Proof theory for deontic logic inspired by Indian Philosophy

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What's the problem?

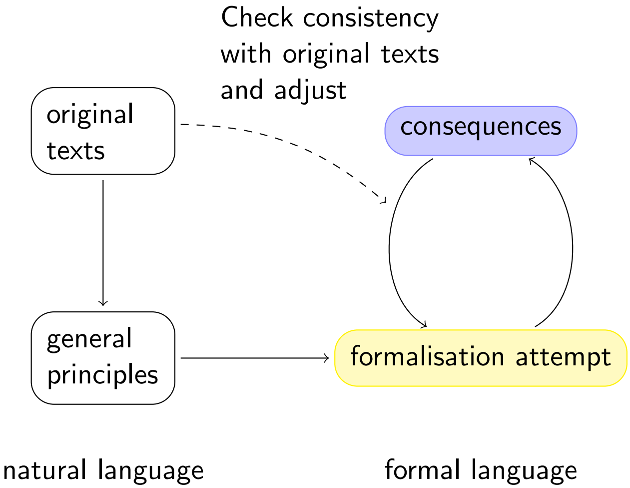
Main idea:

Use logic to formally analyse ancient texts of Indian Philosophy

Expected benefits:

- Indology: A better understanding of the texts and clarification through formalisation
- Logic: New inputs and development of new methods

How do we formalise?



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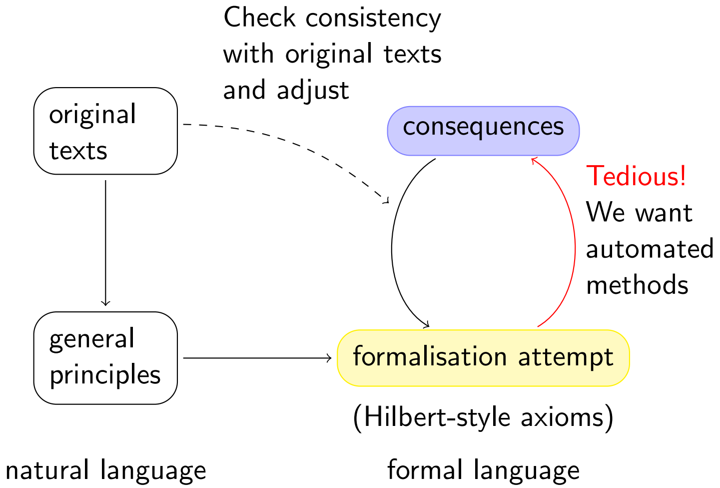

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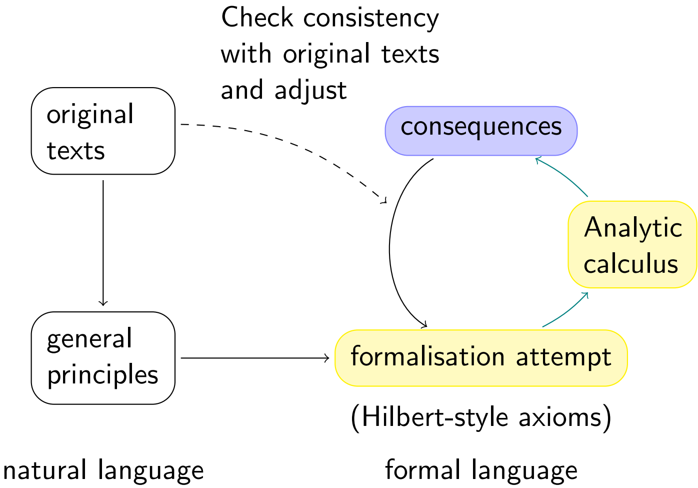

So we use Proof Theory to do the dirty work!

What kind of Indian Philosophy?

We consider texts of the Mīmāṃsā school:

- Main period of activity
  last centuries BCE to beginning of 20th century

- Main focus
  interpretation of the prescriptive portions of the Vedas

- Main tool
  formulation of general rules and interpretative principles (nyāyas) to interpret the texts

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  interpretation of the prescriptive portions of the Vedas
  $\rightsquigarrow$ Deontic Logic
- Main tool
  formulation of general rules and interpretative principles (nyāyas) to interpret the texts
  $\rightsquigarrow$ Axioms

But why this kind of Indian Philosophy – Why Mīmāṃsā?

- Suitability: The clear formulation of Mīmāṃsā interpretative principles lends itself to formalisation
- Historical significance: Mīmāṃsā is one of the main schools of Indian Philosophy and considered early deontic logic
- Importance: The Mīmāṃsā principles are used in Indian court cases even today
- Novelty: Mīmāṃsā texts have scarcely been considered from the modern Western point of view due to:
    - Lack of translations
    - Often highly metaphorical language

Part 1: The logic – formalising the key concepts

Preliminaries: The language

How to model propositional reasoning?

> *When there is a contradiction, at the denial of one alternative, the other is known (to be true).*

(Interpretation of Jayanta's Nyāyamañjarī, 9^{th} c. CE)

$\rightsquigarrow$ classical propositional logic

How to model the deontic concepts?

Mīmāṃsā authors use eligibility conditions for prescriptions, e.g.

> *The one who desires heaven should sacrifice with the Full- and New-moon Sacrifices*

$\rightsquigarrow$ dyadic deontic operators

Note: We base our logic on propositions instead of actions.

Preliminaries: The language, formally

The formulae of deontic logic are given by:

$$p \in \mathsf{Var} \mid \bot \mid \neg A \mid A \wedge B \mid A \vee B \mid A \to B \mid \mathcal{O}(A/B) \mid \mathcal{F}(A/B)$$

They are interpreted in the standard way:

- $\mathcal{O}(A/B) \rightsquigarrow$ "Given that B is the case, it is obligatory that A is the case"
- $\mathcal{F}(A/B) \rightsquigarrow$ "Given that B is the case, it is forbidden that A is the case"

As base calculus we assume the (Hilbert-)axioms and rules of classical propositional logic and the congruence rules

$$\frac{\vdash A \leftrightarrow C \quad \vdash B \leftrightarrow D}{\vdash \mathcal{O}(A/B) \leftrightarrow \mathcal{O}(C/D)} \qquad \frac{\vdash A \leftrightarrow C \quad \vdash B \leftrightarrow D}{\vdash \mathcal{F}(A/B) \leftrightarrow \mathcal{F}(C/D)}$$

The logic: Axioms

> *When, on the other hand, coming into being [of something needed] [...] are not realized by another prescription, [the principal prescription] itself begets the four [stages] of coming into being [...] [of the prescriptions] connected to itself.*

(Rāmānujācārya, *Tantrahasya* IV.4.3.3)

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If a prescription enjoins something which has requirements, then it enjoins the requirements as well.

$$\frac{\vdash A \rightarrow B}{\vdash \mathcal{O}(A/C) \rightarrow \mathcal{O}(B/C)}$$

The logic: More axioms

Given that purposes Y and Z exclude each other, if one should use X for the purpose Y, then it cannot be the case that one should use it at the same time for the purpose Z.

(Interpretation of Kumārila, *Tantravārttika* on PMS 1.3.3)

$$\rightsquigarrow \qquad \frac{\vdash \neg(A \wedge B)}{\vdash \neg(\mathcal{O}(A/C) \wedge \mathcal{O}(B/C))}$$

If conditions X and Y are always equivalent, given the duty to perform Z under conditions X, the same duty applies under Y.

(Interpretation of Śabara, PMS 6.1.25)

$$\rightsquigarrow \qquad \frac{\vdash B \leftrightarrow C}{\vdash \mathcal{O}(A/B) \rightarrow \mathcal{O}(A/C)}$$

Making the logic useful: Sequents

These rules are equivalent to dyadic deontic logic MD:

$(M)\ \ \mathcal{O}(A \wedge B/C) \rightarrow \mathcal{O}(A/C)$

$(D)\ \ \neg(\mathcal{O}(A/B) \wedge \mathcal{O}(\neg A/B))$

$$\frac{\vdash A \leftrightarrow C \quad \vdash B \leftrightarrow D}{\vdash \mathcal{O}(A/B) \leftrightarrow \mathcal{O}(C/D)}\ Cg$$

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(Turning the handle to get analytic calculus)

$$\frac{A \Rightarrow C \qquad B \Rightarrow D \qquad D \Rightarrow B}{\Gamma, \mathcal{O}(A/B) \Rightarrow \mathcal{O}(C/D), \Delta}\ \mathsf{Mon}_1$$

$$\frac{A, C \Rightarrow \qquad B \Rightarrow D \qquad D \Rightarrow B}{\Gamma, \mathcal{O}(A/B), \mathcal{O}(C/D) \Rightarrow \Delta}\ \mathsf{D}$$

$$\frac{A \Rightarrow}{\Gamma, \mathcal{O}(A/B) \Rightarrow \Delta}\ \mathsf{P}$$

A sequent calculus suitable for automatic proof search!

(A sequent $A_1, \ldots, A_n \Rightarrow B_1, \ldots, B_m$ reads as $\bigwedge_{i=1}^{n} A_i \to \bigvee_{j=1}^{m} B_j$)

...so let's try to use the calculus!

Suppose that

- Śūdras must not study the Vedas: $\mathcal{O}(\neg\texttt{std\_vds}/\texttt{sdr})$
- Performing the Agnihotra sacrifice demands studying the Vedas: $\texttt{agnhtr} \to \texttt{std\_vds}$

Question 1: May Śūdras perform Agnihotra?

$$\dfrac{\dfrac{\texttt{agnhtr} \overset{\checkmark}{\Rightarrow} \texttt{std\_vds}}{\neg\texttt{std\_vds} \Rightarrow \neg\texttt{agnhtr}} \quad \dfrac{\checkmark}{\texttt{sdr} \Rightarrow \texttt{sdr}} \quad \dfrac{\checkmark}{\texttt{sdr} \Rightarrow \texttt{sdr}}}{\mathcal{O}(\neg\texttt{std\_vds}/\texttt{sdr}) \Rightarrow \mathcal{O}(\neg\texttt{agnhtr}/\texttt{sdr})}\ \text{Mon}_1$$

Answer: No, they must not!

. . . so let's try to use the calculus!

Suppose that

- Śūdras must not study the Vedas: $\mathcal{O}(\neg\texttt{std\_vds}/\texttt{sdr})$
- Performing the Agnihotra sacrifice demands studying the Vedas: $\texttt{agnhtr} \rightarrow \texttt{std\_vds}$
- Chariot makers are Śūdras: $\texttt{chmk} \rightarrow \texttt{sdr}$

Question 2: What about chariot makers?

$$\dfrac{\dfrac{\checkmark}{\neg\texttt{std\_vds} \Rightarrow \neg\texttt{std\_vds}} \quad \overset{X}{\texttt{sdr} \Rightarrow \texttt{chmk}} \quad \overset{\checkmark}{\texttt{chmk} \Rightarrow \texttt{sdr}}}{\mathcal{O}(\neg\texttt{std\_vds}/\texttt{sdr}) \Rightarrow \mathcal{O}(\neg\texttt{std\_vds}/\texttt{chmk})}\ \mathsf{Mon}_1$$

Answer: We can't derive anything!
(Because the logic is too weak in the second argument.)

Part 2: Reasoning on conditions

How to reason on the conditions?

We cannot introduce full downwards monotony for conditions: $\mathcal{O}(\neg\texttt{std\_vds}/\texttt{sdr})$ and $\mathcal{O}(\texttt{agnhtr}/\texttt{chmk})$ would give

$$\mathcal{O}(\neg\texttt{std\_vds}/\texttt{chmk}) \wedge \mathcal{O}(\texttt{std\_vds}/\texttt{chmk})$$

which is inconsistent with axiom (D).

So we distinguish (prima-facie) assumptions from derived statements and use:

> *Guṇapradhāna / specificity principle*
> *More specific rules override more general ones.*

Discussed by Jaimini (2^{nd} c. BCE)

To make this precise we split the assumptions into:

- propositional assumptions (facts): $\bigwedge_{i \leq n} p_i \rightarrow \bigvee_{j \leq m} p_j$
- deontic assumptions: $\mathcal{O}_{\text{pf}}(A/B)$ with A, B propositional.

Guṇapradhāna / Specificity intuitively

Idea: Use downwards monotonicity in the second argument...

$\texttt{agnhtr} \rightarrow \texttt{std\_vds}$ $\quad$ $\texttt{chmk} \rightarrow \texttt{sdr}$

$\mathcal{O}_{\mathsf{pf}}(\neg\texttt{std\_vds}/\texttt{sdr})$

$\mid\!\sim \mathcal{O}(\neg\texttt{std\_vds}/\texttt{chmk} \wedge \neg\texttt{upnyn})$

Guṇapradhāna / Specificity intuitively

Idea: Use downwards monotonicity in the second argument if the obligation is not overruled by a more specific one…

$\texttt{agnhtr} \rightarrow \texttt{std\_vds}$ $\qquad$ $\texttt{chmk} \rightarrow \texttt{sdr}$

$\mathcal{O}_{\mathsf{pf}}(\neg\texttt{std\_vds}/\texttt{sdr})$

$\lightning$

$\mathcal{O}_{\mathsf{pf}}(\texttt{agnhtr}/\texttt{chmk})$

$\not\mid\!\sim\ \mathcal{O}(\neg\texttt{std\_vds}/\texttt{chmk} \wedge \neg\texttt{upnyn})$

$\mid\!\sim\ \mathcal{O}(\ \texttt{std\_vds}/\texttt{chmk} \wedge \neg\texttt{upnyn})$

Guṇapradhāna / Specificity intuitively

Idea: Use downwards monotonicity in the second argument
if the obligation is not overruled by a more specific one
and if there is no other conflicting one which is not overruled itself.

$$\mathtt{agnhtr} \rightarrow \mathtt{std\_vds} \qquad \mathtt{chmk} \rightarrow \mathtt{sdr}$$

$$\mathcal{O}_{\mathsf{pf}}(\neg \mathtt{std\_vds} / \mathtt{sdr})$$

$$\lightning$$

$$\mathcal{O}_{\mathsf{pf}}(\neg \mathtt{std\_vds} / \neg \mathtt{upnyn})$$

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How to put this into sequents?

We derive an obligation $\mathcal{O}(A/B)$ from deontic assumptions $\mathfrak{L}$ if

- it is entailed by an applicable $\mathcal{O}_{\mathsf{pf}}(C/D) \in \mathfrak{L}$...

$$\{B \Rightarrow D\} \quad \cup \quad \{C \Rightarrow A\}$$

$$\frac{}{\Rightarrow \mathcal{O}(A/B)}\ \mathcal{O}_R^{\mathcal{O}_{\mathsf{pf}}}$$

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- there is no other applicable conflicting obligation which is not overruled itself by a more specific one

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Wait … underivability premisses?

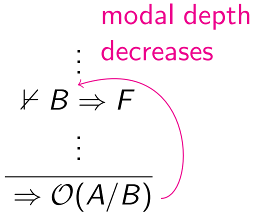
The underivability premisses look fishy and smell like circular definitions…

Fortunately:

Theorem.
Derivability for formulae of modal depth $n+1$ depends only on derivability of formulae of modal depth at most n.

So everything is well-defined, we escape a fixpoint definition and even get

Theorem.
Derivability from assumptions is decidable in polynomial space.



(with $\mathcal{O}_{\mathsf{pf}}(E/F) \in \mathfrak{L}$, hence no $\mathcal{O}_{\mathsf{pf}}$ in F)

Bonus: Saving the twins (well . . . at least one)

Vikalpa / Disjunctive response:
When there is a real conflict between obligations, any of the conflicting injunctions may be adopted as option.

Discussed by Jaimini (2^{nd} c. BCE)

This principle actually follows from the methods and lets us reason about the drowning twins:

$$\texttt{save\_twin1} \wedge \texttt{save\_twin2} \rightarrow \bot$$

$$\mathcal{O}_{\mathsf{pf}}(\texttt{save\_twin1}/\texttt{reach1}) \qquad \mathcal{O}_{\mathsf{pf}}(\texttt{save\_twin2}/\texttt{reach2})$$

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$$\not\vdash \mathcal{O}(\texttt{save\_twin2}\,/\,\texttt{reach1})$$

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Application: Comparing formalisations

The procedure also helps in comparing and adjusting formalisations of deontic assumptions using:

Theorem.
If $\mathcal{O}_{\mathsf{pf}}(A/B) \in \mathfrak{L}$, and $\mathcal{O}(A/B)$ is not derivable, then there is an explicit conflict: a $\mathcal{O}_{\mathsf{pf}}(C/D) \in \mathfrak{L}$ with $\vdash \neg(A \wedge C)$ and $\vdash B \leftrightarrow D$.

Explicit conflicts are bad:

A deontic assumption involved in an explicit conflict
is never applied as written!

This suggests for a list of deontic assumptions:

- count explicit conflicts in a conflict score
- if possible, generate alternative formalisations
- evaluate the formalisations by minimising the conflict score.

Application: Comparing interpretations

Implementation

Interpretation and formalisation

- $\mathcal{O}_{\text{pf}}(A/B)$
- $\mathcal{F}_{\text{pf}}(A/B)$

$\vdots$

$\mathcal{O}(A \vee \neg A/B)$

$\mathcal{F}(A \wedge \neg A/B)$

- Do A in B
- Don't do A in B

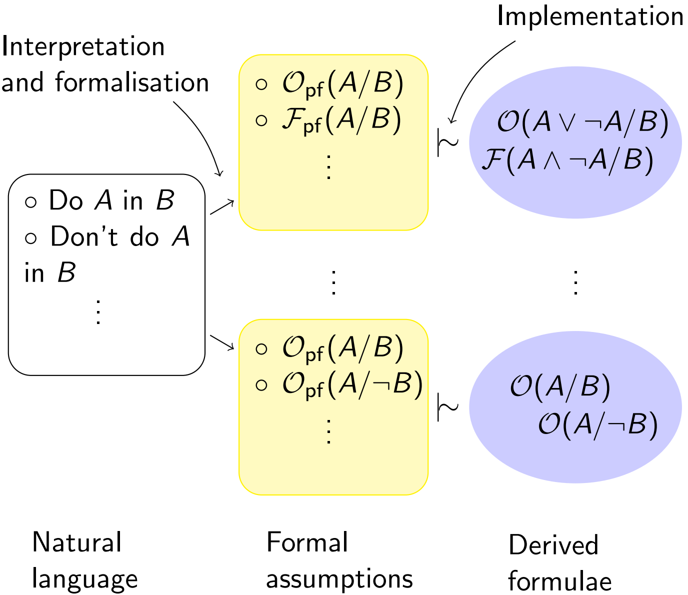
$\vdots$

Natural language

Formal assumptions

Derived formulae

Application: Comparing interpretations



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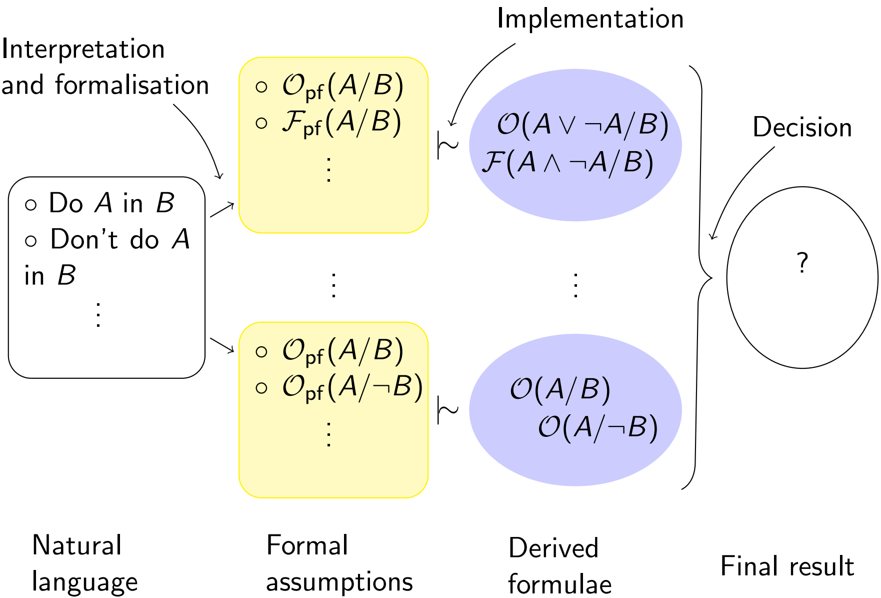

What does this look like in practice?

Let the implementation work that out!

http://subsell.logic.at/bprover/deonticProver/version1.2/

Summing up

In this line of work we have

- investigated proof-theoretic methods for converting axiom systems into analytic (sequent) calculi
- Obtained a known deontic logic from texts of the Mīmāṃsā school of Indian Philosophy
- Introduced a proof-theoretic mechanism for reasoning with conflicting obligations using specificity
- Implemented a tool which also helps to evaluate the formalisation of deontic assumptions

Thank you!

`https://mimamsa.logic.at`