**MUltseq: Sequents, Equations, and Beyond**

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This abstract presents the program *MUltseq*, which can be used to decide the validity of finitely-valued formulas, the consequence relation, and the validity of equations and quasi-equations in certain finite algebras. In its core, *MUltseq* is a generic sequent prover for propositional finitely-valued logics. This means that it takes as input the rules of a many-valued sequent calculus as well as a many-sided sequent and searches – automatically or interactively – for a proof of the latter. For the sake of readability, the output of *MUltseq* is typeset as a \(\LaTeX\) document. Though the sequent rules can be entered by hand, *MUltseq* is primarily intended as a companion for *MUltlog*, a program that computes – among other calculi – optimized rules of a sequent calculus from the truth tables and distribution functions of a finitely-valued logic \([1, 5, 8]\).

Provided the input sequent calculus is both correct and complete for the logic under consideration – which is always the case when the rules were computed by *MUltlog* – *MUltseq* serves as a decision procedure for the validity of formulas and sequents. More interestingly, *MUltseq* can also be used to decide the *consequence relations* associated with the logic and the sequent calculus. The problem of deciding whether a particular formula \(\phi\) is true in all models satisfying a given set of formulas \(\Delta\), i.e., whether \(\phi\) logically follows from \(\Delta\), can be reduced to the problem of proving a certain sequent that depends only on \(\phi\) and \(\Delta\). Similarly, the problem of finding a derivation of a sequent \(\sigma\) from hypotheses \(\Sigma\) can be reduced to proving a particular set of sequents \([4, 7]\).

From the algebraic point of view, it is an interesting problem to determine whether an *equation* or a *quasi-equation* is valid in a finite algebra. If we consider the algebra as a set of truth values and a collection of finitely-valued connectives, and use an appropriate translation of equations and quasi-equations into sequents, the problem again reduces to the provability of many-valued sequents \([2, 3]\).

The decision procedures implemented in *MUltseq* help to get a better intuition and understanding of some theoretical problems. For instance, it is known that each propositional logic between the implication-less fragment of Intuitionistic Propositional Calculus and Classical Propositional Calculus has an algebraic semantics. If we consider the algebraic semantics of all these logics, we obtain a denumerable chain which corresponds to the chain of all subvarieties of the variety of Pseudo-complemented Distributive Lattices \([7]\). Each of these subvarieties is generated by a finite algebra, so the study of the sequent calculi obtained

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by MULTLog for each of these algebras and the decision procedures in MULTseq might help to find algebraizable Gentzen systems for the original logics.

MULTseq is developed by the authors of this abstract within a project titled “Generic Decision Procedures for Many-Valued Logics”. It is written in a subset of Prolog compatible with any standard Prolog interpreter. More details as well as the most recent version of MULTseq can be obtained on the web [6].

References