Axioms vs Hypersequent Rules with Context Restrictions

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Motivation

Fact 1

There are many logics.

Fact 2

The number of logics is growing (almost) everyday.

Problem

Given the specification of a logic, construct an analytic calculus to be used in a decision procedure for it!

In the spirit of a "smart reuse of resources" we would like to have general methods to approach this problem.

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Problem

Given the specification of a logic, construct an analytic calculus to be used in a decision procedure for it!

- Assume the logic is given as a Hilbert-style axiom system.
- Which framework to choose for the calculus: sequents, hypersequents, nested sequents, display, . . . ?
- How to construct the calculus?

We need a general theory of derivation systems including results about which frameworks are appropriate for which logics!

Motivation

Some first results:

- Kracht (display calculi, modal logics);
- Avron, Lahav, Zamansky (sequent calculi, paraconsistent logics);
- Ciabattoni, Galatos, Terui (sequent / hypersequent calculi, substructural logics);
- Ciabattoni, Ramanayake (display calculi, (modal) logics);
- Schröder, Pattinson, L. (sequents, modal logics);
- Lahav (hypersequents, modal logics);

Here we concentrate on hypersequents and propositional normal modal logics (works for non-normal and intuitionistic as well).

Hypersequent calculi

Hypersequent Basics

The formulae of normal modal logics are given by

$$\mathcal{F} \ni A ::= p_i \mid \bot \mid A \wedge A \mid A \vee A \mid A \rightarrow A \mid \Box A \mid \heartsuit A \mid \dots$$

Sequents are tuples $\Gamma \Rightarrow \Delta$ of multisets of formulae read as $\bigwedge \Gamma \rightarrow \bigvee \Delta$, and hypersequents are multisets of sequents written

$$\Gamma_1 \Rightarrow \Delta_1 \mid \cdots \mid \Gamma_n \Rightarrow \Delta_n$$

We consider hypersequent calculi with axioms and structural rules:

$$\frac{}{\mathcal{G} \mid \Gamma, A \Rightarrow A, \Delta}\ \mathrm{Ax} \qquad \frac{\mathcal{G} \mid \Gamma \Rightarrow \Delta}{\mathcal{G} \mid \Gamma, \Sigma \Rightarrow \Delta, \Pi}\ \mathrm{IW} \qquad \frac{\mathcal{G}}{\mathcal{G} \mid \Gamma \Rightarrow \Delta}\ \mathrm{EW}$$

$$\frac{\mathcal{G} \mid A, A, \Gamma \Rightarrow \Delta}{\mathcal{G} \mid A, \Gamma \Rightarrow \Delta}\ \mathrm{IC_L} \qquad \frac{\mathcal{G} \mid \Gamma \Rightarrow \Delta, A, A}{\mathcal{G} \mid \Gamma \Rightarrow \Delta, A}\ \mathrm{IC_R}$$

$$\frac{\mathcal{G} \mid \Gamma \Rightarrow \Delta \mid \Gamma \Rightarrow \Delta}{\mathcal{G} \mid \Gamma \Rightarrow \Delta}\ \mathrm{EC} \qquad \frac{\mathcal{G} \mid \Gamma \Rightarrow \Delta, A \quad \mathcal{G} \mid A, \Sigma \Rightarrow \Pi}{\mathcal{G} \mid \Gamma, \Sigma \Rightarrow \Delta, \Pi}\ \mathrm{Cut}$$

Hypersequent rules with restrictions examples

What could the additional rules look like?
Two (classic) examples from the literature [Avron 1996]:

$$\frac{\mathcal{G} \mid \Gamma, A \Rightarrow \Delta}{\mathcal{G} \mid \Gamma, \Box A \Rightarrow \Delta} \qquad \frac{\mathcal{G} \mid \Box\Gamma, \Sigma \Rightarrow \Box\Delta, \Pi}{\mathcal{G} \mid \Box\Gamma \Rightarrow \Box\Delta \mid \Sigma \Rightarrow \Pi}$$

The characteristic features of these rules are:

- They might introduce one layer of connectives in the active part of the conclusion
- One active component per premiss
- Possibly more than one active component in the conclusion
- They copy a restricted part of the contexts of each component to the premisses

How can we make that precise?

Hypersequent rules with restrictions formally

A context restriction is a tuple $\langle F_\ell; F_r\rangle$ of sets of formulae. It restricts a sequent $\Gamma \Rightarrow \Delta$ by allowing only substitution instances of formulae from F_ℓ (resp. F_r) in Γ (resp. Δ).

Hypersequent rules with context restrictions are of the form

$$\frac{(\Gamma_1 \Rightarrow \Delta_1; \mathcal{C}^1_1 \dots \mathcal{C}^1_n) \quad \dots \quad (\Gamma_m \Rightarrow \Delta_m; \mathcal{C}^m_1 \dots \mathcal{C}^m_n)}{\Sigma_1 \Rightarrow \Pi_1 \mid \cdots \mid \Sigma_n \Rightarrow \Pi_n}$$

with $\mathcal{C}^i_j$ context restrictions and $\Gamma_i, \Delta_i \subseteq Var$ and $\Sigma_i, \Pi_i \subseteq \Box(Var)$. Simple rules use only $\langle\emptyset, \emptyset\rangle, \langle\{p\}, \{p\}\rangle, \langle\{\Box p\}, \emptyset\rangle$.

In an application the premiss with restriction $\mathcal{C}^i_1 \dots \mathcal{C}^i_n$ copies the context of the jth component restricted by $\mathcal{C}^i_j$.

Example:

$$\frac{(\ \Rightarrow\ ; \langle\{\Box p\}; \{\Box p\}\rangle\ \langle\{p\}; \{p\}\rangle)}{\Rightarrow \mid \Rightarrow} \quad \rightsquigarrow \quad \frac{\mathcal{G} \mid \Box\Gamma, \Sigma \Rightarrow \Box\Delta, \Pi}{\mathcal{G} \mid \Box\Gamma \Rightarrow \Box\Delta \mid \Sigma \Rightarrow \Pi}$$

Cut elimination and Decidability

Theorem

Every

set of rules with

restrictions has cut elimination.

Proof idea: Adapt the proof of [Ciabattoni, Metcalfe, Montagna 2010]: push cuts up to the left, then right.

$$\dfrac{\dfrac{\dfrac{. \Rightarrow .,A \mid \Gamma \Rightarrow \Delta}{. \Rightarrow .,A \mid . \Rightarrow .,A}\,R}{. \Rightarrow .,A}\,\mathrm{EC} \qquad \dfrac{\dfrac{\Sigma \Rightarrow \Pi}{A,. \Rightarrow . \mid A,. \Rightarrow .}\,Q}{A,. \Rightarrow .}\,\mathrm{EC}}{.,. \Rightarrow .,.}\,\mathrm{Cut}$$

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$$
\dfrac{
  \dfrac{\cdot \Rightarrow \cdot, A \mid \Gamma \Rightarrow \Delta}{\cdot \Rightarrow \cdot, A \mid \cdot \Rightarrow \cdot, A}\; R
  \qquad
  \dfrac{\dfrac{\Sigma \Rightarrow \Pi}{A, \cdot \Rightarrow \cdot \mid A, \cdot \Rightarrow \cdot}\; Q}{A, \cdot \Rightarrow \cdot}\; \mathrm{EC}
}{
  \dfrac{\cdot, \cdot \Rightarrow \cdot, \cdot \mid \cdot, \cdot \Rightarrow \cdot, \cdot}{\cdot, \cdot \Rightarrow \cdot, \cdot}\; \mathrm{EC}
}\; IH
$$

Cut elimination and Decidability

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Every right-substitutive,

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$$
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}{\cdot, \cdot \Rightarrow \cdot, \cdot \mid \cdot \Rightarrow \cdot, A}\ R
\qquad
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\dfrac{\Gamma \Rightarrow \Pi}{A, \cdot \Rightarrow \cdot \mid A, \cdot \Rightarrow \cdot}\ Q
}{A, \cdot \Rightarrow \cdot}\ \mathrm{EC}
}{
\dfrac{\cdot, \cdot \Rightarrow \cdot, \cdot \mid \cdot, \cdot \Rightarrow \cdot, \cdot}{\cdot, \cdot \Rightarrow \cdot, \cdot}\ \mathrm{EC}
}\ \mathrm{Cut}
$$

Cut elimination and Decidability

Theorem

Every right-substitutive, single-conclusion right, right-contraction closed, set of rules with restrictions has cut elimination.

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$$
\cfrac{
  \cfrac{\cdot,\cdot \Rightarrow \cdot,\cdot \mid \Gamma \Rightarrow \Delta}{\cdot,\cdot \Rightarrow \cdot,\cdot \mid \cdot \Rightarrow \cdot, A}\,R
  \qquad
  \cfrac{
    \cfrac{\cdot,\cdot \Rightarrow \cdot,\cdot \mid \Gamma \Rightarrow \Delta}{\cdot,\cdot \Rightarrow \cdot,\cdot \mid \cdot \Rightarrow \cdot, A}\,R
    \quad
    \cfrac{\Sigma \Rightarrow \Pi}{A,\cdot \Rightarrow \cdot \mid A,\cdot \Rightarrow \cdot}\,Q
  }{A,\cdot \Rightarrow \cdot \mid \cdot,\cdot \Rightarrow \cdot,\cdot \mid \cdot,\cdot \Rightarrow \cdot,\cdot}\,\text{Cut}
}{
  \cfrac{\cdot,\cdot \Rightarrow \cdot,\cdot \mid \cdot,\cdot \Rightarrow \cdot,\cdot \mid \cdot,\cdot \Rightarrow \cdot,\cdot \mid \cdot,\cdot \Rightarrow \cdot,\cdot}{\cdot,\cdot \Rightarrow \cdot,\cdot}\,\text{EC}
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$$

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Theorem

Every right-substitutive, single-conclusion right, right-contraction closed, principal cut closed, mixed-cut permuting set of rules with restrictions has cut elimination.

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$$\dfrac{\dfrac{\dfrac{.,. \Rightarrow .,. \mid \Gamma \Rightarrow \Delta}{.,. \Rightarrow .,. \mid . \Rightarrow .,A}\, R \qquad \dfrac{\dfrac{.,. \Rightarrow .,. \mid \Gamma \Rightarrow \Delta \quad \Sigma \Rightarrow \Pi}{.,. \Rightarrow .,. \mid \Gamma', \Sigma' \Rightarrow \Delta', \Pi'}\,\text{Cut}}{A,. \Rightarrow . \mid .,. \Rightarrow .,. \mid .,. \Rightarrow .,.}\,\text{cut}(R,Q)}{.,. \Rightarrow .,. \mid .,. \Rightarrow .,. \mid .,. \Rightarrow .,. \mid .,. \Rightarrow .,.}\,\text{Cut}}{.,. \Rightarrow .,.}\,\text{EC}$$

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Theorem

Derivability in a cut-free, contraction-closed, bounded conclusion and tractable rule set is decidable in 2EXPTIME.

Proof idea: Modify the rule set to make internal contractions admissible and enumerate the relevant hypersequents.

Axioms and Rules

Axioms and Interpretations

We assume that the specification of a logic is given as a Hilbert system, i.e. by a set $\mathcal{A}$ of axioms and the rules

$$\frac{\vdash A}{\vdash A\sigma}\ \mathsf{US} \qquad \frac{\vdash A \quad \vdash A \to B}{\vdash B}\ \mathsf{MP} \qquad \frac{\vdash A \leftrightarrow B}{\vdash \heartsuit A \leftrightarrow \heartsuit B}\ \mathsf{Cg}$$

We want to interpret a hypersequent as a formula – but the interpretation for $|$ is not clear! So let's make it a parameter:

An interpretation for a logic $\mathcal{L}$ is a set $\{\varphi_n(p_1, \ldots, p_n) : n \in \mathbb{N}\}$ of formulae such that $\models_{\mathcal{L}} \psi$ iff $\models_{\mathcal{L}} \varphi_1(\psi)$ (regularity) and which respects the structural rules:

- $\models_{\mathcal{L}} \varphi_n(\xi_1, \xi_2, \vec{\chi})$ iff $\models_{\mathcal{L}} \varphi_n(\xi_2, \xi_1, \vec{\chi})$
- If $\models_{\mathcal{L}} \varphi_n(\vec{\chi})$ then $\models_{\mathcal{L}} \varphi_{n+1}(\xi, \vec{\chi})$
- similarly for external contraction, cut, etc.

Example: $\iota_{\Box} = \{\bigvee_{i \leq n} \Box p_i : n \in \mathbb{N}\}$ for reflexive normal modal logics or $\iota_{\boxplus} = \{\bigvee_{i \leq n}(p_i \land \Box p_i) : n \in \mathbb{N}\}$ for normal modal logics.

Axioms vs Rules

Consider the axiom for S4.3:

$$(.3)\ \ \Box(\Box p \to q) \vee \Box(\Box q \to p)$$

The $\iota_\Box$-simple axioms corresponding to simple hypersequent rules for $\iota_\Box = \{\bigvee_{i \leq n} \Box p_i : n \in \mathbb{N}\}$ are given by the following grammar:

$$S ::= \varphi_n(L \to R, \dots, L \to R)$$
$$L ::= L \wedge L \mid \heartsuit P_r \mid \psi_\ell \mid \top \mid \bot \qquad R ::= R \vee R \mid \heartsuit P_\ell \mid \psi_r \mid \top \mid \bot$$
$$P_r ::= P_r \vee P_r \mid P_r \wedge P_r \mid P_\ell \to P_r \mid \psi_r \mid p_i \mid \bot \mid \top$$
$$P_\ell ::= P_\ell \vee P_\ell \mid P_\ell \wedge P_\ell \mid P_r \to P_\ell \mid \psi_\ell \mid p_i \mid \bot \mid \top$$

with $\heartsuit \in \Lambda \cup \{\epsilon\}$

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\end{aligned}$$

with $\heartsuit \in \Lambda \cup \{\epsilon\}$ and $\psi_\ell \in \{q_i, \Box q_i : i \in \mathbb{N}\}$, $\psi_r \in \{r_i : i \in \mathbb{N}\}$ such that every ψ_ℓ, ψ_r occurs under φ_n once on the top level and at least once under a modality.

Applications

Applications: Normal modal logics

Example

Modal logic S5 is given by the axioms for S4 and the simple axiom

$$(5)\;\; \Diamond p \to \Box\Diamond p \quad\equiv\quad \Box p \vee \Box\neg\Box p \quad\rightsquigarrow\quad \frac{\mathcal{G} \mid \Gamma, \Box\Sigma \Rightarrow \Pi}{\mathcal{G} \mid \Gamma \Rightarrow \Delta \mid \Box\Sigma \Rightarrow}$$

Example

Modal logic S4.2 of confluent frames is given by the axioms for S4 plus

$$(.2)\;\; \Diamond\Box p \to \Box\Diamond p \quad\equiv\quad \Box\neg\Box p \vee \Box\neg\Box\neg p \quad\rightsquigarrow\quad \frac{\mathcal{G} \mid \Gamma, \Sigma \Rightarrow}{\mathcal{G} \mid \Box\Gamma \Rightarrow \mid \Box\Sigma}$$

(but see the next talk for more on this one!).

Applications: Simple Frame Properties

All the calculi for logics given by simple frame properties based on K or K4 in [Lahav 2013] fit our framework and satisfy the criteria for cut elimination and decidability. Thus we purely syntactically reprove cut elimination and have

Theorem

Logics given by simple frame properties are decidable in 2EXPTIME.

The correspondence between simple rules and $\iota_\Box$-simple axioms also gives (under some conditions) Hilbert systems for these logics.

Applications: Deontic logics

The logic of uniform deontic frames LUDF from [Roy et al, 2012] has modalities $\Box$ ("necessary"), $\mathcal{O}$ ("obligatory") and $\mathcal{P}$ ("permissible") and axioms S5 for $\Box$ plus

$$\begin{array}{lll} \mathcal{P}A \land \mathcal{P}B \to \mathcal{P}(A \lor B) & \mathcal{O}A \to \mathcal{P}A & \mathcal{O}A \to \Box\mathcal{O}A \\ \mathcal{O}A \to (\mathcal{P}B \to \Box(B \to A)) & \mathcal{O}A \to \neg\Box\neg A & \mathcal{P}A \to \Box\mathcal{P}A \end{array}$$

These are turned into the rules $\mathcal{R}_{\mathsf{LUDF}}$ including e.g.

$$\frac{(\vec{p}, \vec{r} \Rightarrow\ ; \mathcal{C}, \mathcal{C}_{\mathsf{id}}) \qquad (\vec{r} \Rightarrow \vec{q}; \mathcal{C}, \mathcal{C}_{\mathsf{id}})}{\vec{\mathcal{O}}p, \vec{\mathcal{P}}q, \vec{\Box}r \Rightarrow\ \mid\ \Rightarrow}$$

where $\mathcal{C} = \langle \{\Box p, \mathcal{O}p, \mathcal{P}p\}, \emptyset \rangle$ and $|\vec{p}| \geq 1, |\vec{q}|, |\vec{r}| \geq 0$ (see paper for the rest).

Theorem

$\mathcal{R}_{\mathsf{LUDF}}$ *has cut elimination and* LUDF *is decidable in* 2EXPTIME.

Summing Up

We have

- identified a general format of rules in a hypersequent calculus for modal logics
- general syntactic criteria for uniform cut elimination and decidability / complexity results
- identified a class of Hilbert axioms corresponding to such rules
- applied these results in the construction of analytic calculi for several logics.

Summing Up

We have

- identified a general format of rules in a hypersequent calculus for modal logics
- general syntactic criteria for uniform cut elimination and decidability / complexity results
- identified a class of Hilbert axioms corresponding to such rules
- applied these results in the construction of analytic calculi for several logics.

Thanks for your attention!