
Fuzzy Logic and Higher-Order Vagueness

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The major reason given in the philosophical literature for dissatisfaction with theories of vagueness based on fuzzy logic is that such theories give rise to a problem of *higher-order vagueness* or *artificial precision*.² In this paper I first outline the problem and survey suggested solutions: fuzzy epistemicism; measuring truth on an ordinal scale; logic as modelling; fuzzy metalanguages; blurry sets; and fuzzy plurivaluationism. I then argue that in order to decide upon a solution, we need to understand the true nature and source of the problem. Two possible sources are discussed: the problem stems from the very nature of vagueness—from the defining features of vague predicates; or the problem stems from the way in which the meanings of predicates are determined—by the usage of speakers together with facts about their environment and so on. I argue that the latter is the true source of the problem, and on this basis that fuzzy plurivaluationism is the correct solution.

1 The problem of artificial precision

Each of the following passages—from Haack, Urquhart and Keefe, respectively—gives a nice statement of the problem of artificial precision:³

[Fuzzy logic] imposes artificial precision. . . [T]hough one is not obliged to require that a predicate either definitely applies or definitely does not apply, one *is* obliged to require that a predicate definitely applies to such-and-such, rather than to such-and-such other, degree (e.g. that a man 5 ft 10 in tall belongs to *tall* to degree 0.6 rather than 0.5). [11, p. 443]

One immediate objection which presents itself to [the fuzzy] line of approach is the extremely artificial nature of the attaching of precise numerical values to sentences like ‘73 is a large number’ or ‘Picasso’s *Guernica* is beautiful’. In fact, it seems plausible to say that the nature of vague predicates precludes attaching precise numerical values just as much as it precludes attaching precise classical truth values. [45, p. 108]

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²The former term is used more widely in the literature, but the same term is also applied to problems which I regard as being rather different in character from the problem for the fuzzy view under discussion here; I shall therefore use the latter term in this paper.

³For further statements of the problem see Copeland [5, pp. 521–2], Goguen [9, p. 332] [10, p. 54], Lakoff [22, pp. 462, 481], Machina [26, p. 187], Rolf [30, pp. 223–4], Schwartz [33, p. 46], Tye [43, p. 11], Williamson [52, pp. 127–8] and Keefe [19, pp. 113–4].

[T]he degree theorist's assignments impose precision in a form that is just as unacceptable as a classical true/false assignment. In so far as a degree theory avoids determinacy over whether a is F , the objection here is that it does so by enforcing determinacy over the *degree* to which a is F . All predications of "is red" will receive a unique, exact value, but it seems inappropriate to associate our vague predicate "red" with any particular exact function from objects to degrees of truth. For a start, what could determine which is the correct function, settling that my coat is red to degree 0.322 rather than 0.321? [18, p. 571]

In a nutshell, the problem for the fuzzy approach is this: it is artificial/implausible/inappropriate to associate each vague *predicate* in natural language with a particular function which assigns one particular fuzzy truth value (i.e. real number between 0 and 1) to each object (the object's degree of possession of the property picked out by that predicate); likewise, it is artificial/implausible/inappropriate to associate each vague *sentence* in natural language with a particular fuzzy truth value (the sentence's degree of truth).

Note that the problem is *not* a problem for pure (mathematical) fuzzy logic: that is, for fuzzy logic *qua* branch of mathematics. It is a problem for theories of vagueness based on fuzzy logic (or more precisely, on fuzzy model theory). Let me explain. Classical logic countenances only two truth values, and classical models are total: every closed well-formed formula (wff) is assigned one or other of these values on each model. This does *not* make it correct, however, to say that it is a commitment of classical logic (model theory) that every statement is either true or false. Such a commitment comes into play only when one seeks to use classical logic to shed light on the semantics of some language (e.g. natural language, or the language of mathematics). It is thus a commitment not of pure classical logic (model theory)—considered as a branch of mathematics—but of model-theoretic semantics (MTS). One who wishes to pursue MTS (in relation to some language), and wishes to make use only of classical model theory, *is* committed to the claim that every statement (made in the language in question) is either true or false.

We therefore need to be careful to distinguish between pure logic (model theory) and MTS. Now note that in order to use classical model theory for the purposes of MTS, one needs to make use of a new notion that does not figure in pure model theory. This is the notion of the *intended model* (or some other notion which plays a similar role—see on). Pure model theory tells us only that a wff is true on this model and false on that one (etc.). In order to obtain a notion of truth *simpliciter*—which is required in MTS—we need a designated model, so that we can say that truth *simpliciter* is truth on the designated model. This is the role played by the intended model.⁴ In MTS, my statement 'Bob is tall' is taken to express some wff. This wff is true on some models and false on others. What we want to know, however, is whether it is true *simpliciter*. It is so if it is true on the model which assigns as referent to the singular term which I expressed as 'Bob' the very guy I was talking about when I said 'Bob' and which assigns as extension to the predicate which I expressed as 'tall' the set of things which have the property I was talking about when I said 'is tall'—that is, the intended model.

⁴Cf. Lepore [23, p. 181]: "A theory of meaning... is concerned only with a single interpretation of a language, the correct or intended one: so its fundamental notion is that of meaning or truth—simpliciter."

Weiner [50, p. 165] sums up the MTS perspective very nicely: “Natural language (or at least a cleaned up version of a fragment of natural language) is to be understood as a formal language along with an intended interpretation. Truth, for sentences of natural language, is to be understood as truth under the intended interpretation.”

Of course, those working in MTS typically do *not* take classical logic and model theory as their starting point. Usually they take some intensional higher-order logic, because they think that this provides the best way of accounting for certain features of natural language. In this paper, we are concerned with those who take fuzzy logic and model theory as their starting point—because they think that this provides the best way of accounting for the *vagueness* of natural language. Now consider the following comment from Hájek [13, p. 368]. The context is a discussion of Shapiro [34]. After mentioning the objection to fuzzy logics from artificial precision and noting Shapiro’s response (a version of the logic as modelling approach to be discussed below), Hájek adds (in parentheses): “Let us comment that mathematical fuzzy logic concerns the possibility of sound inference, surely not techniques of ascribing concrete truth degrees to concrete propositions.” Quite so: the problem of artificial precision is *not* a problem for mathematical fuzzy logic. But it *is* a problem for fuzzy theories of vagueness—for fuzzy logic-based MTS. Such theories *are* concerned with ascribing concrete truth degrees to concrete propositions. The simplest way for them to proceed is to adopt the idea of an intended model. A proposition will be assigned different degrees of truth on different models; the concrete truth degree of a concrete proposition is the degree of truth assigned to it on the *intended* model. In what follows, when I speak of the ‘(basic) fuzzy theory of vagueness’, what I mean is that version of MTS for natural language which says that a vague discourse is to be modelled as a collection of wffs together with a unique intended fuzzy model. It is *this* view—which is committed to the idea that each vague sentence of natural language has a unique fuzzy truth value, namely its truth value on the intended model—that is threatened by the artificial precision problem.

Summing up: pure/mathematical fuzzy logic does not face the problem of artificial precision, because all models are equal in its eyes. Precisely for this reason, however, it does not (on its own) provide a *theory of vagueness* in the sense of a theory which tells us about the (actual) meaning and truth (simpliciter) of claims made in vague natural language in a way which respects our pre-theoretic intuitions about these matters (e.g. that ‘Shaquille O’Neal is tall’ is true to degree 1) and reveals what is wrong with sorites reasoning. In order to get such a theory, we need to add to pure fuzzy logic a notion of some model(s) being special or designated in some way: for any vague discourse, amongst all its *possible* models, there are only *some* that are relevant to questions of the (actual) meaning and truth (simpliciter) of statements in the discourse. The simplest approach is to say that there is just one such model: the ‘intended model’. This is what I call the ‘basic fuzzy theory of vagueness’. It immediately runs into the artificial precision problem. In the next section we examine possible responses to the problem. Some (e.g. fuzzy epistemicism) stick with the basic fuzzy theory of vagueness; some (e.g. fuzzy plurivaluationism) stick with the underlying pure fuzzy model theory, but abandon the notion of a *unique* designated model in favour of a class of such models; some (e.g. blurry sets) abandon the underlying fuzzy model theory in favour of a different kind of pure model theory (while retaining the idea of a unique intended model).

2 Proposed solutions

This section presents six responses to the problem of artificial precision that have been proposed in the literature.

2.1 Fuzzy epistemicism

The fuzzy epistemicist responds to the problem by saying that each vague sentence (e.g. ‘Bill is tall’) does indeed have a unique fuzzy truth value (e.g. 0.4), but we do not (cannot) know what it is.⁵ Our ignorance explains our unease about assigning this or that particular value to a given sentence. Hence the fuzzy epistemicist explains the phenomena behind the objection to the fuzzy view, while defusing the objection: the basic fuzzy theory of vagueness can be retained, complete with the implication that each vague sentence has a unique fuzzy truth value. (Compare the way in which the epistemic account of vagueness would—if it worked—allow us to retain classical MTS for vague natural language.)⁶

2.2 Measuring truth on an ordinal scale

The next response holds that when we assign fuzzy truth values to sentences, the only thing that is meaningful about the assignments is the relative *ordering* of the values assigned. As Goguen puts it:⁷

We certainly do not want to claim there is some *absolute* [fuzzy] set representing ‘short’. . . . Probably we should not expect particular numerical values of shortness to be meaningful (except 0 and 1), but rather their *ordering*. . . degree of membership may be measured by an *ordinal scale*. [9, pp. 331–2]

On this view, while we may assign ‘Bill is tall’ degree of truth 0.5 and ‘Ben is tall’ degree of truth 0.6, these are *not* the uniquely correct value assignments: we could just as well assign any other values where the first is less than the second. We can think of this view as follows. Instead of a unique intended fuzzy model, we have a class of acceptable models, closed under a certain sort of transformation of the truth values: any model which can be obtained from an acceptable model by applying an order-preserving (and endpoint-fixing) transformation to the real interval $[0,1]$ is equally acceptable. On this view, then, a vague predicate is *not* associated with a *unique* function which assigns real numbers between 0 and 1 to objects, and a vague sentence is *not* assigned a *unique* fuzzy truth value—and so the objection from artificial precision is avoided.

⁵Fuzzy epistemicism is mentioned by Copeland [5, p. 522] and developed in more detail by MacFarlane [25]. Machina [26, p. 187, n.8] could also be interpreted as hinting at such a view when he writes of “difficulties about how to assign degrees of truth to propositions”; Keefe [18, p. 571] [19, p. 115] interprets him in this way and criticises his view on this basis.

⁶Advocates of epistemic theories of vagueness include Cargile [3], Campbell [2], Sorensen [39, ch. 6] [40], Williamson [51] [52, ch’s 7–8] and Horwich [16].

⁷See also Sanford [32, p. 29], Machina [26, p. 188], Goguen [10, p. 59], Hájek [12, pp. 162–3], Weatherston [49] and Hyde [17, p. 207].

2.3 Logic as modelling

The most detailed version of this response is Cook's [4].⁸ Cook distinguishes *descriptions* from *models*:⁹ while descriptions may simplify and approximate, the key feature of models is that some aspects of them are not even *intended* to represent—not even in an approximate or simplified way—an aspect of the thing modelled. Such features of a model are called *artefacts*. Cook gives an example: “a model ship might have, deep in its interior, supports situated where the engine room is located in the actual ship. Although the supports do not represent anything real on the actual ship, they are not necessarily useless or eliminable as a result, since they might be crucial to the structural integrity of the model” [4, p. 236]. Cook then argues that the objection from artificial precision depends on viewing the fuzzy theory of vagueness as providing a *description* of the semantics of vague language. If, on the other hand, we view it as providing a *model*—and if, more specifically, we view the particularity of the fuzzy values assigned (i.e. the fact that one particular value—not any other value—gets assigned) as an *artefact* of the model—then the problem dissolves. The objection to the fuzzy approach turns on the assignment of a unique fuzzy truth value to each vague sentence; if the uniqueness of the assignment is not an aspect of the model which is supposed to correspond to anything about vague language—if it is merely an artefact of the model—then the objection misses the mark.

2.4 Fuzzy metalanguages

The next response is expressed as follows by Williamson:¹⁰

If a vague language requires a continuum-valued semantics, that should apply in particular to a vague meta-language. The vague meta-language will in turn have a vague meta-meta-language, with a continuum-valued semantics, and so on all the way up the hierarchy of meta-languages. [52, p. 128]

The idea is first to present a semantics for vague language which assigns sentences real numbers as truth values, and then say that the metalanguage in which these assignments were made is itself subject to a semantics of the same sort. So on this view, statements of the form ‘The degree of truth of “Bob is tall” is 0.4’ need not be simply true or false: they may themselves have intermediate degrees of truth. Thus, rather than exactly one sentence of the form ‘The degree of truth of “Bob is tall” is x ’ being true and the others false, many of them might be true to various degrees. Hence there is a sense in which sentences in natural language which predicate vague properties of objects are *not* each assigned just one particular fuzzy truth value—and so the objection from artificial precision is avoided.

⁸For other versions of the response, see Edgington [6, pp. 297, 308–9] and Shapiro [34, ch. 2, §1].

⁹Note that ‘model’ here is used in the sense it has in, for example, ‘model aeroplane’ and ‘Bohr’s model of the atom’. We have hitherto been using the term in a different sense—the one it has in, for example, ‘model theory’ and ‘model of a first-order language’. In order to avoid confusion, we shall use ‘model’ in Cook’s sense in §2.3 and in the part of §3 which discusses the logic as modelling view; in these parts of the paper, we shall use ‘structure’ in place of ‘model’ in the other sense. Elsewhere in the paper, we use ‘model’ in the sense of ‘model theory’.

¹⁰For discussions or mentions of similar or related views see Cook [4], Edgington [6, pp. 297, 310], Field [8, p. 227], Horgan [14, p. 160], Keefe [19, pp. 117–21], McGee and McLaughlin [27, p. 238], Rolf [30, p. 222], Sainsbury [31, p. 260], Tye [41, p. 551] [42, p. 287] [43, p. 16] [44, pp. 219–20], Varzi [47, pp. 7–9] and Williamson [52, p. 130] [53, p. 695].

2.5 Blurry sets

This response—due to Smith [35]—involves a system in which the truth values, rather than being reals in $[0,1]$, are *degree functions*: functions from $[0,1]^*$ to $[0,1]$.¹¹ Suppose $f : [0,1]^* \rightarrow [0,1]$ is the truth value of ‘Bob is tall’ (B). The idea is that the value which f assigns to the empty sequence—say, 0.5—is a *first approximation* to Bob’s degree of tallness/the degree of truth of (B). The values assigned by f to sequences of length 1 then play two roles. First, they rate possible first approximations. The higher the value assigned to $\langle x \rangle$, the better x is as a first approximation to Bob’s degree of tallness/the degree of truth of (B). If $f(\langle 0.3 \rangle) = 0.5$, then we say that it is 0.5 true that Bob is tall to degree 0.3; if $f(\langle 0.5 \rangle) = 0.7$, then we say that it is 0.7 true that Bob is tall to degree 0.5; and so on. Second, the assignments to sequences of length 1 jointly constitute a second level of approximation to Bob’s degree of tallness/the degree of truth of (B). Together, these assignments determine a function $f_{\langle \rangle} : [0,1] \rightarrow [0,1]$. We regard this as encoding a density function over $[0,1]$, and we require that its centre of mass is at $f(\langle \rangle)$ (Figure 1). The same thing happens again when we move to the values assigned

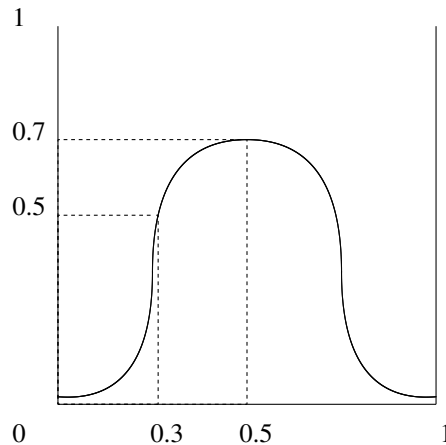


Figure 1. Bob’s degree of tallness: second approximation.

to sequences of length 2: these values play two roles. First, they rate possible ratings of first approximations. The higher the value assigned to $\langle x, y \rangle$, the better y is as a rating of x as a first approximation to Bob’s degree of tallness/the degree of truth of (B). If $f(\langle 0.5, 0.7 \rangle) = 0.8$, then we say that it is 0.8 true that it is 0.7 true that Bob is tall to degree 0.5; if $f(\langle 0.4, 0.5 \rangle) = 0.3$, then we say that it is 0.3 true that it is 0.5 true that Bob is tall to degree 0.4; and so on. Second, the assignments to sequences of length 2 jointly constitute a third level of approximation to Bob’s degree of tallness/the degree of truth of (B). Together, the assignments made by f to sequences $\langle a, x \rangle$ of length 2 whose

¹¹ $[0,1]^*$ is the set of words on the alphabet $[0,1]$; that is, the set of all finite sequences of elements of $[0,1]$, including the empty sequence $\langle \rangle$.

first member is a determine a function $f_{(a)} : [0, 1] \rightarrow [0, 1]$. This can be seen as encoding a density function, and we require that its centre of mass is at $f(\langle a \rangle)$ (Figure 2). And so

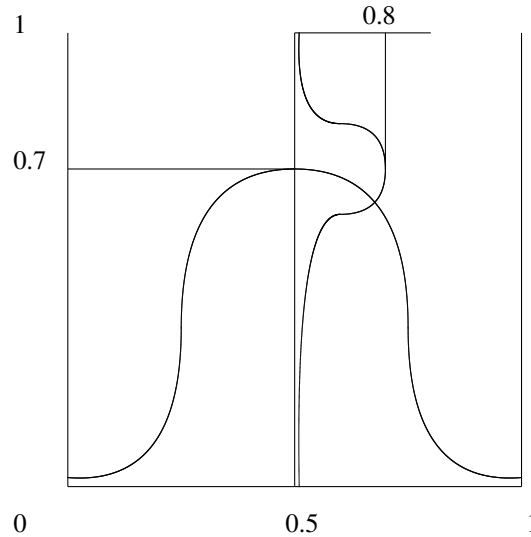


Figure 2. Bob's degree of tallness: third approximation (part view).

the story goes, ad infinitum. Figuratively, we can picture a degree (of truth or property-possession) as a region of varying shades of grey spread between 0 and 1 on the real line. If you focus on any point in this region, you see that what appeared to be a point of a particular shade of grey is in fact just the centre of a further such grey region. The same thing happens if you focus on a point in this further region, and so on. The region is blurry all the way down: no matter how much you increase the magnification, it will not come into sharp focus.

On this view, as on the fuzzy metalanguage view, statements of the form 'The degree of truth of "Bob is tall" is 0.4' need not be simply true or false: they may themselves have intermediate degrees of truth. So rather than exactly one sentence of the form 'The degree of truth of "Bob is tall" is x ' being true and the others false, many of them might be true to various degrees. Thus there is a sense in which sentences in natural language which predicate vague properties of objects are *not* each assigned just one particular fuzzy truth value—and so the objection from artificial precision is avoided.

Note that on both the fuzzy metalanguage and blurry set views, we have a hierarchy of statements, none of which tells us the full and final story of the degree of truth of 'Bob is tall'. However there is a crucial difference between the two views. The fuzzy metalanguage view involves a hierarchy of *assignments* of *simple* truth values. The blurry set view involves a *single* assignment of a *complex* truth value—a truth value which has an *internal* hierarchical structure. On the blurry set view, each vague sentence is assigned a unique degree function as its truth value, and these assignments can be described in a classical, precise metalanguage.

2.6 Fuzzy plurivaluationism

In order to explain this response—due to Smith [37]—we must first explain *classical* plurivaluationism. Recall the classical MTS picture outlined at the end of §1: a discourse in natural language is to be modelled as a bunch of wffs together with a unique designated classical model (the ‘intended model’); a statement (in a discourse) is true (simpliciter) if it is true relative to the model that is designated (for that discourse). The classical plurivaluationist accepts much of this picture. In particular, she countenances *only* classical models. However she denies that there is always a unique intended model of a discourse. As mentioned in §1, MTS requires some notion additional to those found in pure model theory: for we wish to be able to speak of statements being true or false simpliciter, not merely true on this model and false on that one (with no model being more relevant than any other). One option here is to pick *one* model as uniquely relevant (i.e. the ‘intended model’). The plurivaluationist takes a less extreme course, holding instead that sometimes (i.e. for some discourses) there are many acceptable models, none of which is uniquely relevant when it comes to questions of the (actual) meaning and truth (simpliciter) of utterances in the discourse. On this view, when I utter ‘Bob is tall’, I say *many things* at once: one claim for each acceptable model. Thus we have semantic *indeterminacy*—or equally, semantic *plurality*. However, if *all* the claims I make are true (or false)—that is, if the wff I express is true (or false) on every acceptable model—then we can pretend (talk as if) I make only one claim, which is true (or false). Figuratively, think of a shotgun fired (once) at a target: many pellets are expelled, not just one bullet; but if *all* the pellets go through the bullseye, then we can harmlessly *talk as if* there was just one bullet, which went through.

The plurivaluationist view of vagueness—the view that what is happening when we make vague statements is that we are speaking relative to multiple classical models (the ‘acceptable models’) as opposed to a single such model (the ‘intended model’)—is expressed by, amongst others, Lewis and Varzi:

I regard vagueness as semantic indecision: where we speak vaguely, we have not troubled to settle which of some range of precise meanings our words are meant to express. [24, p. 244, n. 32]

Broadly speaking, [plurivaluationism]¹² tells us two things. The first is that the semantics of our language is not fully determinate, and that statements in this language are open to a variety of interpretations each of which is compatible with our ordinary linguistic practices. The second thing is that when the multiplicity of interpretations turns out to be irrelevant, we should ignore it. If what we say is true under all the admissible interpretations of our words, then there is no need to bother being more precise. [48, p. 14]

Note that the classical plurivaluationist view is quite different from a second view, both of which have unfortunately been conflated in the literature under the name ‘supervaluationism’. Plurivaluationism trades only in *classical* models; instead of supposing that for each discourse there is *one* model relevant to questions concerning the (actual) meaning and truth (simpliciter) of utterances in the discourse (the ‘intended model’), it allows that

¹²Varzi actually uses the term ‘supervaluationism’ here; see below for discussion.

classical. It stands to the basic fuzzy theory of vagueness—on which a vague discourse is associated with a unique intended fuzzy model—in just the way that classical pluralism stands to the original classical MTS picture. That is, everything about the original view is retained (so in the classical case, only standard classical models are countenanced, and in the fuzzy case, only standard fuzzy models are countenanced), *except* the idea that each discourse is associated with a unique intended model. The latter idea is replaced with the thought that each discourse is associated with multiple acceptable models.

The situation is summarised in Figure 4. Recall that a system of model-theoretic semantics comprises two ingredients: an underlying system of (pure, mathematical) model theory; and a notion which plays the role of picking out, from amongst all the models countenanced by the underlying model theory, the model(s) relevant to questions of the (actual) meaning and truth (simpliciter) of utterances in a discourse. The table shows two possible choices of first ingredient down the left, and two possible choices of second ingredient across the top. Each of the four possible pairs of choices determines a theory of vagueness: these four theories are shown in the table’s body.

ingredient 2 → ↓ ingredient 1	unique intended interpretation	multiple acceptable interpretations
classical model theory	classical MTS	classical pluralism
fuzzy model theory	basic fuzzy theory of vagueness	fuzzy pluralism

Figure 4. Four theories of vagueness.

The upshot of fuzzy pluralism is that there is *not* one uniquely correct assignment of truth value to ‘Bob is tall’. There are multiple, equally-correct assignments: one in each acceptable model. Thus, the objection from artificial precision is avoided.

3 Choosing a solution

We now have six solutions to the artificial precision problem on the table. Which (if any) is the right one? In this section I begin the process of answering this question by arguing that two of the solutions—the logic as modelling and fuzzy metalanguage views—can be ruled out on methodological grounds.

Consider first the fuzzy metalanguage approach. In light of the distinction between pure fuzzy logic and fuzzy MTS, we can distinguish two different ways of understanding this approach—two different ways of fuzzifying the basic fuzzy theory of vagueness: fuzzify pure fuzzy model theory itself; or fuzzify only fuzzy MTS.¹⁵ The first of these

¹⁵To avoid possible misunderstanding: what I mean here is that there are two ways of spelling out the fuzzy metalanguage approach to the problem of artificial precision sketched in §2.4. I am not saying that these are the only two views that could be described using the term ‘fuzzy metalanguage’; for example, Běhounek [1] presents a distinct view which some might wish to describe in such terms. Note that Běhounek’s view is not intended as a solution to the problem of artificial precision, as presented in §1. (Recall n.2: the term

options involves the idea that the language in which pure fuzzy model theory is presented is vague: the correct semantics for *this* language is fuzzy MTS. The problem with any such approach is well summed-up by Goguen:

Our models are typical purely exact constructions, and we use ordinary exact logic and set theory freely in their development . . . It is hard to see how we can study our subject at all rigorously without such assumptions. [9, p. 327]

We understand pure fuzzy model theory as standard mathematics, presented in the usual precise language of mathematics. If someone were to say at the end of presenting (pure) fuzzy model theory that the language in which he made his presentation was governed by a semantics involving the notions he had just presented—rather than by the standard classical semantics for mathematical language—then we would have to conclude that we had not really understood his presentation after all.¹⁶

The second option avoids this problem: on this option, the language in which pure fuzzy model theory is presented is the ordinary precise language of mathematics. What *is* vague, on this second option, is that part of the basic fuzzy theory of vagueness which goes beyond pure fuzzy model theory: the notion of the ‘intended model’. Sticking to the idea of analyzing vagueness in fuzzy terms, this amounts to the idea that in place of the unique intended fuzzy model posited by the basic fuzzy view, we have a *fuzzy set* of acceptable models: acceptability of a model becomes a graded notion. This view is subject to an immediate difficulty. What are we to say of an utterance which is true to degree (say) 0.7 relative to a model which is acceptable to degree (say) 0.5? That is, how does degree of truth on a model combine with degree of acceptability of that model to yield an absolute (i.e. not model-relative) assessment of an utterance? Perhaps there is something plausible that can be said here (as far as I am aware, no such view has been developed in the literature). In any case, a further problem remains. The view under discussion generalizes fuzzy plurivaluationism: where the latter posits a (crisp) set of acceptable models, the former posits a fuzzy set. As we shall see, however, fuzzy plurivaluationism *solves* the artificial precision problem. Hence, by Ockham’s razor, the view under consideration is to be rejected: its added complexity is unnecessary.¹⁷

Consider next the logic as modelling view. Cook sums up the position as follows:

In essence, the idea is to treat the problematic parts of the degree-theoretic picture, namely the assignment of particular real numbers to sentences, as mere artefacts. . . . If the problematic parts of the account are not intended actually to describe anything occurring in the phenomenon in the first place, then they certainly cannot be *misdescribing*. [4, p. 237]

This seems fine as a defence of the fuzzy view against outright dismissal in the face of the artificial precision problem—but it is essentially a *parry*, a *provisional* defence: matters cannot be left here. For if some parts of the fuzzy view are to be regarded as

¹⁶‘higher-order vagueness’—which appears in the titles of both the present paper and [1]—has been applied in the literature both to the problem of artificial precision and to distinct problems.)

¹⁶For a more detailed argument along these lines, see Smith [37, §6.2.1].

¹⁷This is not to say that it might not be interesting to explore this view—just that, for purposes of solving the artificial precision problem, we have no reason to adopt it.

representing aspects of the semantics of vague discourse, while others are mere artefacts, then we want to know *which* parts are which. Cook recognises this:

although sentences do have real verities [i.e. truth values], these verities are not real numbers but are only *modelled* by real numbers... what sorts of objects are they? ... the natural question to ask is which properties of the reals correspond to actual properties of verities and which do not. [4, pp. 239–40]

One answer here would be that only the *ordering* of the reals corresponds to a fact about the verities. This would take us back to the view that truth is measured on an ordinal scale. Cook favours a different response, according to which not only ordering is significant, but also *large* differences between reals—whereas small differences are often (but not always) artefacts. Cook does not spell out this view in a way analogous to the ordinal view: that is, by specifying a kind of transformation of the real interval $[0,1]$ such that a difference between two fuzzy structures is an artefact if and only if one structure can be obtained from the other by such a transformation. Without some such spelling-out, the view remains incomplete: we do not know which aspects of the fuzzy view are artefacts and which are not. With such a spelling-out, the idea that we use fuzzy structures as *models* gives way to the idea that we can fully *describe* the semantics of a vague discourse by associating it not with a fuzzy structure, but with some fully-specified *class* of fuzzy structures which are the same as one another up to the specified kind of transformation (as on the ordinal view). Either way, the logic as modelling view does not—in the final wash-up—make a distinctive contribution. While valuable as an initial parry to the objection from artificial precision, it is essentially a transitory response: it must—if it is to amount to more than hand-waving—eventually give way to a fully-specified system which *can* then be viewed not merely as a model, but as an accurate description of the semantics of vague discourse.¹⁸

Four proposed solutions remain on the table. How are we to decide which is the right one? In order to do so, we need a clearer idea of the true nature and source of the problem of artificial precision. Consider again the three quotations given in §1, which set out the problem. Haack offers no diagnosis; Urquhart maintains that the *nature of vague predicates* precludes attaching precise numerical values; Keefe asks what could *determine* which is the correct function, settling that her coat is red to degree 0.322 rather than 0.321. I shall argue (§6) that Keefe is on the right track and Urquhart is not: the problem with the fuzzy view turns not on considerations having to do with the nature of vagueness, but rather on considerations having to do with the way in which the meanings of our terms are fixed. In order to make this case, I must first discuss the nature of vagueness (§4) and the question of how meaning is determined (§5).

4 The nature of vagueness

Given some property, object, stuff or phenomenon *P*, we may distinguish between a *surface characterization* of *P*—a set of manifest conditions, possession of which marks out the *P*'s from the non-*P*'s—and a *fundamental definition* of *P*—a statement of the

¹⁸For a more detailed argument along these lines, see Smith [38].

fundamental underlying nature or essence of the P 's, which *explains* why they have such-and-such surface characteristics. For example, a surface characterization of *water* says that it is a clear, tasteless, potable liquid which falls as rain, while the fundamental definition says that it is H_2O . In the case of *vagueness* (of predicates), there is a generally accepted surface characterization: vague predicates are those which have borderline cases; whose extensions have blurred boundaries; and which generate sorites paradoxes. What we do *not* have is a fundamental definition of vagueness. Yet before we can say whether something—for example, assignment of a unique fuzzy truth value to each vague sentence—conflicts with the *nature of vagueness*, we need such a fundamental definition: we need to know what *is* the nature of vagueness. In this section I shall briefly discuss some possible definitions of vagueness and explain why they are inadequate, before presenting a definition which I regard as correct.¹⁹

We might try to define vagueness in terms of possession of borderline cases. This will not do, however, because while it does indeed seem that vague predicates have borderline cases, this is not the *fundamental* fact about them. It cannot be, because we can easily imagine predicates which have borderline cases but which are *not* vague—for example 'is schort', which we define as follows:

1. If x is less than four feet in height, then ' x is schort' is true.
2. If x is more than six feet in height, then ' x is schort' is false.

This predicate has borderline cases, but it does not generate a sorites paradox, nor does its extension have blurred boundaries—hence, it is not vague.

We might try to define vagueness in terms of having an extension that has blurred boundaries—but this characterization is too vague to constitute a fundamental definition.

We might try to define vagueness in terms of sorites susceptibility. This will not do, however, because while it is indeed the case that vague predicates generate sorites paradoxes, this is not the *fundamental* fact about them. It seems clear that vague predicates generate sorites paradoxes *because* they are vague—and so their vagueness cannot *consist in* their generating such paradoxes.

We might try to define vagueness as semantic indeterminacy (of the sort involved in plurivaluationism). Again, however, such indeterminacy cannot be the fundamental fact about vague predicates, because we can easily imagine predicates which exhibit such indeterminacy but which are *not* vague—for example 'gavagai' or 'mass'. If Quine [28, ch. 2] and Field [7], respectively, are right, then these predicates exhibit semantic indeterminacy—but they do not generate sorites paradoxes, nor do their extensions have blurred boundaries: hence they are not vague.

We might—following Wright—try to define vagueness as tolerance, where a predicate F is tolerant with respect to ϕ if there is some positive degree of change in respect of ϕ that things may undergo, which is “insufficient ever to affect the justice with which F is applied to a particular case” [54, p. 334]. The problem with this definition, however, is that given a sorites series for F , F cannot be tolerant, on pain of contradiction. Hence if tolerance is the essence of vagueness, we must either accept true contradictions, or else deny that there are any vague predicates (with associated sorites series).

¹⁹For longer versions of the arguments see Smith [36] or [37, ch. 3].

This brings us to my positive proposal, that F is vague if and only if it satisfies the following condition, for any objects a and b :

Closeness If a and b are very close/similar in respects relevant to the application of F , then ' Fa ' and ' Fb ' are very close/similar in respect of truth.

The principal advantages of this definition are that it accommodates tolerance intuitions, without contradiction; it yields an explanation of *why* vague predicates have the three characteristic surface features mentioned above; and it accommodates intuitions about higher-order vagueness, within the definition of vagueness itself. For details see Smith [36] and [37, ch. 3]; in what follows I shall assume that Closeness provides the correct fundamental definition of vagueness.

5 The determination of meaning

It is generally agreed that whatever semantic facts there are, they are determined by other facts. For Quine [29, p. 38], these other facts are the publicly accessible facts concerning what people say in what circumstances. Because he thinks that such facts do not determine a unique meaning for 'gavagai', he denies that this sentence *has* a unique meaning. For Kripkenstein [21], the class of meaning-determining facts is wider: it includes dispositional facts, and private mental facts. Nevertheless, he thinks that this wider class of facts still does not determine a unique meaning for 'plus'—and so he denies that 'plus' *has* a determinate meaning.

Turning to vagueness, there is widespread agreement in the literature concerning which facts are relevant to determining the semantic facts:

- All the facts as to what speakers actually say and write, including the circumstances in which these things are said and written, and any causal relations obtaining between speakers and their environments.
- All the facts as to what speakers are disposed to say and write in all kinds of possible circumstances.
- All the facts concerning the eligibility as referents of objects and sets.

(I would also add: all the facts concerning the simplicity/complexity of interpretations.) There is also widespread agreement that if these facts are insufficient to determine (unique) meanings for some utterances, then those utterances have no (unique) meanings. In other words, semantic facts are never primitive or brute: they are always determined by the meaning-determining facts—which are as itemized above.²⁰

This generates a constraint on any theory of vagueness: the theory must cohere with the foregoing picture of how meaning is determined. If the theory says that vague predicates have meanings of such-and-such a kind (e.g. functions from objects to classical truth values, or functions from objects to fuzzy truth values), then we must be able to satisfy ourselves that the meaning-determining facts itemized above could indeed determine such meanings for actual vague predicates. To the extent that the meaning-determining facts do *not* appear sufficient to determine meanings for vague predicates of the kind posited by some theory of vagueness, that theory is undermined.

²⁰For a more detailed discussion of these issues, see Smith [37, §6.1.1].

6 Choosing a solution (continued)

Let us now return to the issue—raised at the end of §3—of the true nature and source of the artificial precision problem. Urquhart maintains that the basic fuzzy theory of vagueness—in particular, the aspect of it which involves assigning a unique fuzzy truth value to each vague statement (i.e. in our terms, its degree of truth on the unique intended fuzzy model of the discourse)—conflicts with the *nature of vague predicates*, while Keefe thinks that the view runs into problems concerning what could *determine* the correct assignment (i.e. in our terms, what could determine *which* model is the unique intended one). Now that we have said something about the nature of vagueness and the issue of the determination of meaning, we are in a position to adjudicate this issue.

First, it will be useful to consider the picture afforded by *classical* MTS, according to which a discourse in natural language is to be modelled as a bunch of wffs together with a unique intended classical model. (As mentioned at the end of §2.1, this is the *semantic* picture underlying *epistemic* theories of vagueness.) This view conflicts with the nature of vagueness. Consider a discourse including some claims of the form ‘Point x is red’, for points x on a strip of paper which changes colour continuously from red to orange. Given that some points on the strip are definitely red and some definitely not so, on any candidate intended model of the discourse there will be two points on the strip which are very close in respects relevant to the application of ‘is red’, such that one of them is in the extension of this predicate and the other is not. Hence one of the claims ‘ k is red’ and ‘ k' is red’ (where k and k' are the two points in question) will be true and the other false (Figure 5). This violates Closeness: the classical picture does not allow for the vagueness of ‘is red’. The point generalizes to all vague predicates: thus the classical picture conflicts with the nature of vagueness. The classical view *also*

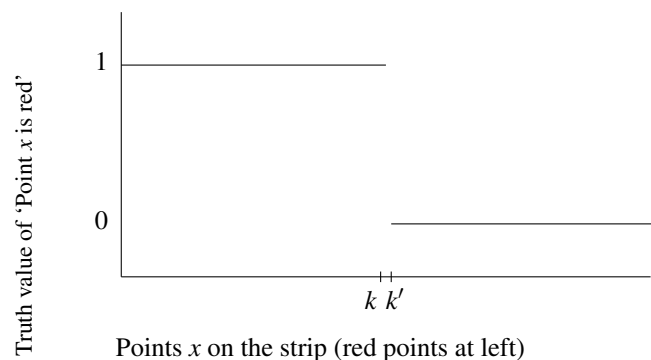


Figure 5. Classical MTS conflicts with the nature of vagueness.

runs into problems concerning the determination of meaning. It seems that the meaning-determining facts itemized in §5 do *not* suffice to pick out a particular height dividing the tall from the non-tall, and so on. Williamson has argued that the classical view is not logically incompatible with the view that usage determines meaning. This is true:

it might just be that the meaning-determining facts are sufficient to determine unique classical extensions for vague predicates (i.e. in our terms, a unique intended classical model for a vague discourse). But we have no idea *how* the trick could be turned—for example, Williamson’s own best suggestion fails—and the most reasonable conclusion seems to be that the meaning-determining facts do *not* suffice to pick out meanings for vague predicates of the kind the classical theory says they have.²¹ The classical view of vagueness therefore faces two distinct problems:

1. The *existence* of a sharp drop-off from true to false in a sorites series: this conflicts with the nature of vagueness.
2. The *particular location* of the drop-off: this conflicts with our best views about how meaning is determined.

I refer to these two problems as the *jolt problem* and the *location problem* respectively.

Let us now turn to the view of vagueness based not on classical MTS, but on fuzzy MTS: that is, the view that I have called the ‘basic fuzzy theory of vagueness’. Does it conflict with the nature of vagueness (as Urquhart claims)? No! Consider again a discourse including some claims of the form ‘Point x is red’, for points x on a strip of paper which changes colour continuously from red to orange. Fuzzy model theory has the resources to make available—as candidates for the intended model—models on which ‘Point x is red’ and ‘Point y is red’ are always very similar in respect of truth whenever x and y are very similar in respects relevant to the application of ‘is red’, even though some claims of this form are definitely true and others are definitely false (see Figure 6).

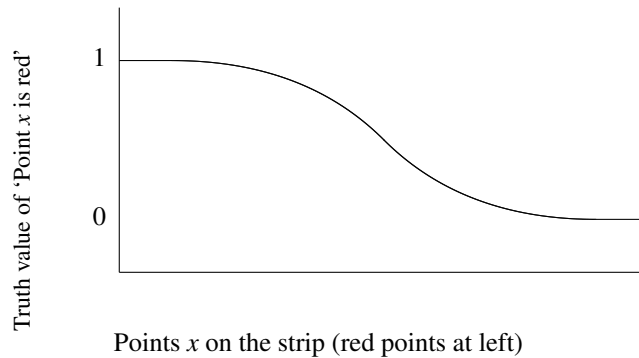


Figure 6. Fuzzy MTS does not conflict with the nature of vagueness.

Hence there is no conflict between the fuzzy view and the nature of vague predicates. Does the fuzzy view run into problems concerning the determination of meaning (as Keefe claims)? Yes! It seems that the meaning-determining facts itemized in §5 do *not*

²¹For detailed discussion of these issues—including the critique of Williamson’s suggestion alluded to in the text—see Smith [37, §2.1.1].

suffice to pick out a particular function from objects to fuzzy truth values representing the extension of ‘is tall’ (and similarly for other vague predicates). So the fuzzy view does not face a version of the jolt problem, but it *does* face a version of the location problem. Indeed, we are now in a position to see that its version of the location problem is nothing other than a more fully articulated version of the artificial precision problem with which we began. It is artificial/implausible/inappropriate to associate each vague sentence in natural language with a particular fuzzy truth value *because* doing so conflicts with our best theories about how the meanings of our words are determined.

We turn now to the four proposed solutions to this problem which remain on the table. Fuzzy epistemicism fails to solve the problem. The problem concerns how there could *be* a unique function which is the extension of ‘is tall’, given that our usage (etc.) does not suffice to pick out a unique such function. Saying that we do not *know* which function it is simply misses the point of the problem.

The proposal that truth is measured on an ordinal scale conflicts with the nature of vagueness. On this view, it *makes no sense* to say that two sentences P and Q are *very close* in respect of truth: it makes sense to say only that one sentence is *truer* than another. (The fact that the two sentences might have truth values which are, in some sense, very close together considered as real numbers—say, 0.8 and 0.800000000001—is irrelevant: the model on which they have these truth values is interchangeable with any model obtainable from it by an order-preserving and endpoint-fixing transformation of the interval $[0,1]$, and there will be such transformations which take P ’s truth value arbitrarily close to 0 and Q ’s arbitrarily close to 1.) But the idea of two sentences being *very close* in respect of truth is at the heart of the Closeness definition—and so a view which makes no room for this notion lacks the resources to distinguish vague predicates (those which satisfy Closeness) from precise predicates (those which violate Closeness).

The blurry set view does not solve the location problem. In just the way that they fail to determine a unique *classical* set (function from objects to classical truth values) or a unique *fuzzy* set (function from objects to fuzzy truth values) as the extension of ‘is tall’, the meaning-determining facts do not suffice to pick out a unique *blurry set* (function from objects to degree functions) as the extension of ‘is tall’.

This brings us to fuzzy pluralism—which *does* solve the location problem. Indeed, it is the *minimal* solution to the problem—for it accepts as its starting point the very idea which constitutes the problem. The problem is that the meaning-determining facts do not suffice to pick out a unique fuzzy model of vague discourse as the intended model. The fuzzy pluralist solution is to abandon the notion of a unique intended model in favour of the idea of multiple acceptable models—where an *acceptable* model is one which is *not* ruled out as *incorrect* by the meaning-determining facts. As the problem is precisely that there is *not* a unique acceptable fuzzy model of vague discourse—because too many models are compatible with the constraints imposed by the meaning-fixing facts—it follows *a fortiori* that fuzzy pluralism—the view that there are multiple equally correct models—is correct. The upshot of fuzzy pluralism is that ‘Bob is tall’ does *not* have a uniquely correct degree of truth: it is assigned multiple different degrees of truth—one on each acceptable model—and none of these is more correct than any of the others. This was the desired result: that it was *not* the case on the original fuzzy view was precisely the problem with which we started.

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