Project proposal

Generic Decision Procedures for Many-valued Logics

Acción Integrada Hispano-Austríacas

Gernot Salzer Ventura Verdú

Remark. The regulations for *Acciones Integradas* are different in Austria and Spain. In Austria, the application is for one year with the possibility to prolong for another year; in Spain, the application is for two years from the outset. Since this project is laid out as a two-years enterprise and our Spanish partners need it anyway, this proposal describes the activities planned for both years.

Contents

1 Abstract

The aims of the project are twofold. On the practical level, the program MUltlog – developed by the Vienna Group for Multiple-valued Logics consisting of M. Baaz, C. Fermüller, G. Salzer and R. Zach – will be extended by methods obtained by the Spanish group, consisting of A. Gil, J. Rebagliato and V. Verdú. In particular, MUltlog will be complemented by a generic theorem prover which in turn will be used to construct decision procedures for validity, for the consequence relation, and for equations and quasi-equations in the context of finitely-valued logics. The provers and the decision procedures will be used to study specific logics; the intuitions gained in this way will help to attack some theoretical problems.

On the theoretical level, several open problems relevant to the research of both groups will be addressed. For finitely-valued logics, these are questions like: Is interpolation, the existence of a deduction theorem and the existence of a Hilbert-type system decidable? Is it possible to express the validity of sequents within the logic itself? For infinitely-valued logics, the relationship between Gödel logic, Łukasiewicz logic and product logic will be investigated.

2 Scientific objectives

One of the main research topics of the Spanish group is the general theory of m-dimensional sequent calculi, with an emphasis on their algebraic characteristics. In particular, A. Gil, J. Rebagliato and V. Verdú developed general methods to deal with consequence relations, equations and quasi-equations in arbitrary finitely-valued logics by using m-dimensional sequent calculi, where m is the number of truth values of the logic. To gain insights and better intuitions it is necessary to experiment with specific logics. Unfortunately, applying the general methods to a particular logic is of a high computational complexity and error-prone when done by hand. For these reasons further research depends on an adequate implementation of the algorithms.

It turned out that the MUltlog system is a good starting point. MUltlog is a program maintained by the Viennese group (with support from the Austrian science foundation); it implements algorithms developed as part of the Viennese research on many-valued logics [2, 6, 27]. In its current version, MUltlog takes as input the specification of a finitely-valued first-order logic and generates several types of calculi, among them m-dimensional Gentzen-style sequent calculi. The main task of MUltlog is the computation of optimal calculi, i.e., of rules with a minimal branching degree. For the sake of better readability the output is in the form of a scientific paper typeset in LaTeX (for a detailed description see [7]).

In this project, MUltlog will be extended to provide the rules also in machine-readable form. A generic sequent-style theorem prover (to be implemented) will use this information to adapt itself automatically to particular logics. Subse-

quently the theorem prover will serve as the basis for several decision procedures, which will also be implemented in the project. The decision problems are:

(a) Is a given formula valid in a particular finitely-valued logic?

(b) Is a given sequent provable in the calculi generated by MUltlog?

From the algebraic point of view, it is an interesting problem to determine whether an equation or a quasi-equation is valid in a finite algebra. If we consider the algebra as a set of truth values and a collection of finitely-valued connectives, and use an appropriate translation of equations and quasi-equations into sequents, the procedures above can be extended to decide the following problems:

(c) Is a given equation valid in a particular finite algebra?

(d) Is a given quasi-equation valid in a particular finite algebra?

The theoretical foundations for these applications have been laid by the Barcelona group in [16, 17, 18, 19, 24, 25].

The prover and the decision procedures will help to get a better understanding of the theoretical problems. For instance, it is known that each propositional logic between the implication-less fragment of Intuitionistic Propositional Calculus (IPC) and Classical Propositional Calculus (CPC) has an algebraic semantics. If we consider the algebraic semantics of all these logics, we obtain a denumerable chain which corresponds to the chain of all subvarieties of the variety of Pseudo-complemented Distributive Lattices (PCDL) (see [25]). Each of these subvarieties is generated by a finite algebra, so the study of the sequent calculi obtained by MUltlog for each of these algebras and the application of the decision procedures will help to find algebraizable Gentzen systems for the original logics.

Using MUltlog for adapting the decision procedures to particular logics will also serve as a test bed for improved optimization algorithms, which are developed in Vienna in a master thesis on the optimization of many-valued circuits [22]. Another application for the extended MUltlog system is teaching: at both universities there are courses on many-valued logics. By observing how calculi change when modifying the logic students will get a deeper understanding of the relationship between logics and inference systems.

The research of both the Spanish as well as the Austrian group can be described as the search for general properties of many-valued logics, though from different viewpoints (algebraic vs. logical). Because of these similarities, several theoretical questions have emerged which are of interest to both groups:

(e) Is it decidable whether the interpolation theorem holds in a finitely-valued logic? If it holds, what is a good algorithm for effectively computing interpolating formulas?

(f) Is it decidable whether there exists a Hilbert-style system for a given finitely-valued logic? How can it be obtained algorithmically?

(g) Is it decidable whether a given finitely-valued logic allows to express the validity of sequents in its own language?

(h) To which extent is it possible to generalize the results for finitely-valued logics to infinitely-valued ones? Most probably the answer will be negative for many questions. However, there are some positive results, too, which can serve as a starting point: according to Hay [21], infinitely-valued Łukasiewicz logic can be partially axiomatized. It seems possible that this method can also be applied to other logics like Gödel logics, and that such partial axiomatizations may be automatically derived from a presentation of the set of truth values.

It is planned to address – and to solve, hopefully – these questions jointly during meetings of the two groups in Vienna or Barcelona. Positive answers to these questions are not only interesting from a theoretical point of view, but will also give rise to future extensions of MUltlog, allowing further experiments.

3 Methodology and detailed research plan

The project will be organized around four workshops (or rather, plenary meetings), which will give all members of the two groups the opportunity to meet and discuss. The workshops will take place twice a year alternately in Vienna and Barcelona. Inbetween the participants will work independently on the goals described above; e-mail and other Internet services will serve as main communication tools.

The first year will be devoted to the extension of the MUltlog kernel to produce machine readable output, to the implementation of a generic theorem prover, and to the construction of decision procedures for validity and provability. First experiments will be done with this new tool. On the theoretical side, work will concentrate on the first two questions concerning interpolation and Hilbert-style calculi.

In the second year the algebraic decision procedures will be implemented and used for experiments. Provided that the first two theoretical problems will have been solved satisfactorily, the remaining two will be addressed, too.

The first workshop will take place in spring 1998, presumably in Barcelona. The agenda will be:

- Agreement on a common platform and strategy for developing the extensions of MUltlog. MUltlog is written in Prolog; the same language will be used for the extensions. Reasonable choices are SWI-Prolog or BinProlog; both are freely available for Unix as well as for DOS and Windows. Portability is an important issue since the programs should not only be

offered to other colleagues worldwide, but should also be used by students for pedagogical purposes.

- Specification of the interface between MUltlog and the decision procedures, i.e., how to represent the optimized rules such that other programs can use it. Specification of the functionality and rough structure of the theorem prover and the decision procedures.
- Agreement on a set of test examples, i.e., a test suite. Good choices are finitely-valued Łukasiewicz or Gödel logics, as they provide not just single examples but a whole series parameterized by the number of truth values; additionally, the results for these logics are also interesting from a theoretical point of view.
- Discussion of the first two theoretical problems, i.e., how to decide interpolation and the existence of Hilbert-style calculi.

In the time until the next meeting, the main implementational work will be done on both sides. In Vienna, MUltlog will be extended to produce machine-readable output; additionally, the optimization algorithm will be improved using methods from the minimization of many-valued circuits. The Barcelona group will implement the generic theorem prover as well as the decision procedures for validity and provability. Both groups will do research on the theoretical problems.

The second workshop will take place in autumn 1998, presumably in Vienna. The agenda will be:

- Integration tests of the programs developed since the last workshop. Fine tuning of the interfaces. Evaluation of the test suite.
- Outline of a project for using MUltlog and the new programs in teaching. Prof. Gilles Hunault (Université Angers, France), has already shown interest in using MUltlog for his lectures.
- First experiments with the new programs, discussion of the output.
- Continuation of the theoretical studies. Discussion and presentation of the results obtained since the first meeting.
- Outline of a joint paper summarizing the results obtained so far, to be submitted to a conference or journal. Preparation of an intermediate report required to prolong this project for another year (necessary at least on the Austrian side).

After this meeting both groups will work on the preparation of the joint paper. Furthermore, the project will be documented on the web, making MUltlog and its extensions available to the public.

The third meeting will take place in spring 1999. The preliminary agenda is:

- The general aspects of the decision procedures for equations and quasi-equations in a finite algebra will be discussed. These procedures can be reduced to the procedures implemented during the first year of the project.

- We will look at some theoretical questions which the new decision procedures can help to answer. In particular we will work one some open problems in the field of algebraic logic which can be formulated in terms of equations and quasi-equations, and which can be solved using the output of the new procedures, like the search for algebraizable Gentzen systems described in section 2.

- On the theoretical side, the remaining open questions will be addressed. According to our plans, this will be the question, under which conditions a logic is expressive enough to state the validity of its sequents. Furthermore, we will start investigations on how to extend the results obtained for finitely-valued logics to the infinitely-valued case.

- Preparation of the final version of our joint paper.

The time until the last workshop will be used to implement the decision procedures for (quasi-)equations. Furthermore, the user interfaces of MUltlog have to be adapted to the extended functionality of the system. The WWW interface of MUltlog is of particular importance. People interested in experimenting with MUltlog and the decision procedures will be able to do this via a standard web browser like Netscape, without the effort of installing MUltlog on their own machines.

The final workshop will take place in autumn 1999. The tentative program is:

- Integration of the programs developed during summer, final tests including the extended interfaces. Preparation of an installation package to make the system available to the public.

- Continuation of the theoretical studies started in the third workshop: discussion of topics in algebraic logic, supported by the new system; presentation of the results obtained for the open theoretical problems, like generalizations to infinitely-valued logics.

- Outline of the final report on the project; possibly drafting of a second joint paper.

- Evaluation of the pedagocial potentials of the system. Which parts of the system can be used for which lectures (including the needs of our 'French connection' Gilles Hunault), what kinds of modifications are necessary to make the prototype student-safe?

4 Contacts to other research groups

(a) *Prof. R. Hähnle,* Universität Karlsruhe. He is an authority on tableaux-based deduction and automated theorem proving for many-valued logics. His text on *Automated Deduction in Multiple-valued Logics* is a standard. R. Hähnle and G. Salzer both worked on the optimization problem of many-valued calculi [20, 27]; the joint discussions lead to essential improvements in both papers. R. Hähnle is regularly visiting the VGML and recently got his habilitation from Technische Universität Wien.

(b) *Prof. P. Hajek,* Czech academy of sciences (Prague). He has done significant work on the theory and applications of many-valued and fuzzy logics, and has collaborated with M. Baaz on the topic of axiomatizability of infinite-valued logics.

(c) *Prof. J. Krajíček,* Oxford University. He is a renowned logician and an expert on complexity theory. Recently, he published an article on the embedding of logics into product logic jointly with M. Baaz and P. Hajek.

(d) *Prof. W. Blok,* University of Illinois at Chicago, and *Prof. D. Pigozzi,* Iowa State University. They have created the theory of algebraizable deductive systems, which has been the inspiration for the theory of algebraizable Gentzen systems developed by the Spanish group. Prof. Pigozzi has spent several sabbatical terms at the "Centre de Recerca Matemàtica" in Barcelona. Jordi Rebagliato spent one year at the University of Illinios at Chicago doing research with W. Blok.

(e) *Prof. D. Mundici,* Università degli Studi di Milano. Prof. D. Mundici is an authority on MV-algebras, which is a theoretical research topic of the Spanish group [19]. He has also close ties to the Vienna group, since he presided for several years over the Kurt-Gödel-Society, which is located in Vienna, with M. Baaz belonging to the executive board.

(f) *COST action 15.* This action, titled *Many-valued Logics for Computer Science Applications,* comprises virtually all European research groups working in many-valued logics. The European Commission provides funding for workshops held at regular intervals, to promote cooperations between different countries. In fact, the cooperation between the Barcelona group and the Viennese group was initiated by a COST workshop.

5 Prospective cooperation after the end of this Acción Integrada

The expected results of the decision procedures will make it possible to study general problems associated with finitely-valued logics – like the existence of

axiomatizations, the search for minimal axiomatizations, and the existence of interpolands and a deduction theorem – and to provide counterexamples to certain open problems. With the results of this first project we will have material to analyze the possibilities of extending MUltlog to deal with infinitely-valued logics. Furthermore, a program that is in some sense the inverse of MUltlog will be implemented. It will take as input an arbitrary sequent calculus and determine whether if corresponds to a finitely-valued logic [8].

6 Suitability according to national and international priorities

The study of many-valued logics and their applications is the main topic of COST Action no. 15 of the European Commission. The research described in this proposal will be of importance to the whole community:

- Apart from a few experimental provers there are no implementions yet. Therefore MUltlog and its extensions can be expected to be a valuable tool for experimentation as well as teaching.
- The theoretical problems to be solved in this project are of general interest. The aim is to solve the problems on a general basis for all finitely-valued logics at once, which will eliminate the patchwork of specific results for single logics.
- There is only little work on the minimization of many-valued circuits up to now. In the long run it can be expected that the research on the optimization of many-valued calculi also leads to improved algorithms for many-valued circuits.

7 Related research grants and projects

Both groups participate in COST Action 15 by the European Commission, titled *Many-valued Logics for Computer Science Applications*. In this framework two *Short Term Scientific Missions* were granted: C. Fermüller visited Barcelona in May 1996, and A. Gil visited Vienna in April 1997. The latter meeting also served the purpose of preparing this proposal.

The development of MUltlog was supported by the Austrian research foundation, FWF grant P10282-MAT. The proposal for a follow-up project oriented towards infinitely-valued logics (including suitable extensions to MUltlog) is currently in the reviewing stage.

The Spanish group has several theoretical grants: DGICYT project "Lógica Algebraica" (1995-98, PB94-0920, main researcher Ventura Verdú), and a project "Lògica Algebraica" (1995SGR-00046) from the Generalitat de Catalunya.

8 The research groups and applied-for funding

8.1 The Austrian group

The Vienna Group for Many-valued Logics (VGML) consists of M. Baaz, C. Fermüller, G. Salzer and R. Zach and is affiliated to the 'Technische Universität Wien' (Vienna university of technology). The research topics of the past five years concerning many-valued logics are:

- Systematic construction of sequent calculi, natural deduction systems, clause formation rules, many-valued resolution, and tableaux systems from the specification of a finitely-valued logic; general properties of these calculi, like cut elimination. [2, 6, 8, 9, 10, 11]
- Optimization algorithms for minimizing the branching degree of calculi [22, 27]
- Development of the MUltlog system [7, 26], coordinated by G. Salzer. The actual programming was done by G. Salzer (kernel, basic algorithms), A. Leitgeb (interface for X-windows), W. Nix (interface for DOS), and M. Schranz (interface for WWW).
- Properties and comparison of infinitely-valued logics [1, 12, 13, 14]
- Approximation of logics by sequences of finitely-valued logics [15], finitely-valued intuitionistic logics [3, 4], finitely-valued default reasoning [5].

R. Zach is currently at the university of Berkley. The Austrian group therefore consists of:

Gernot Salzer, Christian Fermüller
Institut für Computersprachen 185.2
Technische Universität Wien
Resselgasse 3
A-1040 Wien

Matthias Baaz
Institut für Algebra und Diskrete Mathematik 118.2
Technische Universität Wien
Wiedner Hauptstraße 8–10
A-1040 Wien

8.2 The Spanish group

One of the main research topics of the Spanish group is the general theory of m-dimensional sequent calculi, with an emphasis on their algebraic characteristics, where m is the number of truth values of the logic [16, 17, 18, 19, 24, 25]. The Spanish group consists of:

Angel J. Gil Estallo
Dept. d'Economia
Estudis Empresarials
Universitat Pompeu Fabra
Trias Fargas 23–25
E-08005 Barcelona

Ventura Verdú
Facultat de Matemàtiques
Universitat de Barcelona
Gran Via 585
E-08005 Barcelona

Jordi Rebagliato
Facultat de Matemàtiques
Universitat de Barcelona
Gran Via 585
E-08005 Barcelona

8.3 Applied-for funding

	Austrian side	Spanish side
Meeting spring 1998 in Barcelona	3 weeks/3 journeys	
Meeting autumn 1998 in Vienna		3 weeks/3 journeys
Meeting spring 1999 in Barcelona	3 weeks/3 journeys	
Meeting autumn 1999 in Vienna		3 weeks/3 journeys

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