An extended framework for specifying and reasoning about proof systems

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1. Introduction
2. SELLF
3. Encoding
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Motivation

There are several logics: classical logic, intuitionistic logic (and fragments), modal logics, paraconsistent logics...
Developed for the most varied applications: theorem provers, knowledge representation, proof carrying code...

These logics need proof systems for reasoning.

These proof systems should have nice properties, such as:

- cut-elimination
- admissibility of non-atomic axioms
- invertibility of rules

But proving *each* property for *each* system by hand can be very time-consuming and error-prone...

Our approach

Provide a framework that can prove these properties in a uniform and automatic way to various proof systems.

Sequent Calculus Proof System	$\Longrightarrow$	**Logical Framework**	Are cuts admissible? Are non-atomic axioms admissible? Which are the invertible rules?

Logical Framework $\equiv$ Linear Logic with Subexponentials

Linear Logic

Resource-aware logic:

- Classical formulas: "marked" with the exponential operators (! and ?)
- Linear formulas: are consumed when used

Refinement of classical logic:

	Additive	Multiplicative
Conjunction ($\wedge$)	$\&$	$\otimes$
Disjunction ($\vee$)	$\oplus$	$\invamp$

$$\frac{\vdash \Theta : \Gamma, P \quad \vdash \Theta : \Gamma, Q}{\vdash \Theta : \Gamma, P \& Q}\ [\&] \qquad \frac{\vdash \Theta : \Gamma, P \quad \vdash \Theta : \Delta, Q}{\vdash \Theta : \Gamma, \Delta, P \otimes Q}\ [\otimes]$$

Subexponentials [Danos, et al 1993, Nigam and Dale, 2009]

Operators can be **canonical**:

$$\boxed{A \mathbin{\&}^a B \equiv A \mathbin{\&}^b B}$$

Exponentials are not canonical (all others are):

$$\boxed{!^a F \not\equiv\, !^b F \text{ and } ?^a F \not\equiv\, ?^b F}$$

$!^a$ and $!^b$ are different operators in linear logic, they are called *sub*exponentials.

$$\vdash \Theta_a : \Theta_b : \Gamma \equiv \vdash \mathcal{K} : \Gamma$$

Subexponentials [Danos, et al 1993, Nigam and Dale, 2009]

One may declare as many subexponentials as needed, organized in a pre-order.

i allows contraction and weakening	$\Rightarrow$	Θ_i is a set
i does not allow contraction and weakening	$\Rightarrow$	Θ_i is a multi-set

Note: The logics specified may have contexts that behave as set or multi-set. Interesting... :)

Subexponentials [Danos, et al 1993, Nigam and Dale, 2009]

$$\frac{\vdash \mathcal{K} \leq_l : \cdot \Uparrow A}{\vdash \mathcal{K} : \cdot \Downarrow !^l A}\ [!^l, \text{ s.t. } \mathcal{K}[\{x \mid l \not\preceq x \wedge x \notin \mathcal{U}\}] = \emptyset] \qquad \frac{\vdash \mathcal{K} +_l A : \Gamma \Uparrow L}{\vdash \mathcal{K} : \Gamma \Uparrow L, ?^l A}\ [?^l]$$

Rule $?^l$: stores a formula in a context.
Rule $!^l$: very useful for the restrictions on the context.

- smaller or not related "linear" subexponentials must be empty
- smaller or not related "classical" subexponentials are made empty

Focusing

Focused proofs are the normal form of proofs for proof search

- Sound and complete proof search strategy for linear logic
- Based on the division of linear logic's connectives:
  - **Asynchronous** (negative): $\parr$, $\&$, $?^i$, $\top$, $\bot$, $\forall$
  - **Synchronous** (positive): $\otimes$, $\oplus$, $!^i$, 1, 0, $\exists$

Asynchronous $\Rightarrow$ invertible rules $\Rightarrow$ *apply eagerly*

Synchronous $\Rightarrow$ non-invertible rules $\Rightarrow$ *apply when no negative formula is left*

Focusing

Focused proofs are composed by the alternation of negative and positive phases.

Each *phase* is a collection of rules of the same polarity that can compose one or more macro-rule:

$$\frac{\vdash \mathcal{K} : \Gamma \Downarrow A_i}{\vdash \mathcal{K} : \Gamma \Downarrow A_1 \oplus A_2} \oplus_i \qquad \frac{\vdash \mathcal{K} : \Gamma \Downarrow A_1 \quad \vdash \mathcal{K} : \Delta \Downarrow A_2}{\vdash \mathcal{K} : \Gamma, \Delta \Downarrow A_1 \otimes A_2} \otimes \qquad \frac{\vdash \mathcal{K} : \Gamma \Uparrow N}{\vdash \mathcal{K} : \Gamma \Downarrow N} R \Downarrow$$

$N_1 \oplus (N_2 \otimes N_3) \rightsquigarrow$

$$\frac{\vdash \mathcal{K} : \Gamma \Uparrow N_1}{\vdash \mathcal{K} : \Gamma \Downarrow N_1 \oplus (N_2 \otimes N_3)} \quad \text{or} \quad \frac{\vdash \mathcal{K} : \Gamma \Uparrow N_2 \quad \vdash \mathcal{K} : \Delta \Uparrow N_3}{\vdash \mathcal{K} : \Gamma, \Delta \Downarrow N_1 \oplus (N_2 \otimes N_3)}$$

Encoding Sequent Calculus Systems in LL

Types:

o	linear-logic formulas
$\mathtt{form}$	object-logic formulas
$\mathtt{term}$	object-logic terms

Propositions:

$\lceil\cdot\rceil$	$\mathtt{form} \rightarrow o$
$\lfloor\cdot\rfloor$	$\mathtt{form} \rightarrow o$

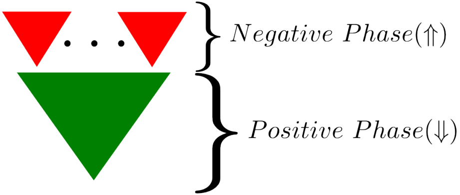
$$\underbrace{B_1, \ldots, B_n \vdash C_1, \ldots, C_m}_{\text{Object-logic}} \quad \rightsquigarrow \quad \underbrace{\vdash \lfloor B_1 \rfloor, \ldots, \lfloor B_n \rfloor, \lceil C_1 \rceil, \ldots, \lceil C_m \rceil}_{\text{Meta-logic (SELLF)}}$$

Bipoles

Monopole: atoms and negative connectives.

Bipole: negated atoms, monopoles and positive connectives.

A bipole derivation contains a single alternation of phases:



Adequacy of enconding

Bipole-derivation $\equiv$ **object-logic rule**

$$\frac{\Gamma, A \longrightarrow B}{\Gamma \longrightarrow A \supset B, \Delta}\ \supset_r \qquad (\supset_r):\ \lceil A \supset B\rceil^{\perp} \otimes !^{l}(?^{l}\lfloor A\rfloor \mathbin{⅋} ?^{r}\lceil B\rceil)$$

$$\dfrac{\dfrac{}{\vdash \Theta \overset{\vdots}{\infty} \lfloor\Gamma\rfloor \overset{\vdots}{l} \lceil\Delta, A \supset B\rceil \overset{\vdots}{r} \cdot \Downarrow \lceil A \supset B\rceil^{\perp}}\ I \qquad \dfrac{\dfrac{\vdash \Theta \overset{\vdots}{\infty} \lfloor\Gamma, A\rfloor \overset{\vdots}{l} \lceil B\rceil \overset{\vdots}{r} \cdot \Uparrow}{\vdash \Theta \overset{\vdots}{\infty} \lfloor\Gamma\rfloor \overset{\vdots}{l} \cdot \overset{\vdots}{r} \cdot \Uparrow ?^{l}\lfloor A\rfloor \mathbin{⅋} ?^{r}\lceil B\rceil}\ ⅋, ?^{r}, ?^{l}}{\vdash \Theta \overset{\vdots}{\infty} \lfloor\Gamma\rfloor \overset{\vdots}{l} \lceil\Delta, A \supset B\rceil \overset{\vdots}{r} \cdot \Downarrow !^{l}(?^{l}\lfloor A\rfloor \mathbin{⅋} ?^{r}\lceil B\rceil)}\ !^{l}}{\dfrac{\vdash \Theta \overset{\vdots}{\infty} \lfloor\Gamma\rfloor \overset{\vdots}{l} \lceil\Delta, A \supset B\rceil \overset{\vdots}{r} \cdot \Downarrow \lceil A \supset B\rceil^{\perp} \otimes !^{l}(?^{l}\lfloor A\rfloor \mathbin{⅋} ?^{r}\lceil B\rceil)}{\vdash \Theta \overset{\vdots}{\infty} \lfloor\Gamma\rfloor \overset{\vdots}{l} \lceil\Delta, A \supset B\rceil \overset{\vdots}{r} \cdot \Uparrow}\ D_{\infty}, 2 \times \exists}\ \otimes$$

Adequacy *on the level of derivations*

Another example
System G3K

$$\frac{y : A, x : \Box A, xRy, \Gamma \Rightarrow \Delta}{x : \Box A, xRy, \Gamma \Rightarrow \Delta}\ \Box_l \qquad (\Box_l)\ \lfloor x : \Box A \rfloor^{\perp} \otimes \exists y.(!^R R(x,y)^{\perp} \otimes ?^l \lfloor y : A \rfloor)$$

$$\dfrac{\Xi \quad \dfrac{\dfrac{\vdash \cdot \overset{\cdot}{\infty} \mathcal{R} \overset{\cdot}{R} \cdot \overset{\cdot}{l} \cdot \overset{\cdot}{r} \cdot \Uparrow R(a,b)^{\perp}}{\vdash \mathcal{L}_{G3K} \overset{\cdot}{\infty} \mathcal{R} \overset{\cdot}{R} \lfloor a : \Box A, \Gamma \rfloor \overset{\cdot}{l} \lceil \Delta \rceil \overset{\cdot}{r} \cdot \Downarrow !^R R(a,b)^{\perp}}\ !^R \quad \dfrac{\vdash \mathcal{L}_{G3K} \overset{\cdot}{\infty} \mathcal{R} \overset{\cdot}{R} \lfloor a : \Box A \rfloor, \lfloor b : A, \Gamma \rfloor \overset{\cdot}{l} \lceil \Delta \rceil \overset{\cdot}{r} \cdot \Uparrow \cdot}{\vdash \mathcal{L}_{G3K} \overset{\cdot}{\infty} \mathcal{R} \overset{\cdot}{R} \lfloor a : \Box A, \Gamma \rfloor \overset{\cdot}{l} \lceil \Delta \rceil \overset{\cdot}{r} \cdot \Downarrow ?^l \lfloor b : A \rfloor}\ R \Downarrow, ?^l}{\vdash \mathcal{L}_{G3K} \overset{\cdot}{\infty} \mathcal{R} \overset{\cdot}{R} \lfloor a : \Box A, \Gamma \rfloor \overset{\cdot}{l} \lceil \Delta \rceil \overset{\cdot}{r} \cdot \Downarrow (!^R R(a,b)^{\perp} \otimes ?^l \lfloor b : A \rfloor)}\ \otimes}{\dfrac{\vdash \mathcal{L}_{G3K} \overset{\cdot}{\infty} \mathcal{R} \overset{\cdot}{R} \lfloor a : \Box A, \Gamma \rfloor \overset{\cdot}{l} \lceil \Delta \rceil \overset{\cdot}{r} \cdot \Downarrow \lfloor a : \Box A \rfloor^{\perp} \otimes \exists y.(!^R R(a,y)^{\perp} \otimes ?^l \lfloor y : A \rfloor)}{\vdash \mathcal{L}_{G3K} \overset{\cdot}{\infty} \mathcal{R} \overset{\cdot}{R} \lfloor a : \Box A, \Gamma \rfloor \overset{\cdot}{l} \lceil \Delta \rceil \overset{\cdot}{r} \cdot \Uparrow \cdot}\ D_{\infty}, \exists}\ \otimes, \exists$$

Where Ξ is a derivation containing only the initial rule.

This system + a subset of the labels' relations captures different modal logics, such as T, 4, B, S4, TB, S5.

Proof Systems Theories

1. Identity rules (cut and initial)

$$\text{Cut} = \exists A.!^a?^b\lfloor A\rfloor \otimes !^c?^d\lceil A\rceil$$

$$\text{Init} = \exists A.\lfloor A\rfloor^\perp \otimes \lceil A\rceil^\perp$$

2. Structural rules

$$\exists A.[\lfloor A\rfloor^\perp \otimes (?^i\lfloor A\rfloor \parr \cdots \parr ?^i\lfloor A\rfloor)]$$

$$\exists A.[\lceil A\rceil^\perp \otimes (?^j\lceil A\rceil \parr \cdots \parr ?^j\lceil A\rceil)]$$

3. Introduction rules

$$\exists x_1 \ldots \exists x_n[(\lfloor \diamond(x_1, \ldots, x_n)\rfloor)^\perp \otimes B]$$

$$\exists x_1 \ldots \exists x_n[(\lceil \diamond(x_1, \ldots, x_n)\rceil)^\perp \otimes B]$$

Systems encoded

and the subexponentials used

- G1m (minimal logic): l, r both linear
- mLJ (multi-conclusion LJ): l, r both classical
- LJQ* (focused sequent calculus for LJ): f linear, l, r classical
- S4 (modal logic):
    - l, r: classical
    - $\Box_L, \Diamond_R$: classical (holds formulas marked with $\Box$ or $\Diamond$ on the left or right)
    - e: classical ("dummy" subexponential to specify structural properties)
- Lax Logic (intuitionistic modal logic):
    - l classical, r linear
    - $\circ_r$ linear
- G3K + relation rules (modal logics T, 4, B, S4, TB, S5): l, r, R classical

Proving cut-elimination

1. Reduction to principal cuts
    - Permute cut rules upwards
    - Permute introduction rules downwards
    - Transform one cut into another (no general procedure was found yet)
2. Reduction to atomic cuts
3. Elimination of atomic cuts

Proving cut-elimination

Step 1: Reduction to principal cuts

- Permute cut rules upwards

$$\dfrac{\Gamma \longrightarrow A \quad \dfrac{\Gamma', A, F \longrightarrow G}{\Gamma', A \longrightarrow F \supset G}\,\supset_R}{\Gamma, \Gamma' \longrightarrow F \supset G}\,\mathrm{Cut} \quad \rightsquigarrow \quad \dfrac{\dfrac{\Gamma \longrightarrow A \quad \Gamma', A, F \longrightarrow G}{\Gamma, \Gamma', F \longrightarrow G}\,\mathrm{Cut}}{\Gamma, \Gamma' \longrightarrow F \supset G}\,\supset_R$$

- Permute introduction rules downwards

$$\dfrac{\varphi \quad \dfrac{\dfrac{\Gamma, A, B, F \longrightarrow G}{\Gamma, A \wedge B, F \longrightarrow G}\,\wedge_L}{\Gamma, A \wedge B \longrightarrow F \supset G, \Delta}\,\supset_R}{\Gamma \longrightarrow F \supset G, \Delta}\,\mathrm{Cut} \quad \rightsquigarrow \quad \dfrac{\varphi \quad \dfrac{\dfrac{\Gamma, A, B, F \longrightarrow G}{\Gamma, A, B \longrightarrow F \supset G, \Delta}\,\supset_R}{\Gamma, A \wedge B \longrightarrow F \supset G, \Delta}\,\wedge_L}{\Gamma \longrightarrow F \supset G, \Delta}\,\mathrm{Cut}$$

Permutations:
Depend on the subexponentials and their relations.

Proving cut-elimination

Step 1: Reduction to principal cuts
Proof by static analysis of subexponentials.
Example: Cut $= \exists A.!^a?^b\lfloor A \rfloor \otimes !^c?^d\lceil A \rceil$

$$
\dfrac{\dfrac{\dfrac{\Xi_1}{\vdash \mathcal{K}_1 \leq_a +_b\lfloor A \rfloor : \cdot \Uparrow \cdot}}{\vdash \mathcal{K}_1 : \cdot \Downarrow !^a?^b\lfloor A \rfloor}\ !^a, ?^b \qquad \dfrac{\dfrac{\dfrac{\dfrac{\Xi_2'}{\vdash \mathcal{K}_2 \leq_{c,s} +_d\lceil A \rceil +_t B : \cdot \Uparrow \cdot}}{\vdash \mathcal{K}_2 \leq_c +_d\lceil A \rceil : \cdot \Downarrow !^s?^t B}\ !^s, ?^t}{\vdash \mathcal{K}_2 \leq_c +_d\lceil A \rceil : \cdot \Uparrow \cdot}\ D_\infty}{\vdash \mathcal{K}_2 : \cdot \Downarrow !^c?^d\lceil A \rceil}\ !^c, ?^d}{\dfrac{\vdash \mathcal{K}_1 \otimes \mathcal{K}_2 : \cdot \Downarrow !^a?^b\lfloor A \rfloor \otimes !^c?^d\lceil A \rceil}{\vdash \mathcal{K}_1 \otimes \mathcal{K}_2 : \cdot \Uparrow}\ D_\infty, \exists}\ \otimes
$$

Case: $s \not\preceq d$ impossible (otherwise rule $!^s$ could not be applied).

Proving cut-elimination

Step 1: Reduction to principal cuts

Case: $s \preceq d$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Xi_1}{\vdash \mathcal{K}_1 \leq_{s,a} +_b \lfloor A \rfloor : \cdot \Uparrow \cdot}}{\vdash \mathcal{K}_1 \leq_s : \cdot \Downarrow !^a ?^b \lfloor A \rfloor} \; !^a, ?^b \quad \dfrac{\dfrac{\Xi_2'}{\vdash \mathcal{K}_2 \leq_{s,c} +_t B +_d \lceil A \rceil : \cdot \Uparrow \cdot}}{\vdash \mathcal{K}_2 \leq_s +_t B : \cdot \Downarrow !^c ?^d \lceil A \rceil} \; !^c, ?^d}{\vdash \mathcal{K}_1 \otimes \mathcal{K}_2 \leq_s +_t B : \cdot \Downarrow !^a ?^b \lfloor A \rfloor \otimes !^c ?^d \lceil A \rceil} \; \otimes}{\vdash \mathcal{K}_1 \otimes \mathcal{K}_2 \leq_s +_t B : \cdot \Uparrow \cdot} \; D_\infty, \exists}{\vdash \mathcal{K}_1 \otimes \mathcal{K}_2 : \cdot \Downarrow !^s ?^t B} \; !^s, ?^t}{\vdash \mathcal{K}_1 \otimes \mathcal{K}_2 : \cdot \Uparrow} \; D_\infty
$$

This permutation is possible given:

- $s \preceq a \;\Rightarrow\; \mathcal{K}_1 \leq_{s,a} = \mathcal{K}_1 \leq_a$
- $c \preceq t \;\Rightarrow\; !^c$ is allowed

Proving cut-elimination

Step 2: Reduction to atomic cuts [Miller and Pimentel, 2012]

Left and right introduction rules must be dual.

Introduction rules for a connective $\diamond$:

$$\exists \bar{x}(\lfloor \diamond(\bar{x}) \rfloor^{\perp} \otimes B_l) \quad \text{and} \quad \exists \bar{x}(\lceil \diamond(\bar{x}) \rceil^{\perp} \otimes B_r)$$

They are called dual the following can be proved in sellf:

$$\vdash \text{Cut} : \cdot \Uparrow \forall \bar{x}(B_l^{\perp} \parr B_r^{\perp})$$

Proving cut-elimination

Step 2: Reduction to atomic cuts [Miller and Pimentel, 2012]
Proof:

$$
\dfrac{\dfrac{\dfrac{\dfrac{\Pi_1}{\vdash \mathcal{X}, Cut, \Psi; \Delta_1 \Downarrow B_l}}{\vdots} \quad \dfrac{\dfrac{\Pi_1}{\vdash \mathcal{X}, Cut, \Psi; \Delta_2 \Downarrow B_r}}{\vdots}}{\quad} \otimes}{\dfrac{\vdots}{\vdash \mathcal{X}, Cut, \Psi; \Delta_1, \Delta_2 \Uparrow \cdot}} D_2 \qquad \rightsquigarrow \qquad \boxed{\text{Cut on object logic}}
$$

Proving cut-elimination

Step 2: Reduction to atomic cuts [Miller and Pimentel, 2012]
Since B_l and B_r are dual:

$$\dfrac{\dfrac{\tilde{\Pi}_2}{\vdash ?\mathcal{X}, ?Cut, ?\Psi, \Delta_2, B_r} \quad \dfrac{\dfrac{\tilde{\Pi}_1}{\vdash ?\mathcal{X}, ?Cut, ?\Psi, \Delta_1, B_l} \quad \dfrac{\Pi'}{\vdash ?\mathcal{X}, ?Cut, ?\Psi, B_r^{\perp}, B_l^{\perp}}}{\vdash ?\mathcal{X}, ?Cut, ?\Psi, \Delta_1, B_r^{\perp}}\ cut}{\vdash ?\mathcal{X}, ?Cut, ?\Psi, \Delta_1, \Delta_2}\ cut$$

$\tilde{\Pi}_1$ and $\tilde{\Pi}_2$ are the proofs Π_1 and Π_2 transformed to unfocused proofs.
Cut-elimination on meta-level: decides on object level cuts may still exist, but on simpler formulas than B_l and B_r.

Proving cut-elimination

Step 3: Elimination of atomic cuts

Further restrictions needed on the subexponentials used for the cut rule:

$$
\begin{prooftree}
\AxiomC{Ξ}
\noLine
\UnaryInfC{$\vdash \mathcal{K}_1 \leq_a +_b \lfloor A \rfloor : \cdot \Uparrow \cdot$}
\doubleLine
\RightLabel{$!^a, ?^b$}
\UnaryInfC{$\vdash \mathcal{K}_1 : \cdot \Downarrow !^a ?^b \lfloor A \rfloor$}
\AxiomC{}
\UnaryInfC{$\vdash \mathcal{K}_2^1 : \cdot \Downarrow \lceil A \rceil^\perp$}
\AxiomC{}
\UnaryInfC{$\vdash \mathcal{K}_2^2 : \cdot \Downarrow \lfloor A \rfloor^\perp$}
\RightLabel{$\otimes$}
\BinaryInfC{$\vdash \mathcal{K}_2 \leq_c +_d : \cdot \Downarrow \lceil A \rceil^\perp \otimes \lfloor A \rfloor^\perp$}
\doubleLine
\RightLabel{$D_\infty, \exists$}
\UnaryInfC{$\vdash \mathcal{K}_2 \leq_c +_d \lceil A \rceil : \cdot \Uparrow \cdot$}
\doubleLine
\RightLabel{$!^c, ?^d$}
\UnaryInfC{$\vdash \mathcal{K}_2 : \cdot \Downarrow !^c ?^d \lceil A \rceil$}
\RightLabel{$\otimes$}
\BinaryInfC{$\vdash \mathcal{K}_1 \otimes \mathcal{K}_2 : \cdot \Downarrow !^a ?^b \lfloor A \rfloor \otimes !^c ?^d \lceil A \rceil$}
\doubleLine
\RightLabel{$D_\infty, \exists$}
\UnaryInfC{$\vdash \mathcal{K} : \cdot \Uparrow \cdot$}
\end{prooftree}
$$

$\boxed{\mathcal{K}_1 \subset \mathcal{K} \text{ and } \lfloor A \rfloor \in \mathcal{K}} \Rightarrow \boxed{\lfloor A \rfloor \text{ must be in } s \text{ such that } b \preceq s}$

Note: A formula may be moved to an upper subexponential without affecting provability.

Proving cut-elimination

Theorem: Given a proof system's specification in SELLF, all conditions for the admissibility of cuts described are decidable.

- Permutation of rules and elimination of atomic cuts: static check of the subexponentials used.
- Duality of introduction rules: proved in $v + 2$ steps, where v is the maximum number of premisse atoms in the body of the introduction clauses.

Note: Some cut-elimination cases cannot yet be identified, such as the transformation of one cut into another.

Proving admissibility of non-atomic identities

[Miller and Pimentel, 2012]

Introduction rules for a connective $\diamond$:

$$\exists \bar{x}(\lfloor \diamond(\bar{x}) \rfloor^{\perp} \otimes B_l) \quad \text{and} \quad \exists \bar{x}(\lceil \diamond(\bar{x}) \rceil^{\perp} \otimes B_r)$$

They are called initial-coherent the following can be proved in sellf:

$$\vdash \text{Init} : \cdot \Uparrow \forall \bar{x}(?^{\infty} B_l \parr ?^{\infty} B_r)$$

In a system with initial coherent introduction rules, the initial rule can be restricted to its atomic version.

Proving the invertibility of rules

Follows from the facts:

- object-logic rules $\Rightarrow$ bipoles in SELLF
- bipoles in SELLF $\Rightarrow$ bodies are (purely) negative formulas
- negative formulas $\Rightarrow$ negative rules are invertible in SELLF
- invertible rules $\Rightarrow$ permutable rules
- permutable rules in meta-logic + adequacy on the level of derivations $\Rightarrow$ permutable rules in the object-logic
- object-logic rules are invertible

Conclusion

Given a sequent calculus system's specification in SELLF, we can:

- Prove cut-elimination (if the proof is not very involved)
- Prove admissibility of non-atomic initial rules
- Check the invertibility of rules

Implemented and online at
`http://www.logic.at/people/giselle/tatu`.

Thank you for your attention!

Questions?