Linearization of certain non-trivial equations in oligomorphic clones

Libor Barto, Michael Kompatscher*, Mirek Olšák, Trung Van Pham, Michael Pinsker

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* Theory and Logic group
  TU Wien
CSPs and non-trivial equations
Constraint satisfaction problems

Let $\mathfrak{A} = (A, R_1, \ldots, R_n)$ be a relational structure.

CSP($\mathfrak{A}$)

INPUT: A primitive positive sentence

$$\phi = \exists x_1 \ldots, x_n R_{i_1}(\ldots) \land \cdots \land R_{i_j}(\ldots)$$

QUESTION: $\mathfrak{A} \models \phi$?
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Conjecture (Feder, Vardi ’98; Bulatov, Jeavons, Krokhin ’02)

Let $\mathbf{A}$ be finite and $\text{Pol}(\mathbf{A})$ be idempotent. Then either

1. There is a clone homomorphism $\xi : \text{Pol}(\mathbf{A}) \to 1$
   (and CSP($\mathbf{A}$) is NP-complete)
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→ in 2: study of non-trivial equations.
Let $C$ be a finite idempotent clone. Then TFAE:

1. $C$ has no clone homomorphism to $\mathbf{1}$
2. $C$ has a Taylor operation
3. $C$ has a weak near unanimity operation
   \[ w(y, x, \ldots, x) = w(x, y, x, \ldots, x) = \ldots = w(x, x, \ldots, y) \]
4. $C$ has a Siggers operation
   \[ s(x, y, x, z, y, z) = s(y, x, z, x, z, y) \]
5. $C$ has a cyclic operation
   \[ c(x_1, x_2, \ldots, x_n) = c(x_2, \ldots, x_n, x_1) \]
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2-5 are examples of linear non-trivial equations: no nesting
Why linear equations?

In contrast to general equations, linear equations are preserved under all the standard CSP reductions:
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Let $\mathbb{A}$ be finite and $\mathbb{B}$ be homomorphic equivalence to some pp-power of $\mathbb{A}$. Then there is an h1 clone homomorphism $\text{Pol}(\mathbb{A}) \to \text{Pol}(\mathbb{B})$, i.e. a mapping preserving linear equations.
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**Conjecture**

Let $\mathbb{A}$ be finite. Then either

1. There is an $h_1$ clone homomorphism $\xi : \text{Pol}(\mathbb{A}) \to 1$ (and $\text{CSP}(\mathbb{A})$ is NP-complete)
2. or $\text{Pol}(\mathbb{A})$ satisfies a non-trivial linear equation and $\text{CSP}(\mathbb{A})$ is in $\text{P}$.
Oligomorphomic clones
The dichotomy conjecture for infinite CSPs

Old conjecture (Bodirsky, Pinsker)
Let $\mathbb{A}$ be a reduct of a finitely bounded homogeneous structure and $\mathbb{A}^c$ its model-complete core. Then either

1. There is a uniformly continuous clone homomorphism $\xi : \text{Pol}(\mathbb{A}^c, a_1, \ldots, a_n) \to 1$ (and CSP($\mathbb{A}$) is NP-complete)
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In those cases $\text{Aut}(\mathbb{A})$ is **oligomorphic**: The action $\text{Aut}(\mathbb{A}) \acts A^n$ has finitely many orbits for every $n$. 
Non-trivial equations in oligomorphic clones

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**Theorem (Barto, Pinsker ’16)**

$\mathcal{C}$... oligomorphic clone and model-complete core. Then either

- Some stabilizer $(\mathcal{C}, a_1, \ldots, a_n) \rightarrow 1$ uniformly continuous or
- $\mathcal{C}$ contains a pseudo-Siggers operation $s$:

$$e_1 \circ s(x, y, x, z, y, z) = e_2 \circ s(y, x, z, x, z, y), \quad e_1, e_2 \in \mathcal{C}.$$
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**Potential approach**

Is $e_1 \circ s(x, y, x, z, y, z) = e_2 \circ s(y, x, z, x, z, y)$ equivalent to a set of linear non-trivial equations?
Linearization with 🎨
Example: the clone of injective functions

For oligomorphic clones: non-trivial equations $\not\rightarrow$ Taylor operations
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For oligomorphic clones: non-trivial equations $\not\Rightarrow$ Taylor operations

**Example**

Let $O^{\text{inj}}$ be the clone generated by all injective operations $\mathbb{N}^n \to \mathbb{N}$. 
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Let $O^{\text{inj}}$ be the clone generated by all injective operations $\mathbb{N}^n \to \mathbb{N}$.

Let $f(x, y) : \mathbb{N}^2 \to \mathbb{N}$ be a bijection, $f \in O^{\text{inj}}$. Then $e : f(x, y) \to f(y, x)$ is a bijection, $e \in O^{\text{inj}}$.

$O^{\text{inj}}$ satisfies the non-trivial equation $f(y, x) = e \circ f(x, y)$. 
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For oligomorphic clones: non-trivial equations $\nRightarrow$ Taylor operations

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But, by injectivity $O^{\text{inj}}$ contains no Taylor operation.
$\Rightarrow$ we need more than one operation!
Lemma
Let $k > 2$ and $g_1(x, y), \ldots, g_{2k-1}(x, y) \in \mathcal{C}$. Assume that for every tuple $I = (i_1 < \cdots < i_k)$, there is an $f_I(x_1, \ldots, x_k) \in \mathcal{C}$, such that $\forall n:$

$$f_I(x, \ldots, x, y, x, \ldots, x) = g_{i_n}(x, y).$$

Then this set of linear equations is non-trivial.
Lemma
Let $k > 2$ and $g_1(x, y), \ldots, g_{2k-1}(x, y) \in C$. Assume that for every tuple $I = (i_1 < \cdots < i_k)$, there is an $f_I(x_1, \ldots, x_k) \in C$, such that $\forall n$:

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Proof
Assume there is a clone homomorphism $\xi : C \to 1$. For the binary functions $g_i(x, y)$, there are only two possible images $\pi_1^2(x, y)$ and $\pi_2^2(x, y)$.
Lemma

Let \( k > 2 \) and \( g_1(x, y), \ldots, g_{2k-1}(x, y) \in C \). Assume that for every tuple \( l = (i_1 < \cdots < i_k) \), there is an \( f_l(x_1, \ldots, x_k) \in C \), such that \( \forall n : \)

\[
f_l(x, \ldots, x, y^\uparrow_n, x, \ldots, x) = g_{i_n}(x, y).
\]

Then this set of linear equations is non-trivial.

Proof

Assume there is a clone homomorphism \( \xi : C \to 1 \). For the binary functions \( g_i(x, y) \), there are only two possible images \( \pi_1^2(x, y) \) and \( \pi_2^2(x, y) \).

By there is an \( l \), with \( \xi(g_i(x, y)) = \text{const.} \).
Lemma

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Proof

Assume there is a clone homomorphism $\xi : C \to 1$. For the binary functions $g_i(x, y)$, there are only two possible images $\pi_1^2(x, y)$ and $\pi_2^2(x, y)$.

By there is an $I$, with $\xi(g_{i_j}(x, y)) = \text{const.}$

But then $\xi(f_I(x_1, \ldots, x_k))$ cannot be a projection!
Examples of CSP classifications

Successful CSP classifications for reducts of finitely bounded homogeneous structures:

- $(\mathbb{N}, =)$ (Equality CSPs; Bodirsky, Kára '06)
- $(\mathbb{Q}, <)$ (Temporal CSPs; Bodirsky, Kára '08)
- the random graph (Graph-SAT problems; Bodirsky, Pinsker '11)
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Theorem (BKOPP ’16)
If $\mathbb{A}$ is a reduct of one of the above then either

- $\text{Pol}(\mathbb{A}^c, a_1, \ldots, a_n) \rightarrow 1$ and $\text{CSP}(\mathbb{A})$ is NP-complete
- or $\text{Pol}(\mathbb{A})$ satisfies a set of non-trivial linear equations $\Rightarrow$ and $\text{CSP}(\mathbb{A})$ is in P
Theorem (pseudo-nu operations)
Let $\mathbb{D}$ be a finitely bounded homogeneous structure, and let $f$ be a strong polymorphism of $\mathbb{D}$ with

$$e(x) = e_1 \circ f(y, x \ldots, x) = e_2 \circ f(x, y, x \ldots, x) = \ldots = e_n \circ f(x, \ldots, x, y).$$

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Theorem (totally symmetric operations)
Let $\mathcal{A}$ be a reduct of a finitely bounded homogeneous structure $\mathcal{D}$, $k$ big enough and let $f(x_1, \ldots, x_k) \in \text{Pol}(\mathcal{A})$ be totally symmetric modulo outer embeddings of $\mathcal{D}$: $\forall \rho \in \text{Sym}(k)$:

$$e_{1,\rho} f(x_1, \ldots, x_k) = e_{2,\rho} f(x_{\rho(1)}, \ldots, x_{\rho(k)})$$

Then Pol($\mathcal{A}$) contains a set of non-trivial linear equations.
More linearization

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Then $\text{Pol}(\mathbb{A})$ contains a set of non-trivial linear equations.

Note: assumptions on the structural side!
The two conjectures are equivalent.
The bad news

The bad news (BKOPP ’16)

For $\mathbb{B}$, the countable atomless Boolean algebra (extended by $\neq$):

- $\text{Pol}(\mathbb{B})$ satisfies the equation $e_1 \circ f(x, y) = e_2 \circ f(y, x)$ and
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Here $\text{Pol}(\mathbb{B})$ is oligomorphic, but $\mathbb{B}$ is not reduct of a finitely bounded homogeneous structure:
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Here $\text{Pol}(\mathbb{B})$ is oligomorphic, but $\mathbb{B}$ is not reduct of a finitely bounded homogeneous structure:

$\text{Aut}(\mathbb{B})$ has double exponential orbit growth.
The good news (BKOPP ’16)

Let $\mathcal{A}$ be such that $\text{Pol}(\mathcal{A})$ is oligomorphic, mc core and

- $\text{Pol}(\mathcal{A})$ has a pseudo-Siggers operation and
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Then $\text{Aut}(\mathcal{A})$ has at least double exponential orbit growth.
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The orbit growth of reducts of finitely bounded homogeneous structures has orbit growth $\leq 2^{p(n)}$. 
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**Corollary:** The two conjectures are equivalent!
1. Under which structural assumptions can we linearize pseudo-Siggers operations?
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2. Understand better the relation between equations in $\text{Pol}(A)$ and orbit growth of $\text{Aut}(A)$. 
Questions

1. Under which structural assumptions can we linearize pseudo-Siggers operations?
2. Understand better the relation between equations in $\text{Pol}(\mathbb{A})$ and orbit growth of $\text{Aut}(\mathbb{A})$.
3. When does $\xi : \text{Pol}(\mathbb{A}) \to 1$ h1-clone homomorphism imply that there is also a uniformly continuous $\xi' : \text{Pol}(\mathbb{A}) \to 1$?
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*Equations in oligomorphic clones and the Constraint Satisfaction Problem for $\omega$-categorical structures*

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Thank you!