Evaluation Games for Shapiro's Logic of Vagueness in Context

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In [1] S. Shapiro presents a model for reasoning with vague propositions focusing on the Sorites paradox [7]. He maintains that the extension of such vague predicates as *bald* and *red* also depends on the conversational context, i.e. on statements made by the participants in a given conversational situation. Moreover one should take into account that statements may be, explicitly or implicitly, withdrawn from the so-called *conversational record*.

This contribution introduces a Hintikka-style dialogue game for evaluating formulas according to Shapiro's model of vagueness. This is motivated by the following observations:

- Shapiro's main setting of the Sorites paradox already includes dialogue situations and conversational records. A dialogue game to evaluate composite propositions just is a natural consolidation of this concept.
- It provides an explicit mechanism for the evaluation of formulas. We will see that in particular Shapiro's falsehood conditions for some connectives and quantifiers are rather indirect. The dialogue rules provide a much more direct characterization of truth and validity, respectively.

Shapiro's model uses a Kripke-like tree structure, called *frame*, combined with a three-valued logic where propositions can be either true, false or indefiniteto represent different states of a conversation. The logical connectives $\neg$, &, $\vee$, and $\rightarrow$ adhere to the standard Kleene truth tables. At the root node of a frame only the externally determined propositions are fixed; moving alongside a branch away from the root corresponds to precifying asserted statements. Hence the nodes are also called *sharpenings* or *partial interpretations*. During a conversation in a Sorites situation withdrawing statements amounts to jumping to another branch in the frame.

Additionally to the standard logical connectives and quantifiers, which are evaluated only locally, Shapiro introduces new non-local ones operating on whole subframes instead. For example, a new non-local implication $\Phi \Rightarrow \Psi$ is true at a sharpening N if at each sharpening of N it holds that if Φ is true, then also Ψ is true.

At each stage of the corresponding game exactly one formula is asserted by the verifier to be true, false or indefinite at a given sharpening. The dialogue rules then define how this formula is to be further reduced and which player has to make which choices based on the outmost connective or quantifier. For the local connectives this amounts to formulating game rules for the Kleene truth tables, whereas for connectives operating on subframes the rules entail choices of further sharpenings.

For characterising validity Shapiro introduces the notion of *forcing*: A formula Φ is forced at a sharpening N (under a certain variable assignment) if for each sharpening N_1 of N there is a sharpening N_2 of N_1 such that Φ is true at N_2. Validity or, more generally, consequence is then defined in terms of forcing: $\Gamma_1, \Gamma_2, \ldots \models \Phi$ if Φ is forced at every sharpening in every frame in which all of $\Gamma_1, \Gamma_2, \ldots$ are forced.

The game itself is restricted to checking the truth of a formula at a given sharpening (under a fixed variable assignment). However, we show how to reduce the problem of checking the forcing conditions to this case by an additional dialogue rule.

We also present an evaluation game in the spirit of dialogue games defined by Lorenzen [6] and, more specifically, R. Giles [5] for Łukasiewicz logic. This game can be used as well to characterise truth and falsehood of a formula in a frame. First the formula is stepwise decomposed into its atomic subformulas, which are evaluated in a second step. We observe that truth of a formula Φ coincides with the existence of a winning strategy for the player asserting Φ in the beginning, while falsehood coincides with the existence of a winning strategy for the other player. Note that for indefinite formulas neither player has a winning strategy. Unfortunately, this game does not cover the whole logic as defined by Shapiro. We will see that the new (non-local) implication $\Rightarrow$ is not expressible in this framework. However, the fragment of this logic without $\Rightarrow$, i.e., based on the connectives $\vee, \&, \rightarrow, \neg, -$(non-local negation) and the quantifiers $\forall, \exists$, A, and E (local and non-local quantifiers, respectively), is fully covered.

References

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