Vagueness in Quantity: Two Case Studies from a Linguistic Perspective

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1 Introduction

Much work on vagueness has focused on adjectives such as *thin*, *bald* and *red* as the paradigm case of the phenomenon. But vagueness is prevalent also in many other domains of natural language, and it is one of these other domains—namely expressions of quantity—that is the focus of the present paper. Quantificational expressions present a diverse range of examples of vagueness, imprecision and underspecification. Some quantifiers, such as *many*, *few*, *much* and (as will be seen below) *most*, are inherently vague. Others have definite but underspecified truth conditions; for example, *some dogs* must be at least two, but does not specify the number further. Vagueness in quantity may be signaled explicitly via modifiers such as *roughly* (*roughly 50 books*) and *approximately* (*approximately 1000 residents*). And even without modification, seemingly precise expressions such as number words may be interpreted approximately; for example, *'there were 100 people in the audience'* is typically understood to mean 'roughly 100'.

In this paper, I examine two case studies of vagueness within the realm of quantity expressions: the vague quantifiers *many*, *few*, *much* and *little*, which pattern in some respects with gradable adjectives, but also show some illuminating differences; and the quantifier *most*, which along with *more than half* provides a minimal pair of a vague and a non-vague expression with otherwise overlapping semantics.

My goal in delving into this topic for a contribution to a volume on reasoning with vague information is first of all to explore what we can learn about vagueness by examining its manifestation in the expression of quantity. I hope to show in what follows that facts from this general area are able to shed some new light on approaches and mechanisms that have been applied to the analysis of vagueness.

But secondly, as I am a linguist, my contribution necessarily comes from the perspective of that field. Perhaps the most central goal of linguistics is to account for the (mostly unconscious) linguistic knowledge of native speakers of a language. This includes, among other things, knowledge about whether a string of words is or is not a grammatical sentence of the language, and about the interpretation(s) that a grammatical sentence can receive. A variety of data sources can be brought to bear on these questions, including the linguist's own intuitions, judgments elicited from other speakers, attested

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examples of speech or writing, and performance by subjects on structured experimental tasks. In this paper, I also aim to illustrate what the careful examination of linguistic data such as these can tell us about the nature and the proper treatment of vagueness.

For a preview of what is to come, the first of the two case studies discussed below will strengthen the case for the relevance of comparison classes in the interpretation of vague, gradable expressions (though suggesting that the traditional notion of a comparison class must be broadened somewhat). The second raises the possibility that vagueness is, in at least some cases, associated with a less informative underlying scale structure than that typically assumed in degree-based semantic analyses.

2 How many is *many*?

For the first case study, I consider the so-called 'vague quantifiers' or 'vague numerals' *many* and *few*, and their mass counterparts *much* and *little*:

(1) a. <u>Many</u> people I know like jazz.
b. <u>Few</u> students came to the lecture.
c. I don't have <u>much</u> money.
d. There's <u>little</u> water left in the bucket.

Many and *few* are quantifiers that combine with plural countable nouns, expressing roughly 'a large number of' and 'a small number of'. *Much* and *little* occur with non-countable (mass) nouns, and correspondingly express something like 'a large amount of' and 'a small amount of'. But what is meant by a large or small amount or quantity is of course vague.

It can readily be shown that words of the *many* class pattern in a variety of respects with the better-studied class of vague gradable adjectives such as *tall*, *thin*, and *expensive*. As seen in (2) and (3), they have comparative and superlative forms, and combine with a range of degree modifiers (the defining characteristics of gradable expressions):

(2) a. Betty has <u>many</u> friends/<u>more</u> friends than Sue/<u>the most</u> friends.
b. Betty has <u>few</u> friends/<u>fewer</u> friends than Sue/<u>the fewest</u> friends.
(cf. tall/taller/tallest)

(3) a. Fred drank <u>too much</u> wine.
b. Barney drank <u>very little</u> wine.
c. Betty read <u>as many</u> books <u>as</u> Wilma.
d. I'm surprised Wilma read <u>that few</u> books.
(cf. too tall, very tall, as tall as, etc.)

Beyond this, their interpretation is context sensitive, just as in the case of gradable adjectives. For example, the number of students in attendance that would suffice to establish the truth of (4) depends among other things on the context or situation of utterance: Are we, for example, talking about an in-class lecture in a university seminar, or a campus-wide lecture by a prominent and popular figure such as Bill Clinton? And even with the context fixed, the boundaries for *many* and its counterparts remain fuzzy, resulting in borderline cases. We might consider 1000 students attending Clinton's lecture to be a clear case of *many*, and 5 in attendance to be a clear case of *not many*, but what about 50? 100?

(4) Many students attended the lecture.

These patterns—context dependence and fuzzy borders/borderline cases—are just what is observed with gradable adjectives. For example, the truth or falsity of *Fred is tall* depends on the context (is Fred an adult man? an 8-year-old boy? a professional basketball player?), and for any context, some cases will seem to fall into a grey area.

Finally, as with other instances of vague predicates, these words give rise to the Sorites paradox. For example, the two reasonable-sounding premises in (5a,b) seem to lead to the unquestionably false (5c):

(5) a. 1000 students attending Clinton's lecture counts as *many*.
b. If n students attending Clinton's lecture counts as *many*, then $n-1$ students attending also counts as *many*.
c. 3 students attending Clinton's lecture counts as *many*.

In light of this long list of parallels between gradable adjectives on the one hand and the *many* class on the other, it is desirable that whatever analysis is developed for the former case be extendable to the latter. That is my goal here.

As an aside, the reader might at this stage wonder why, if words like *many* behave so similarly to gradable adjectives, we don't simply conclude that they are ordinary gradable adjectives, and thus require no special treatment. In [1], I show that there are a number of significant distributional differences between the two classes, which give reason to think that they belong to distinct semantic types. As one example, words of the *many* class can occur as modifiers in comparatives (6a,b). Ordinary gradable adjectives cannot (6c); even the adjective *numerous*, which otherwise has much the same semantic content as *many*, is extremely awkward in this position (6d):[2]

(6) a. Many more than 100 students attended the lecture.
b. John is much taller than Fred.
c. *John is tall taller than Fred.
d. ?Numerous more than 100 students attended the lecture.

In [1], I argue that differences of this sort can be accounted for by analyzing *many* and like words as predicates over scalar intervals (sets of degrees), in contrast to ordinary gradable adjectives, which are analyzed as predicates over individuals (I refer the reader to that work for further justification of this claim, and for details of the implementation). For present purposes, however, we can abstract away from this distinction.

[2]Here I follow the linguist's convention of using a star (*) to mark a sentence that is ungrammatical, in the sense of not being a possible sentence in the language, and a question mark (?) to mark a sentence that is degraded but not outright ungrammatical.

Let us turn now to the semantic analysis of vague predicates such as these. A long tradition holds that sentences involving vague adjectives, such as those in (7), should be analyzed with reference to a **comparison class** that in some way serves to provide a frame of reference or standard of comparison (see especially [2, 3, 4, 5, 6, 7]; though see [8] for an opposing position). On this approach, (7a) would be interpreted as saying that Fred's height exceeds the standard for some set of individuals of which Fred is a member (adult American men, 8-year-old boys, basketball players, etc); (7b) is true if the cost of Sue's apartment exceeds the standard for some relevant set of apartments; and so forth.

(7) a. Fred is tall.
 b. Sue's apartment is expensive.

This view is made more plausible by the fact that the comparison class may be made overt via a *for*-phrase, as in (8):

(8) a. Fred is tall for a jockey.
 b. Sue's apartment is expensive for a place on this street.

A comparison class analysis can be implemented in a variety of ways. On the delineation-based approach developed by Klein [4], gradable adjectives are taken to introduce a partitioning of the comparison class into three sets, a positive extension, a negative extension and an extension gap. Sentences of the form in (7) and (8) are true if the subject falls in the positive extension (with respect to the comparison class), false if he or she falls into the negative extension, and undefined otherwise. Alternately, a comparison class analysis can be given a degree-based implementation (see e.g. [6]), according to which gradable adjectives are analyzed as expressing relations between individuals and degrees on the scale associated with some dimension of measurement. Sentences such as (7) and (8) are then true if the degree associated with the subject exceeds some standard degree (or set of degrees) derived from the comparison class.

Putting aside for now a discussion of the relative strengths and weaknesses of these two approaches, and provisionally adopting the latter option, examples such as (8a) can be analyzed as having the following truth conditions:

(9) $[\![\text{Fred is tall for a jockey}]\!] = 1$ iff $HEIGHT(fred) > R_{Std}$.

(10) $[\![\text{Fred is short for a jockey}]\!] = 1$ iff $HEIGHT(fred) < R_{Std}$,
 where $R_{Std} = median_{x:jockey(x)}(HEIGHT(x)) \pm n$, for some value n.

Here *HEIGHT* is a measure function that associates individuals with their degree of height. On this view, the standard of comparison R_{Std} is a range around the median value in the comparison class, whose width is determined by the value n. The effect is that 'tall for a jockey' receives the interpretation 'taller than most jockeys', and 'short for a jockey' is correspondingly 'shorter than most jockeys'; this seems to correctly capture our intuitions. Note that it is necessary to define the standard as a range rather than a single point to account for the acceptability of sentences such as *'Fred isn't tall for a jockey, but he isn't short either'*. In [9], I argue that the value n, and thus the width of the standard range, is actually a function of the degree of dispersion within the comparison class. As evidence, speakers tend to agree that the truth or falsity of a sentence such as (8b) is dependent not just on the average price for apartments on this street, but also on how prices are distributed. If most apartment prices are clustered closely around the average, then a price only moderately above that price would count as expensive; but if there is a broader range of variation in apartment prices, the standard for what counts as expensive (for an apartment on this street) shifts upward. For the sake of simplicity, I will not represent this here, and will just use the value *n*.

With the formulation in (9), we have a framework for expressing (if not an actual explanation for) both the context sensitivity of words such as *tall* as well as the existence of borderline cases. Context sensitivity derives from the possibility of selecting different sets for the comparison class; this may be specified via a *for* phrase, or may be left to the context. Borderline cases arise as a result of the underspecification of the value *n*.

Let us turn now to words of the *many* class. There have been a number of attempts to capture their interpretive variability (see especially [10, 11, 12, 13, 14, 15, 16, 17]), but as discussed further below, these typically capture some but not all of the readings available to them. That they, too, should be analyzed with reference to a comparison class is suggested by the fact that they also occur with *for*-phrases:

(11) a. Fred has <u>many</u> friends.
b. For a politician, Fred has <u>many</u> friends.

This can be captured with essentially the same approach as was applied to the case of *tall* above, simply by replacing the function *HEIGHT* by the function *NUMBER*, which associates sets of individuals with their cardinality:[3]

(12) ⟦For a politician, Fred has many friends⟧ = 1 iff

$$NUMBER(Fred\text{'}s\,friends) > R_{Std},$$

where $R_{Std} = median_{x:politician(x)}(NUMBER(x\text{'}s\,friends)) \pm n$.

It appears, then, that the comparison class analysis of gradable predicates meets the desired goal of being extendable to the case of words such as *many*. But now consider examples such as the following, where there is no subject whose characteristics may be considered in relation to a comparison class:

(13) <u>Few</u> students attended the lecture.

Here, it helps to consider a little more closely the interpretations that a sentence like this may receive. As has often been noted in the literature (see e.g. [12]), examples such as (13) tend to have multiple distinct readings, which can be brought out by different continuations, as in (14). One possible reading is that the number of students who attended the lecture is smaller than the number of members of some other group(s) who did so (cf. (14a)); a second is that the number of students attending the lecture is smaller than

[3] This is what we need for *many* and *few*, where the dimension in question is cardinality. In the case of *much* and *little*, which occur with non-countable nouns, we require a function that associates entities with their measure on some other dimension, e.g. volume, weight, etc.

the number of students who participated in other activities (cf. (14b)). (Below, I will consider some further possibilities beyond these two.)

(14) a. Few students attended the lecture, as compared to the number of faculty members there.

b. Few students attended the lecture, as compared to the number who went to the football match.

This suggests that the comparison class invoked is a set of alternatives to the domain of quantification, or to the sentential predicate. For example, the two readings of (13) noted above might be analyzed with reference to the following comparison classes:

(15) a. $C = \{\mathit{students}, \mathit{faculty\ members}, \mathit{townspeople}, \ldots\}$.

b. $C = \{\mathit{went\ to\ the\ lecture}, \mathit{went\ to\ the\ football\ match}, \mathit{studied}, \ldots\}$.

The proposal that some uses of words of the *many* class must be analyzed with reference to some such sets of alternatives is found for example in the work of Lappin [12]. My proposal here is that we can view this as a variant of a comparison class based analysis. But note that in doing so, we have extended the notion of a comparison class from a set of individuals (as needed for examples such as (7) and (11)) to include also sets of sets of individuals.

Note also that the approach proposed here is similar in some respects to those under which words of the *many* class are analyzed with respect to alternatives generated via focus (e.g. [14, 16]). The difference is that here, the set of alternatives (i.e. the comparison class) plays a direct role in setting the standard of comparison, rather than contributing to the logical form of the sentence, or having a purely pragmatic effect.

A further possibility is suggested by the following example:

(16) There are few cars in the parking lot today.

What comparison class might be involved here? Again, consider what would be conveyed by an utterance of (16). On its most natural interpretation, (16) does not seem to mean that the number of cars in the lot is smaller than the number of most other types of things in the lot, or smaller than the number of cars in most other places (though both of these readings are at least marginally possible, particularly with stress on *cars* and *parking lot*, respectively). Rather, it seems to me that the preferred interpretation is that the number of cars in the parking lot today is smaller than the number there at most other relevant times. This suggests that we have a comparison class over times.

The possibility that we have comparison classes of times in addition to comparison classes of individuals is supported by the fact that *for*-phrases can refer explicitly to times (17a). Furthermore, this possibility is not limited to words like *many* (17b), though this option seems to be more readily available with this class than with ordinary gradable adjectives.

(17) a. For a Sunday, there are few cars in the parking lot.

b. For a Sunday, the parking lot is crowded.

The comparison class approach developed up to this point can be extended to capture these cases as well, by assuming, as is standardly done, that the logical forms corresponding to sentences of natural language include an argument or interpretation parameter that ranges over times. The truth conditions of a relevant example can then be expressed as follows (where t is a variable that ranges over times, and t^* is the time of utterance):

(18) $[\![$For a Sunday, there are few cars in the parking lot$]\!]$ = 1 iff
$$NUMBER(\textit{cars in the lot at } t^*) < R_{Std},$$
where $R_{Std} = median_{t:Sunday(t)}(NUMBER(\textit{cars in the lot at } t)) \pm n.$

By this means, the comparison class analysis is able to capture a further interpretation available to words of the *many* class.

Consider now the following variant of an earlier example:

(19) <u>Few</u> students attended the lecture today.

A moment's introspection will show that this sentence also has a variety of readings. It can be interpreted as saying alternately: the number of students who came to the lecture was smaller than the numbers of students who participated in most alternative activities; smaller than the number of members of most other groups who came to the lecture; or smaller than the number who came to corresponding lectures at most other times. As discussed above, these readings could be modeled in terms of comparison classes consisting of alternatives to 'came to the lecture', alternatives to 'students', or alternatives to 'today'. But (19) also has another reading, perhaps the most salient one, according to which it expresses that the number of students who came to today's lecture is smaller than the speaker expected.

To capture this last reading, it is necessary to introduce some notion of alternate possible situations, or ways the world might have been. But this is of course nothing particularly unusual in semantic analysis. The necessary step is to move from an extensional to an intensional semantics, and introduce a variable or interpretation parameter that ranges over possible worlds. This is the same step that is typically taken, for example, in the analysis of modals such as *must* and *can*.

The truth conditions of (19) on this reading can then be formalized as follows (where w^0 is the actual world):

(20) $[\![$Few students attended the lecture$]\!]$ = 1 iff
$$NUMBER(\textit{students who attended the lecture in } w^0) < R_{Std},$$
where $R_{Std} = median_w(NUMBER(\textit{students who attended the lecture in } w)) \pm n.$

It seems, then, that in addition to comparison classes over individuals, sets of individuals and times, we also need comparison classes over worlds.

In fact, the approach represented in (20) is very close to a well known intensional analysis of *many* proposed by Fernando & Kamp [15]. Starting from the basic insight that n C's count as many iff the number of C's 'could well have been' less than n, these authors propose an interpretation for *many* that involves a probability function over worlds:

(21) n-IS-MANY$_x(\chi)$ iff "it is probable that" $(\exists_{<n}x(\chi))$
iff $p(\{w : |\{x : \chi \ in \ w\}| < n\}) > c$.

This same insight is captured in (20), which introduces the more specific requirement that the number must be lower in most relevant worlds. Thus the intensional analysis can be derived as a special case of a more general comparison class approach.

A further special case of comparison classes is illustrated by examples such as the following:

(22) Few faculty children attended the departmental picnic.

As has been pointed out numerous times in the literature, and discussed in particular by Partee [13], examples such as these are ambiguous between two distinct interpretations, which have come to be known as 'proportional' and 'cardinal'. On the proportional reading, the sentence means that a small proportion of all faculty children attended the picnic. On the cardinal reading, it means that the number of faculty children who attended was small in the absolute sense, regardless of what proportion of all faculty children this group makes up. These two interpretations are truth conditionally distinct: If it is the case that there are only a small number of faculty children, and they all attended the picnic, (22) is false on the proportional reading but true on the cardinal reading.

As evidence that this ambiguity is not just a dramatic instance of context sensitivity, note that the availability of one or the other of these interpretations is grammatically constrained. With a certain class of predicates that express more or less permanent properties (so-called individual-level predicates), only the proportional reading is available. For example, (23a) must mean that a small proportion of the lawyers I know are honest (the proportional reading); if I know only two or three lawyers and they are all honest, the sentence is false. Conversely, sentences with predicates of existence strongly favor the cardinal reading. Thus (23b) means that there are only a small number of two-headed snakes, not that of all the two-headed snakes, only a small proportion exist.

(23) a. Few lawyers I know are honest.

b. Few two-headed snakes exist.

The factors responsible for the pattern exemplified in (23a), whereby certain contexts allow only the proportional reading, have been the subject of considerable discussion in the linguistics literature (see e.g. [18]). My goal here is not to delve into this matter, but rather to show that the proportional interpretation of words of the *many* class can also be handled by a variant of the comparison class analysis. Specifically, for a sentence of the form in (24a), let the comparison class C be a set of subsets of the set denoted by the noun phrase sister of *many/few*, as in (24b):

(24) a. Many/few A B

b. $C = (A \cap B) \cup \{X : X \subset A \wedge X \cap (A \cap B) = \emptyset\}$

Then when the comparison class analysis developed above is implemented, *many A* will be interpreted as a large proportion of the *A*s, and *few A* will be interpreted as a small proportion of the *A*s. For example, (25) receives the interpretation that the set of honest lawyers I know is smaller than most other (non-overlapping) sets of lawyers I know:[4]

(25) $[\![$*Few lawyers I know are honest*$]\!]$ = 1 iff

$$NUMBER(honest\ lawyers\ I\ know) < R_{Std},$$

where $R_{Std} = median_{X \in C}(NUMBER(X)) \pm n$ and
$C = \{honest\ lawyers\ I\ know\} \cup \{X : X \subseteq \{dishonest\ lawyers\ I\ know\}\}$.

Interestingly, Klein [10] proposes an analysis very much along these lines as the basic semantics for *many* and *few*. I would suggest that such an analysis is too narrow, in that a comparison class of the specific form in (24) is not able to capture the time- and world-based readings of these words. But the preceding discussion shows that this interpretation can be derived as a special case of a more general comparison class analysis, which is also able to handle these other types of interpretations.

To summarize the main theme of this case study, investigating patterns in the interpretation of words of the *many* class provides further support for the role of comparison classes in the semantics of vague predicates. To be sure, we have seen that the notion of a comparison class must be expanded from how it is typically understood: Not only do we have comparison classes whose members range over individuals, but also those whose members range over sets of individuals, times and worlds. But with this broader view, we are able to capture a range of readings available to words such as *many*, in doing so subsuming several more specific analyses of their meaning under a single more general framework. While I do not claim to have provided evidence that all uses of *many* words (or vague predicates in general) should be analyzed with respect to a comparison class, a case can be made for the central importance of this construct.

3 A tale of *most Americans*

For the second case study, I consider a word that is not generally categorized as vague, namely the quantifier *most*. In this section, I will show that *most* does in fact exhibit a classic hallmark of vagueness, namely a fuzzy boundary, and that it furthermore gives us an insight into the properties that distinguish a vague expression from an otherwise equivalent precise expression.

If we were to consult an introductory semantics textbook (e.g. [20]), we might find that a sentence of the form in (26a) was given a logical form along the lines of (26b), where $|X|$ is the cardinality of the set X:

(26) a. <u>Most</u> Americans have broadband internet access.

b. $|Americans\ with\ broadband\ internet\ access| >$
$|Americans\ without\ broadband\ internet\ access|$

[4] The restriction to non-overlapping sets in (24) and (25) is necessary to set the cut-offs for *many* and *few* at the appropriate proportions. If we instead considered all subsets of the domain set A, then *many* would be restricted to proportions over 50%, i.e. equivalent to *most*, which is not consistent with speakers' intuitions. The present definition sets the cut-off lower (how low depends on *n*). See [21] for another case where truth conditions must be stated in terms of non-overlapping subsets of a set, this one involving the quantifier *most*.

With the semantics given here, *most* has a precise lower bound at 50%, such that any proportion greater than 50% counts as *most*. But this is at odds with the usual intuition of native speakers that a narrow majority is not sufficient to establish the truth of a sentence involving *most*. That is, speakers commonly judge that *most* is more than 'more than 50%'.

The following example provides nice illustration of the intuition that proportions just slightly over 50% do not support the use of *most*. It is a fact about the U.S. population that it has a slightly female skew (in 2008, the figures were 50.7% female versus 49.3% male). But the following sentence is not at all a felicitous way to describe this situation:

(27) <u>Most</u> Americans are female.

It is not entirely clear whether (27) should be considered false in the present situation, or true but in some way pragmatically deviant. But in either case there is nothing about the analysis represented in (26) that explains its oddness.

If *most* isn't lower bounded at 50%, what is its lower bound? The answer is that a precise value cannot be given: While the intuition seems to be that the minimum standard for *most* must be somewhat greater than 50%, there is no value *n* such that speakers judge any proportion over *n*% to be *most*, while proportions less than or equal to *n*% are not *most*. Thus *most* has a fuzzy boundary, just as is observed in the case of vague adjectives such as *tall (for a jockey)*. We can imagine this giving rise to a version of the Sorites paradox as well (if *n*% does not count as *most*, then *(n+0.01)*% also does not count as *most*; and so forth).

Interestingly, English has another way of expressing a proportion greater than 50%, namely the complex quantifier *more than half*. The difference is that in this case the lower bound is sharp, not fuzzy. For example, (28a) is true if just slighly more than 50% of (relevant) Americans have broadband, and (28b) is, in the actual situation, clearly true.

(28) a. <u>More than half</u> of Americans have broadband internet access.
b. <u>More than half</u> of Americans are female.

It is perhaps not entirely appropriate to characterize *more than half* as precise, in that it specifies a range of proportions rather than a single value. But the extent of that range can be specified precisely, and in this respect we have a clear contrast to *most*.

Thus in *most* and *more than half* we have a pair of expressions with essentially overlapping semantics, but which differ in that one is vague (in the sense of having a fuzzy boundary) while the other is precisely defined. I would like to propose that in examining the behavior of this pair, we have the opportunity to uncover other properties that might distinguish vague from non-vague expressions more generally. This is the goal of this section. (Note that at the end of the section, I will suggest that there is another class of pairs that exhibit the same contrast, and that point to a similar conclusion.)

In examining this topic, I draw on attested examples of the use of *most* and *more than half*, sourced from the Corpus of Contemporary American English (COCA) [19], a 410+ million word (approximately 20 million words per year for the years 1990–2010) corpus equally divided among spoken language, fiction, popular magazines, newspapers, and academic texts.

As of mid 2009, there were 432,830 occurrences of *most* and 4857 occurrences of *more than half* in COCA. In what follows, I focus on the use of *most* and *more than half* as quantifiers, as in examples (26) and (28). I put aside the so-called relative proportional use of *most* (e.g. *Anna read the most books*) and its use in adjectival superlatives (e.g. *Anna is the most diligent student*), as well as the adverbial use of *more than half* (e.g. *Our work is more than half finished*).

A look at the corpus data demonstrates deep distributional differences between *most* and *more than half*.

First, the corpus data substantiates the intuition that *more than half* is felicitously used for proportions just slightly greater than 50%, while *most* is not. Of course, in the majority of corpus examples involving the use of a quantifier, it is not possible to determine what the actual proportion is. However, there is one particular type of example, found quite commonly in the reporting of survey data, where a quantifier is used in conjunction with an exact percent; this usage gives us the sort of data we need to investigate the proportions which are described by the two quantifiers in question. In (29) and (30) we see typical examples of this sort for *most* and *more than half*, respectively. Note that the cited percentages are considerably higher in the former than the latter case.

(29) a. The survey showed that most students (81.5%) do not use websites for math-related assignments. (Education, 129(1), pp. 56–79, 2008)

b. Most Caucasian grandparents were married (67%), had attained an education level above high school (64%), and lived on an annual household income above $20,000 (74%). (Journal of Instructional Psychology, 24(2), p. 119, 1997)

c. Most respondents (92.6 percent) had completed high school. (Journal of Environmental Health, 69(5), pp. 26–30, 2006)

(30) a. More than half of respondents (55%) say that making money is more important now than it was five years ago. (Money, 21(3), p. 72, 1992)

b. More than half of the respondents (60%) earned Ph.D. degrees. (Physical Educator, 53(4), p. 170, 1996)

c. Booz Allen Hamilton, a technology consultancy, concluded in a study that more than half of new hires (51 per cent) were found through the Internet. (Christian Science Monitor, 2000)

This pattern is confirmed quantitatively. Figure 1 tallies all of these numerical examples found in the corpus for *more than half*, and the corresponding examples, involving the same nouns, for *most*. These data show that *more than half* is typically used for proportions between 50% and 65%; in over a third of cases, the percentage in question is less than 55%. By contrast, *most* is rarely used for proportions below 60%, and quite common up to over 90%.

What is interesting is that this difference in the range of proportions conveyed goes hand-in-hand with several other more fundamental differences in the distribution of *most* and *more than half*.

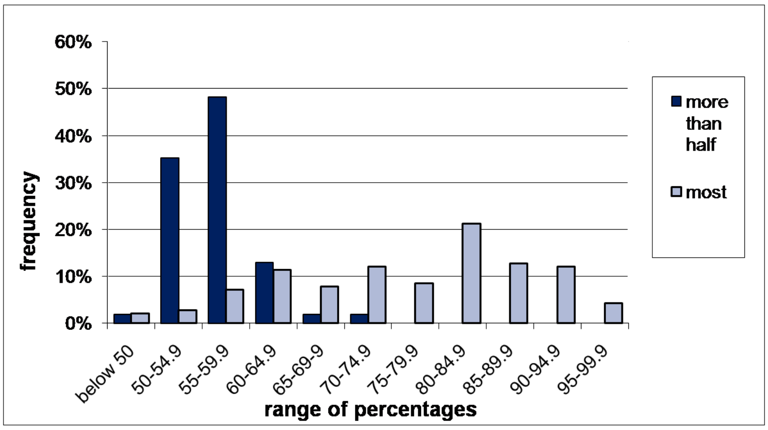

Figure 1. Percentages associated with *most* and *more than half*.

Plural nouns and generic interpretations As seen in the corpus examples in (31), *most* can readily be followed directly by a plural noun phrase, and in this case tends to take on a generic interpretation. (31a), for example, says something about the behavior of people in general; (31b) tells us something about teens in general.

(31) a. Most people follow the moral judgments of those around them. (Writer, 121(7), pp. 30–33, 2008)

b. Most teens want to fit in with their peers. (CNN YourHealth, 31/8/2002)

By contrast, *more than half* has a very different feel. As illustration, when *most* in natural examples of this sort is replaced by *more than half*, as in (32), the results are decidedly odd, and the generic flavor is lost entirely. Rather, to the extent that the resulting sentences are acceptable, they have what might be termed a 'survey results' interpretation; (32b), for example, seems to report on some sort of survey conducted among teens.

(32) a. ?More than half of people follow the moral judgments of those around them.

b. ?More than half of teens want to fit in with their peers.

Numerically, it is actually rare that *more than half* is followed directly by a plural noun, this occurring in only 8% of cases (68 out of 860 randomly selected tokens), compared to 57% of cases for *most* (516 out of 909 tokens). Instead, *more than half* usually (79% of cases) occurs in a partitive construction, followed by a definite description or pronoun:

(33) a. More than half of the tornadoes in November were produced on the 27th by a line of strong thunderstorms that stretched from northeast Oklahoma to northern Illinois. (Weatherwise, 44(2), p. 19, 1991)

b. Dumars scored 31 points and made more than half of his shots in a 102-94 victory over the Bucks. (New York Times, 17/2/1999)

c. In Illinois, the organ bank reports nearly 4,000 people on the organ transplant waiting list—more than half of them waiting for kidneys.
(Chicago Sun Times, 11/8/1999)

What ties together the examples in (33) is that the noun phrases following *more than half* (e.g. *the tornadoes in November*; *his [i.e. Dumars'] shots*) denote sets of individuals or entities localized in space and time. Furthermore, this generalization carries over to the minority of cases where *more than half* is followed directly by a plural noun. While (32b) is odd, (34) is perfectly felicitous; the difference is that while *teens* in (32b) is most readily interpreted as referring to teens in general, *teens surveyed* in (34) denotes a particular group of teens.

(34) More than half of teens surveyed said they are "not too careful or not at all careful" to protect their skin. (Todays Parent, 23(7), p. 154, 2006)

Supporting data The intuition that *more than half* (in contrast to *most*) has a 'survey results' interpretation is corroborated by examining the degree to which some sort of data is cited to support the use of the two quantifiers. To assess this, six plural nouns were selected that occurred multiple times in the corpus with *more than half*; for each, the proportion of cases was tallied in which there was mention of some sort of supporting data in the immediate context. For comparison, the same analysis was done for the first 100 occurrences of each of the the same six nouns with *most*.[5] As seen in Table 1, the difference is dramatic: In 80% of cases with *more than half*, there is some reference to supporting data; but this is true for just 19% of the cases with *most*. To put this differently, *more than half* is typically used when actual numerical data is being reported; *most* is not.

Table 1. Presence of supporting data.

	Cases with supporting data mentioned	
	More than half	Most
Americans	5/12	13/100
Men	4/6	5/100
Women	4/5	7/100
Students	5/5	36/100
Patients	5/5	39/100
Families	1/2	11/100
TOTAL	28/35	111/600
	80%	19%

Uncountable domains A further, and particularly intriguing, difference between *most* and *more than half* relates to the sort of domains over which they quantify. Consider corpus examples such as the following:

(35) a. But like most things, obesity is not spread equally across social classes.
(Mens Health, 23(7), p. 164, 2008)

[5] The restriction to 100 tokens per noun was necessitated by the very large number of occurrences of *most* with these nouns, e.g. over 2000 occurrences of *most Americans*.

b. Most beliefs, worries, and memories also operate outside awareness.
(Science News, 142(16), 1992)

c. But he had enough material on his truck to handle most problems.
(Contractor, 47(4), p. 30, 2000)

All of these sentences are entirely felicitous, but when one stops to think about it, there is something quite odd here. To take (35a) as an example, the 'things' whose distribution across social classes is mentioned do not seem to be the sort of entities that we could put on a list and count. The same could be said about 'beliefs, worries and memories' and the 'problems' faced by contractors. Yet quantification with *most* is nonetheless acceptable.

The same cannot be said about *more than half*. When *most* in the attested examples is replaced by *more than half*, as in (36), the result is once again peculiar:

(36) a. ?But like more than half of things, obesity is not spread equally across social classes.

b. ?More than half of beliefs, worries, and memories also operate outside awareness.

c. ?But he had enough material on his truck to handle more than half of problems.

The source of the oddness is the implication of enumerability that results: (36a), for example, seems to imply that we have in fact made a list of 'things', and gone down that list to count how many are spread equally across social classes. Thus *more than half* requires a domain that is enumerable; *most*, apparently, does not.

The examples given in (35) involve plural nouns, and the dimension in question is number. A similar contrast is observed in the mass domain, where another dimension of measurement is involved. The felicitous corpus example (37a) contrasts with the awkward (37b); the issue here is that racism is not something that can be quantitatively measured. On the other hand, *more than half* is acceptable in examples such as (38), the difference being that energy use can receive a numerical measure (expressed e.g. in kilowatt hours).

(37) a. But black activists acknowledge that most racism is not so blatant.
(Associated Press, 16/9/1991)

b. ?But black activists acknowledge that more than half of racism is not so blatant.

(38) More than half of home energy use goes to space heating and cooling.
(Popular Mechanics, 184(6), p. 79, 2007)

As the preceding discussion shows, *most* and *more than half*—despite their superficially similar meanings—are used very differently by speakers. There is a common thread that connects the three contrasts discussed above. All have something to do with number and countability or measurability. *More than half* is restricted to quantifying over domains that are enumerable (i.e. whose members can be individuated and counted) or otherwise measurable. It typically combines with noun phrases denoting sets that are spatially and temporally bounded, and thus countable or measurable, and its use is frequently supported by some sort of numerical data. These restrictions do not apply to *most*. To put it differently, the felicitous use of *more than half* rests in some way on the possibility, at least in principle, of assigning numerical measures to the entities or groups in question, while that of *most* does not.

Thus the distinction between a precise boundary (*more than half*) and a vague or fuzzy boundary (*most*) goes hand-in-hand with a second distinction, that between a requirement for numerical representability and the lack of such a requirement. I would like to propose that these two patterns are in fact related, and that the second provides a clue to the first.

In implementing this idea, I build on a proposal by Hackl [21], who argues that *most* and *more than half* have distinct logical forms that, while truth-conditionally equivalent, give rise to different verification strategies in sentence processing. In somewhat generalized form, Hackl's logical forms are the following:

(39) a. $[\![\textit{more than half}]\!](A)(B) = 1 \text{ iff } \mu_{DIM}(A \cap B) > \mu_{DIM}(A)/2.$

b. $[\![\textit{most}]\!](A)(B) = 1 \text{ iff } \mu_{DIM}(A \cap B) > \mu_{DIM}(A - B).$

Here μ_{DIM} is a measure function, that is, a function that associates an entity or set with a degree on the scale associated with some dimension of measurement DIM.

But while both the formulae in (39) are based on μ_{DIM}, there is an important difference in the structure of the measurement scale that is assumed in the two cases. In (39a), μ_{DIM} must yield a value that can be meaningfully divided by two. To borrow a concept from measurement theory (see especially [22]), this means that here μ_{DIM} encodes measurement at the ratio level, and thus that the underlying numerical scale is a ratio scale, featuring a fixed zero point and a fixed unit of measure. Examples of ratio scales include weight measured in pounds or kilograms, length measured in meters or feet, or set cardinality represented via the natural numbers, all familiar cases of quantitative measurement. Thus by virtue of its logical form, the felicitous use of *more than half* presupposes measurement at the ratio level. From this follows the requirement that the domain of quantification be enumerable or otherwise quantitatively measurable.

The logical form for *most* in (39b), on the other hand, places much weaker requirements on the structure of the scale that serves as the range of μ_{DIM}. For example, suppose that we start with a simple qualitative ordering of individuals or sets, one which could be represented numerically by assigning whole numbers to those individuals/sets in an order-preserving manner (similar to assigning place numbers to finishers in a race, without recording their actual finishing times). In measurement-theoretic terms, this is ordinal level measurement. An ordinal-level measure function μ_{DIM} is sufficient to support the evaluation of the formula in (39b), which will come out as true if the rank order assigned to $A \cap B$ precedes or exceeds that assigned to $A - B$. To return to an earlier example, even if the problems faced by contractors are not enumerable or countable in a traditional sense, (35c) could be evaluated as true if the problems the individual in question could handle were ranked above those he couldn't with respect to a qualitative (i.e. ordinal) ranking of sets or entities. Put differently, *most*, unlike *more than half*, does not presuppose something as informative as ratio-level measurement, and thus may be felicitously used in situations where *more than half* is not supported.

Furthermore, even ordinal level measurement is not necessary. Imagine that we start with a 'tolerant' ordering of individuals based on a 'significantly greater than' relationship. By this, I mean an ordering where the difference relationship $\succ$ is transitive but the indifference relationship $\sim$ is not, such that we might have $x \sim y$ and $y \sim z$ but $x \succ z$. In formal terms, such an ordering relationship is a semi-order (see [23, 7]). A tolerant ordering can be given a numerical representation via a measure function μ_{DIM} that assigns each set or entity a range of values rather than a single point, with the 'greater than' relationship $>$ holding between two degrees (i.e. ranges) only if there is no overlap between them. Such a measure function is sufficient to support the logical form in (39b), which will hold true if $A \cap B$ has a 'significantly' greater measure with respect to the relevant dimension than $A - B$. But as discussed above, this is precisely the situation when *most* is used, namely when the number or measure of As that are B is significantly greater than the number/measure of As that are not B. That is, the absence of a sharp lower bound for *most* can be related to the possibility for its interpretation relative to a tolerant ordering.

There is in fact considerable support for the psychological reality of the encoding of cardinalities and other measures as ranges of a sort, whose difference is a function of their degree of (non-)overlap. Findings from the psychology of number cognition (see especially [24]) demonstrate that in addition to the ability to represent precise number, humans (and other animals) possess a separate and more primitive approximate number system (ANS). In the ANS, numerical magnitudes are thought to be encoded as ranges of activation on the equivalent of a mental number line. The representations generated by the ANS support simple arithmetic and, importantly for the present case, the comparison of quantities. In this function, its operation is ratio-dependent: Two numbers can be reliably distinguished via the ANS only if the ratio between them is sufficiently large, that is, if the overlap between their ranges is sufficiently small (cf. the above discussion of tolerant orderings).

A connection between the ANS and the semantics of *most* has been made previously by Pietroski et al. [25], who show that verification of sentences with *most* exhibits a characteristic pattern of ratio dependence, and Halberda et al. [26], who demonstrate that young children who lack full numerical competence may still be able to evaluate *most* approximately. The present discussion suggests that it is the logical form of *most* that allows its evaluation via the ANS. Furthermore, since the ANS is the more basic of humans' two systems for processing number (emerging evolutionarily and developmentally before the ability to represent precise number), it is reasonable to assume that it serves as the default option in the interpretation of *most*. In this way we can account for the tendency for *most* to be used in an imprecise manner even when precise number is available.

The picture that emerges is the following. The precise *more than half* encodes a comparison of points on a ratio scale (an aspect of its meaning that also restricts its use to cases where ratio-level measurement is possible). The vague *most*, on the other hand, assumes a less informative scale structure, perhaps one based on a tolerant ordering (which accounts for both its broader distribution relative to *more than half*, as well as its essentially fuzzy lower bound). In this case, then, precision versus vagueness corresponds to a difference in what level of measurement, and thus what sort of scale structure, is involved.

One wonders whether this distinction might characterize precision versus vagueness more generally. Here, it is helpful to consider another sort of example. The contrast in interpretation between *more than half* and *most* has a parallel in the contrast between so-called explicit and implicit comparatives, as exemplified in (40):

(40) a. John is taller than Fred. **Explicit**

b. John is tall compared to Fred. **Implicit**

As discussed in particular by Kennedy [8], (40a) but not (40b) supports crisp comparisons. That is, if John is only very slightly (say, 1 cm) taller than Fred, the explicit comparative (40a) is true, but the implicit comparative (40b) is false. This recalls the acceptability of *more than half*, but not *most*, for proportions just slightly greater than 50%. Van Rooij [7] proposes that while explicit comparatives such as (40a) can be modeled via weak orders (which include ratio-level scales as a special case), implicit comparatives such as (40b) should be modeled via semi-orders, just as is proposed above for *most*. Thus once again, the vague member of the pair is associated with a tolerant ordering structure.

Importantly, the claim here is not that dimensions such as height, number and so forth <u>cannot</u> be measured at the ratio level. This is obviously not the case. Rather, the idea is that these dimensions can also be measured at a less informative level, specifically relative to a semi-ordered scale structure of the type discussed above, and it is this level of measurement that is assumed by vague expressions that reference these dimensions (including the quantifier *most*, and perhaps gradable adjectives in implicit comparatives).

If such a generalization can be established more generally, the implication would be that the formal modeling of vague expressions should make reference to other types of degree structures than the totally ordered scales often assumed in degree semantics. I leave this more general possibility as a topic for further investigation.

4 Concluding remarks

The goal of this paper was to take a close look at linguistic data relating to vagueness in the expression of quantity, with a view to exploring what this less studied area might be able to tell us about the nature and proper treatment of vagueness more generally. Most of the discussion above has focused on particular lexical items in English: words of the *many* class in the first case study, the quantifier *most* in the second. But both of these case studies suggest some broader implications for the formal analysis of vagueness, the first reinforcing the relevance of comparison classes in the interpretation of vague expressions, the second raising the possibility that vagueness is, in at least some cases, associated with a tolerant scale structure.

In closing, I would like to suggest that similar insights may be available through the in depth investigation of vagueness as it occurs in other domains of natural language, including in particular vague nouns, as well expressions of temporal and spatial relationships such as prepositions.

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