MUltseq: Sequents, Equations, and Beyond*

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This abstract presents the program MUltseq, which can be used to decide the validity of finitely-valued formulas, the consequence relation, and the validity of equations and quasi-equations in certain finite algebras. In its core, MUltseq is a generic sequent prover for propositional finitely-valued logics. This means that it takes as input the rules of a many-valued sequent calculus as well as a many-sided sequent and searches – automatically or interactively – for a proof of the latter. For the sake of readability, the output of MUltseq is typeset as a LaTeX document. Though the sequent rules can be entered by hand, MUltseq is primarily intended as a companion for MUltlog, a program that computes – among other calculi – optimized rules of a sequent calculus from the truth tables and distribution functions of a finitely-valued logic [1, 5, 8].

Provided the input sequent calculus is both correct and complete for the logic under consideration – which is always the case when the rules were computed by MUltlog – MUltseq serves as a decision procedure for the validity of formulas and sequents. More interestingly, MUltseq can also be used to decide the *consequence relations* associated with the logic and the sequent calculus. The problem of deciding whether a particular formula ϕ is true in all models satisfying a given set of formulas Δ, i.e., whether ϕ logically follows from Δ, can be reduced to the problem of proving a certain sequent that depends only on ϕ and Δ. Similarly, the problem of finding a derivation of a sequent σ from hypotheses Σ can be reduced to proving a particular set of sequents [4, 7].

From the algebraic point of view, it is an interesting problem to determine whether an *equation* or a *quasi-equation* is valid in a finite algebra. If we consider the algebra as a set of truth values and a collection of finitely-valued connectives, and use an appropriate translation of equations and quasi-equations into sequents, the problem again reduces to the provability of many-valued sequents [2, 3].

The decision procedures implemented in MUltseq help to get a better intuition and understanding of some theoretical problems. For instance, it is known that each propositional logic between the implication-less fragment of Intuitionistic Propositional Calculus and Classical Propositional Calculus has an algebraic semantics. If we consider the algebraic semantics of all these logics, we obtain a denumerable chain which corresponds to the chain of all subvarieties of the variety of Pseudo-complemented Distributive Lattices [7]. Each of these subvarieties is generated by a finite algebra, so the study of the sequent calculi obtained

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by `MUltlog` for each of these algebras and the decision procedures in `MUltseq` might help to find algebraizable Gentzen systems for the original logics.

`MUltseq` is developed by the authors of this abstract within a project titled *"Generic Decision Procedures for Many-Valued Logics"*. It is written in a subset of Prolog compatible with any standard Prolog interpreter. More details as well as the most recent version of `MUltseq` can be obtained on the web [6].

References

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