at a sharpening N without $\phi(x)$ being forced for any particular x.
- Solution: introduce the *global* quantifiers E, A (analogously):

Truth condition for E

A formula $Ex.\phi(x)$ is true at N in F if and only if there exists x such that $\phi(x)$ is forced at N in F.

Falsity condition for E (stable failure)

A formula $Ex.\phi(x)$ is false at N in F if and only if there is no sharpening N' of N in F such that $Ex.\phi(x)$ is true at N in F.

- other connectives: (intuitionistic-style) negation $-$, conditional $\Rightarrow$

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Motivation:

- Shapiro's logic already explicitly refers to conversational situations. Why suddenly leave this context for evaluating the truth a formula?
- To provide an explicit mechanism for the evaluation of formulas. Dialogue rules provide a much more direct characterization of truth, falsity and indefiniteness as Shapiro's definitions.

- The game is a pure evaluation game.
- Given a formula ϕ, a frame F and a sharpening N, decide whether ϕ is forced at N in F.
- Note: ϕ being forced at N in F is identified with *truth* of ϕ at N in F .

A Hintikka-style evaluation game

Overview

- Two players:

  P Proponent of ϕ

  O Opponent, tries to falsify ϕ

- Game rules define which player's turn it is and which options there are based on the outmost connective.
- Possible moves include choosing domain elements and further sharpenings.
- ... until only an atomic formula is left.
- Truth of ϕ at *N* in *F* corresponds to *P* having a winning strategy for the game.

A Hintikka-style evaluation game

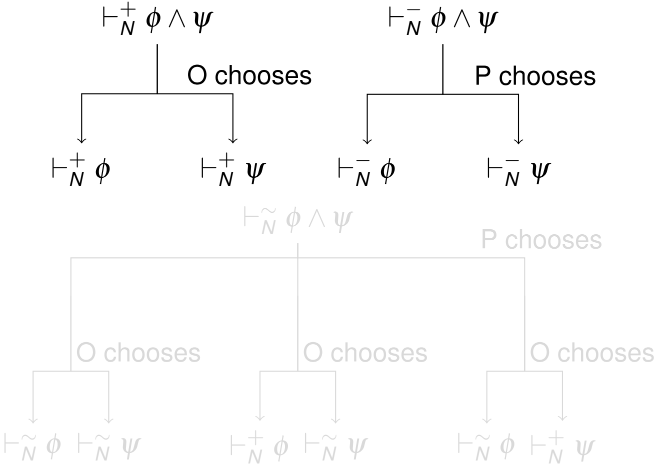
Start of the game

Given ϕ, N and F the game proceeds as follows:

1. O chooses a sharpening N' of N (denoted $N' \succeq N$)
2. P chooses a sharpening N'' of N'
3. P asserts that ϕ is (locally) true at N'' by stating $\vdash_N^+ \phi$
4. The adequate game rules are applied repeatedly
5. ...
6. If an atomic formula $P(a)$ is reached then P is declared the winner if
    1. he is stating $\vdash_N^+ P(a)$ and $P(a)$ is true in N, or
    2. he is stating $\vdash_N^- P(a)$ and $P(a)$ is false in N, or
    3. he is stating $\vdash_N^\sim P(a)$ and $P(a)$ is indefinite in N.

A Hintikka-style evaluation game

Game Rules: Conjunction



- encode the corresponding truth table into game rules

A Hintikka-style evaluation game

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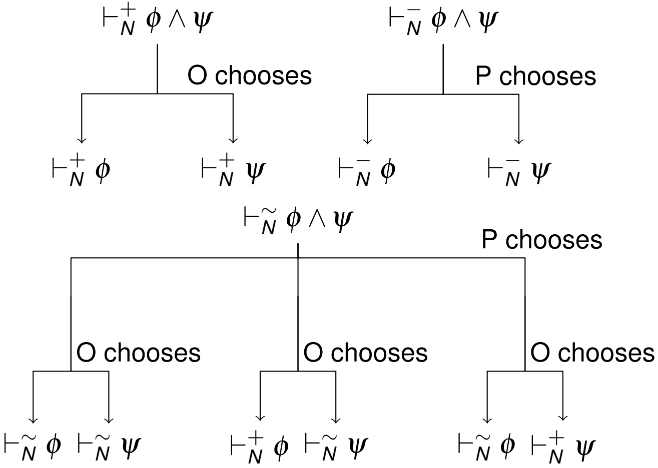

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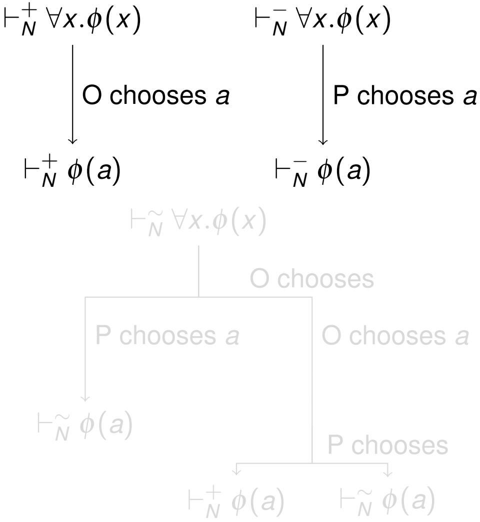
A Hintikka-style evaluation game

Game Rules: $\forall$ – *quantifier*



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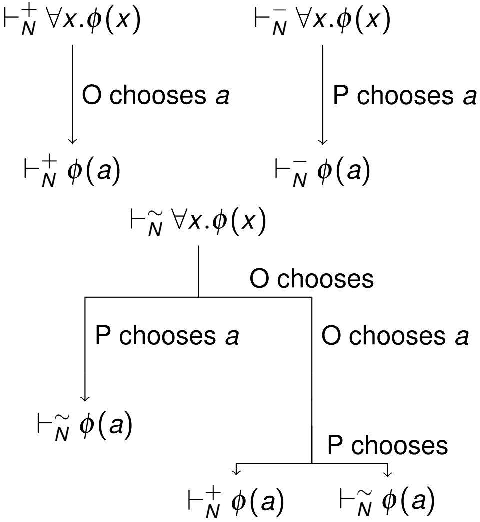


A Hintikka-style evaluation game

Game Rules: *A*-quantifier I

$\vdash^{+}_{N} Ax.\phi(x)$
- O chooses a
- O chooses $N' \succeq N$
- P chooses $N'' \succeq N'$

$\vdash^{+}_{N''} \phi(a)$

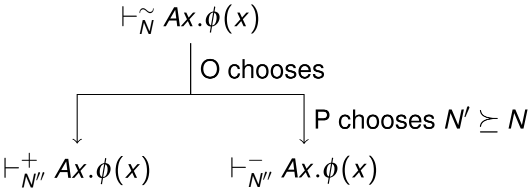
$\vdash^{-}_{N} Ax.\phi(x)$
- O chooses $N' \succeq N$
- P chooses a
- P chooses $N'' \succeq N'$
- P chooses

$\vdash^{-}_{N''} \phi(a)$ — O chooses $N''' \succeq N''$ — $\vdash^{\sim}_{N'''} \phi(a)$

- rule for $\vdash^{-}_{N} Ax.\phi(x)$ is much more explicit than the definition via stable failure

A Hintikka-style evaluation game

Game Rules: *A*-quantifier II



- indefinite case only implicitly defined by Shapiro
- $Ax.\phi(x)$ is indefinite iff there are both further sharpenings where $Ax.\phi(x)$ is true and further sharpenings where $Ax.\phi(x)$ is false.

Giles-style evaluation games

Short overview

- Also possible to define other evaluation games.

Characteristics of Giles style games

- Seperate deconstruction of complex formulas into their atomic subformulas.
- Seperate evaluation of atomic subformulas.
- Both players may assert a (multi)set of formulas.

- Possible to characterise truth, falsity and indefiniteness using such games.
- But: the conditional $\Rightarrow$ is not (easily) expressible in that framework.

That's it

Thanks for your attention!

Any questions?