Permission in a Kelsenian perspective

Agata CIABATTONI [a] Xavier PARENT [a] and Giovanni SARTOR [b]
[a] *Vienna University of Technology*
[b] *Cirsfid-Alma AI, University of Bologna; EUI, Florence*

Abstract. Although permissions are of crucial importance in several settings, they have garnered less attention within the deontic logic community than obligations. In previous work we showed how to reconstruct deontic logic using Kelsen's quasi-causal conception of norms, restricting ourselves to the notion of obligation. Here we extend the account to permission, and show how to analyse the notion of strong permission through a Kelsenian lens. In our framework various forms of conflicts between obligation and permission are disentangled.

1. Introduction

For the legal theorist Hans Kelsen [1] the basic form of legal norms –which directly reflects the nature of law– consist in a connection between an illicit (an unlawfulness - *unrecht*), i.e., an unwanted behaviour (accompanied by its circumstances) and an adverse reaction by the legal system (the sanction). From this perspective something is prohibited if it is connected to that reaction. The connection at stake, which Kelsen calls "imputation" has a quasi-causal nature. In [2] we have formalised this idea. This resulted in a simple deontic logic possessing interesting properties, and capable to avoid the deontic paradoxes that have beset SDL [3].

Here we extend our approach to permissions. Due to space limitation, we only cover the notion of strong permission−a classical topic in deontic logic−and its bilateral version; the latter formalize the idea of Kelsen's positive and negative permissions [1].

In accordance with Kelsen's positivist perspective, according to which all norms originate from acts of legal authorities, we distinguish two distinct categories of deontic norms, which express the authoritative disapproval or approval for certain states of affairs. This is achieved by establishing a causal-like connection between these states of affairs and corresponding approval or disapproval atoms. These concepts can be correlated with Kelsen's emphasis on the pivotal role of sanctions in the law. When an authority disapproves a particular state of affairs, it implies a commitment to punish it, whereas approval implies a commitment to refrain from punishment.

Our approach to model obligation differs from Anderson's [4] as (a) we model the connection between a sanction (actually, a disapproval) and its triggering condition as a causal-like relationship (using input/output (I/O) logic [5]), rather than as a necessity relationship, and (b) different unlawful facts in our approach might lead to different disapprovals. This feature enables us to address contrary-to-duty CTD obligations, i.e., obligations which are applicable only if other obligations are violated.

The use of a designated approval constant for permission is already present in [6] (and recently revived in [7]). Our approach diverges in several key aspects. First, we use multiple approval constants rather than a single one. The intuition is that each (new) state of affairs requires a (distinct) approval. Moreover, we analyse the imputation link as a causal-like relationship within the framework of I/O logic [5], while [6,7] use conditional logic and relatedness logic, respectively. Last, but not least, our logic allows to talk and reason about the interplay between permission and obligation, which is a very useful feature missing in the other approaches.

The idea of using multiple violation constants was anticipated by Anderson [8, p. 201], and developed further for the first time by Wyner [9,10] under the label "polynormativity". His work does not cover strong permission, and uses dynamic logic. Modal logic with its possible worlds semantics is incompatible with Kelsen's view that norms do not bear a truth-value. We recall that I/O logic originates as an attempt to analyse deontic concepts without relying on truth-values, and it is closely related to causal calculus [11].

2. Disapproval and approval norms and their logic

Our normative language includes three kinds of norms:

- *disapproval norms*, linking a disapproval (different for each such norms) to the realisation of a state of affairs
- *approval norms*, linking an approval (different for each such norms) to the realisation of a state of affairs
- *constitutive norms*, linking an institutional fact to a brute fact.

Definition 1 (Language, norms, normative systems)**.** *Let L be an ordinary propositional language containing the classical connectives and constants* $\{\wedge, \vee, \neg, \rightarrow, t, f\}$*, and two additional disjoint sets of atoms DIS and APP, where each* $\delta_i \in DIS$ *and* $\alpha_j \in APP$ *denotes a particular disapproval and approval, respectively.*

A norm *is a conditional* $\phi \Rightarrow \psi$*, where* ϕ *is a boolean formula and* ψ *is* δ_i *(disapproval norm), or* α_j *(approval norm), or a boolean formula (constitutive norm).*

A norm code *is a finite set of norms.*

Example 1 (Park rules, norm code N_1)**.** *The norm code* N_1 *will be used as running example through the paper. It consists of norms 1, 5, 6, 8 expressing disapprovals, norms 3,4,7, expressing approvals, and the constitutive norm 2.*

1. motorVehicle $\Rightarrow \delta_1$	2. motorBike $\Rightarrow$ motorVehicle	3. walk $\wedge$ lawn $\Rightarrow \alpha_1$
4. dog $\wedge$ leash $\Rightarrow \alpha_2$	5. walk $\wedge$ flowerBed $\Rightarrow \delta_2$	6. dog $\wedge \neg$leash $\Rightarrow \delta_3$
7. bike $\wedge$ pathway $\Rightarrow \alpha_3$	8. kill $\Rightarrow \delta_4$	

Different norms establish different disapprovals or approvals. This feature captures the fact that each unlawful or lawful situation triggers a distinct response by a legal and moral system (disapproval or approval of a state of affairs does not entail that other states of affairs are equally approved or disapproved). (See Wyner [9, p. 264] for a similar point.) We now recall our normative logic, that specifies a causal-like entailment for norms, in the spirit of Kelsen. This logic was applied in [2] (and called there violation logic) to constitutive and disapproval norms only.

Definition 2 (Normative logic)**.** *A normative inference relation is a binary relation* $\Rightarrow$ *between the set of propositions in L satisfying the following rules (*$\models$ *is the semantical consequence in classical propositional logic):*

(Truth) $t \Rightarrow t$*;*

(Strengthening) If $A \models B$ *and* $B \Rightarrow C$*, then* $A \Rightarrow C$*;*

(Weakening) If $A \Rightarrow B$ *and* $B \models C$*, then* $A \Rightarrow C$*;*
(And) If $A \Rightarrow B$ *and* $A \Rightarrow C$ *then* $A \Rightarrow B \wedge C$*;*
(Cut) If $A \Rightarrow B$ *and* $A \wedge B \Rightarrow C$*, then* $A \Rightarrow C$*;*
(Or) If $A \Rightarrow C$ *and* $B \Rightarrow C$*, then* $A \vee B \Rightarrow C$*.*

This logic enables us to derive norms from the primitive ones. A distinct feature of causal relations is that they are not reflexive, that is, they do not satisfy the postulate $A \Rightarrow A$, for all A. It is a particular case of the Input/Ouput logic out_4 in [5], and it differs from the causal calculus in [11] from the presence in the latter of the axiom $f \Rightarrow f$.

3. Strong permission

A deontic logic of obligations was introduced in [2] on the base of the normative logic. Although simple, the resulting logic was shown to behave well w.r.t. the main paradoxes introduced in the deontic literature. The idea is that a proposition is obligatory iff its complement causes a violation. The key notion was that of illicit, i.e., a state of affairs that minimally entails (a disjunction of) disapprovals.

We now extend the logic for obligations with permissions. While the core tenet of Kelsen's theory of norms [1] revolves around obligations, permissions also play an important role in his theory. Like most contemporary approaches in deontic logic, Kelsen distinguishes two notions of permissions: *permission in the negative sense* and *permission in the positive sense*. The latter is a form of strong (explicit) permission, while the former results from the fact that the legal system abstains from regulating certain action:

> "Human behaviour is regulated negatively by a normative order if this behaviour is not forbidden by the order, without being positively permitted [. . .], and therefore is permitted only in a negative sense." [1]

Unlike von Wright [3]'s permissions, Kelsen's permissions are incompatible not only with prohibitions but also with obligations. This is due to the fact that for Kelsen permissions are bilateral: always cover both an action and its omission. In other terms, when Kelsen says that an action is "permitted", he means that both the action and is omission are acceptable (in the weak or in the strong sense). As a consequence his "permission" of A is incompatibile not only with the prohibition of A, but also with A's obligation. We prefer to stick with von Wright's concept, and model accordingly Kelsen's bilateral permissions as the conjunction of two unilateral ones, one for an action and one for its negation. We show how to formalize in our quasi-causal framework positive (i.e. strong) permissions, and negative (i.e., weak) permission.

Weak permission is modelled as usual in deontic logic: a state of affairs is weakly permitted iff it does not entail any disapproval. To capture strong permissions we follow the approach used earlier for obligations. As disapproval norms grounded the notion of illicit (the state of affairs that triggers a disjunction of disapprovals) here we define the corresponding notion of licit, namely, the state of affairs that minimally triggers a disjunction of approvals: a state of affair is permitted iff it is a licit.

We are aware that Kelsen did not connect strongly permitted states of affairs to approvals. In fact, he did not provide a clear conceptualisation of strong permissions. This is why in his late work he tended to merge the idea of permission with that of a derogation. We however think that our approach is in the spirit of Kelsen's work, and can even be seen as an extension of his fundamental ideas —-a positivist approach to the law and a quasi-causal model of legal norms—-in ways that he did not consider.

Definition 3 (Licit)**.** *A Boolean formula A is a licit relatively to a norm code N iff for* $X = \bigvee S$*, where* $S \subseteq APP$*:* (1) $A \Rightarrow X$*, and* (2) *there is no B such that* $A \models B$*,* $B \not\models A$ *and* $B \Rightarrow X$.

The notion of licit corresponds to that for illicit (see [2]). A licit is not only a sufficient, but also a minimal condition for the (disjunction of the) approvals to take place. Clause 2 makes A the weakest formula leading to the approval in question.

Example 2. $\mathrm{walk} \wedge \mathrm{lawn}$ *is a licit, and so is* $(\mathrm{walk} \wedge \mathrm{lawn}) \vee (\mathrm{bike} \wedge \mathrm{pathway})$.

Definition 4 (Noncontextual permission)**.** *A Boolean formula A is (the content of) a noncontextual permission relative to a norm code N, denoted as* $\mathbf{P}_N A$*, iff A is a licit relatively to N.*

Henceforth we will omit the norm code N from the notation of *all* notions of permission, when no ambiguity occurs.

Example 3. *We have the strong permissions* $\mathbf{P}(\mathrm{walk} \wedge \mathrm{lawn})$*, and* $\mathbf{P}[(\mathrm{walk} \wedge \mathrm{lawn}) \vee (\mathrm{bike} \wedge \mathrm{pathway})]$

Note that clause (2) in Def. 3 blocks the intuitively invalid conjunction introduction rule: $\mathbf{P}_N A \Rightarrow \mathbf{P}_N(A \wedge B)$. This rule would be inherited by the presence of Strengthening in the base approval logic (cf. [6, p. 306], who observes the problem in a definition of strong permission attributed to Anderson, in terms of strict implication).

Remark 1. *Multiple approval constants are also (technically) needed. Without them our condition of minimality would be satisfied only by a single permitted state of affairs, namely, the disjunction of the antecedents of all approval norms: given n approval rules* $A_i \Rightarrow \alpha$*, with* $1 \leq i \leq n$*, the only licit, and thus strongly permitted state, is* $\bigvee_{i=1}^{n} A_i$.

The notion of Kelsen's positive permission (following Hart [12] we will refer to it as a "liberty") is then:

Definition 5 (Bilateral strong permission or liberty)**.** *A consistent Boolean formula A is a bidirectional strong permission, also called a strong liberty, relatively to a norm set N if and only if* $\mathbf{P}_N A \wedge \mathbf{P}_N \neg A$*, which we denote as* $\mathbf{L}^s_N A$.

Similarly, we define contextual permissions, based on the idea of contextual licits – minimal propositions which generate new approvals in the given context.

Definition 6 (Contextual licit)**.** *A Boolean formula A is a contextual licit relative to a norm code N and context C iff the following holds, where* $X = \bigvee S$*, and* $S \subseteq APP$*:*

1. *(i)* $C \not\models \neg A$*, (ii)* $\bigwedge C \not\Rightarrow X$*, and (iii)* $\bigwedge(C \cup \{A\}) \Rightarrow X$
2. *There is no B such that: (i)* $\bigwedge(C \cup \{B\} \Rightarrow X$*, (ii)* $A \models B$ *and* $B \not\models A$*, and (iii)* $\bigwedge_{c \in C}(\neg c) \not\models B$

Example 4. *In Ex. 1, in the context* $\{\mathrm{pathway}\}$*,* bicycle *is a contextual licit. The minimality condition for licit has a similar rationale as for illicit. On the one hand, we do not want that, in context* $\{lawn\}$*,* $walk \wedge bicyle$ *or* $walk \wedge kill$ *are licit. On the other hand, we want that, in context* $\{lawn\}$*, walk is a licit, instead of the weaker formula* $\neg lawn \vee walk$.

Definition 7 (Contextual permission)**.** *A Boolean formula A is a contextual permission relatively to a norm set N and a context C, denoted as* $\mathbf{P}_{(N,C)}(A)$*, if and only if A is a licit relatively to N and C.*

Example 5. *In Ex. 1 we have* $\mathbf{P}_{\{pathWay\}}(\text{bicycle})$.

Our logic for permissions allows to separate the elements of a conjunctive permission in an appropriate context. The lack of (unrestricted) separability is one of the distinctive features of our account of permission, similar to our account of obligation. We are here in full agreement with other recent approaches to strong permission, for a discussion of the notion of separability, see e.g. [13,14,15].

Proposition 1. *From* $\mathbf{P}(A_1 \wedge \cdots \wedge A_n)$ *follows* $\mathbf{P}_{\{A_1,\ldots,A_k\}}(A_{k+1} \wedge \cdots \wedge A_n)$*, whenever* $\{A_1,\ldots A_k\} \not\models \neg(A_{k+1} \wedge \cdots \wedge A_n)$.

Proof. For ease of readability, we only show the case where $n = 2$. Assume $\mathbf{P}(A_1 \wedge A_2)$. Given: $A_1 \wedge A_2 \Rightarrow \vee S$, and there is no B weaker than $A_1 \wedge A_2$ such that $B \Rightarrow \vee S$. Clause (1.i) in Def. 6 is met by assumption. Clause (1.ii) in Def. 6 is met by minimality. Clause (1.iii) holds trivially. To show: there is no D weaker than A_2 such that $A_1 \wedge D \Rightarrow \vee S$ and $\neg A_1 \not\models D$. Assume by contradiction that there is such D. We have $A_2 \models D$, and so $A_1 \wedge A_2 \models D$. Also $D \not\models A_1 \wedge A_2$ since $D \not\models A_2$. By (Or), $(A_1 \wedge A_2) \vee (A_1 \wedge D) \Rightarrow \vee S$. Obviously, $(A_1 \wedge A_2) \vee (A_1 \wedge D)$ is weaker than $A_1 \wedge A_2$. Contradiction. Hence $\mathbf{P}_{\{A_1\}} A_2$. □

The converse does not hold, and hence a contextual permission cannot be reduced to a non-contextual permission. Indeed,

Example 6. *Consider the context* $\{lawn, bike\}$ *and the norms in Ex. 1. It is true that* $\mathbf{P}_{\{\text{lawn, bike}\}}\text{walk}$. *But it is not the case that* $\mathbf{P}(\text{walk} \wedge \text{bike} \wedge \text{lawn})$.

4. Conflict involving obligations or strong permissions

In our logic we can identify four basic conflicts: obligations incompatible with other obligations (cases 1 and 2), and with permissions (cases 3 and 4).

1. OO-absolute incoherence. There are obligations $\mathbf{O}A$ and $\mathbf{O}B$ such that A and B are inconsistent. A disapproval is inevitable, since complying with one obligation leads to violate the other. Consider for instance $\mathbf{O}(\neg dog)$ and $\mathbf{O}(dog \wedge leash)$.
2. OO-relative incoherence. There are obligations $\mathbf{O}A$ and $\mathbf{O}B$ such that there is a context C, consistent with each of A and B such that $\{C, A, B\}$ is inconsistent. In this case a disapproval is inevitable in context C, but it may be avoided when C is not the case. Consider for instance the case in which there is a prohibition to bring dogs in the park, but a dog sitter may agree to do that with the dog owner, and therefore be under the obligation to do so ($\mathbf{O}(\neg dog)$ and $\mathbf{O}(promiseDog \rightarrow dog)$). The conflict only arises in the context *promiseDog*.
3. OP-absolute incoherence. There is an obligation $\mathbf{O}A$ and a permission $\mathbf{P}B$ where B and A are inconsistent. In this case the exercise of the permission leads to the violation of the obligation. Under such conditions one has to renounce to an approval, if one wants to avoid the disapproval. E.g., to comply with $\mathbf{O}(\neg dog)$ one has to renounce to exercise $\mathbf{P}(dog \wedge leash)$.

4. OP-relative incoherence. There is a permission $\mathbf{P}_C B$ and an obligation $\mathbf{O}_C A$ such that A and B are inconsistent. Intuitively, one cannot realize a permission in a given context without violating an obligation arising in the same context. This is the same as OP-absolute incoherence, but relativized to a context. An example of an incoherent code would be one that both prohibits and permits to enter the park with a large leashed dog: $\mathbf{O}_{\{large,leash\}}(\neg enter)$ and $\mathbf{P}_{\{large,leash\}}(enter)$.

The last two cases are called cross-(in)coherence in [16].

5. Conclusion

Permission has consistently remained a contentious concept within legal theory and has received less attention in deontic logic compared to obligations. Our Kelsenian approach, which establishes causal-like connections between states of affairs and disapproval or approval atoms, offers an intuitive foundation for formalizing norms that grant permissions. The framework further enables the identification and precise characterization of different types of conflicts between obligations and permissions. In our forthcoming research, we intend to analyse strong and weak permissions and to delve into potential resolutions for these conflicts, drawing upon techniques from non-monotonic logics.

Acknowledgments: Work supported by the H2020 ERC Project "CompuLaw" (G.A. 833647) and the Austrian Research Funds (FWF) [M 3240 N, ANCoR project].

References

[1] Kelsen H. The Pure Theory of Law. University of California Press; 1967.
[2] Ciabattoni A, Parent X, Sartor G. A Kelsenian deontic logic. In: Erich S, editor. Legal Knowledge and Information Systems - JURIX 2021. vol. 346 of Frontiers in Artificial Intelligence and Applications. IOS Press; 2021. p. 141–150.
[3] von Wright GH. Deontic logic. Mind. 1951 01;LX(237):1–15.
[4] Anderson AR. The logic of norms. Logique et Analyse. 1958;1:84–91.
[5] Makinson D, van der Torre LWN. Input/output logics. J Phil Log. 2000;29(4):383–408.
[6] Asher N, Bonevac D. Free choice permission is strong permission. Synthese. 2005;145:303–323.
[7] Glavanicová D, Jarmuzek T, Klonowski M, Kulicki P. Free choice permission, legitimization and relating semantics. Log J IGPL. 2023;31(3):574–597.
[8] Anderson A. The formal analysis of normative systems. In: Rescher N, editor. The Logic of Decision and Action. University of Pittsburgh Press; 1967. p. 147–215.
[9] Wyner AZ. Maintaining obligations on stative expressions in a deontic action logic. In: Deontic Logic in Computer Science, DEON 2004. vol. 3065. Springer; 2004. p. 258–274.
[10] Wyner AZ. Sequences, obligations, and the contrary-to-duty paradox. In: Deontic Logic and Artificial Normative Systems, DEON 2006. vol. 4048. Springer; 2006. p. 255–271.
[11] Bochman A. On laws and counterfactuals in causal reasoning. In: Proc. 16 Intern. Conf. on Principles of Knowledge Representation and Reasoning (KR 2018); 2018. p. 494–503.
[12] Hart H. Essays on Bentham. OUP; 1982.
[13] Hansen J. Imperatives and Deontic logic. University of Leipzig; 2008.
[14] Olszewski M, Parent X, van der Torre L. Input/output logic with a consistency check - the case of permission. In: Deontic Logic and Normative Systems DEON 2020/21. College publications; 2021. p. 358–375.
[15] Olszewski M, Parent X, Van der Torre L. Permissive and regulative norms in deontic logic. J of Logic and Computation. 2023 05:exad024.
[16] Makinson D, van der Torre L. Permission from an I/O perspective. J of Phil Logic. 2003;32(4):391–416.