Comparing Context Updates in Delineation and Scale Based Models of Vagueness

CHRISTOPH ROSCHGER[1]

1 Introduction

Although there exists a multitude of different approaches to vagueness in linguistics, there seems to be a general agreement that vagueness involves one or another kind of context dependence. E.g. Lewis [17] gives a high level overview of the use of contexts in natural language and illustrates that they are essential for modeling the shifting of vague standards. Therefore a vague sentence cannot be interpreted in isolation from the rest of a discourse; instead its truth conditions depend on—implicit or explicit—assumptions made by the conversationalists so far. Models of vagueness can be roughly divided into two groups: scale based and delineation models. Whereas scale based models explicitly model vague standards as cut off points on some appropriate scale, delineation models take another route. Vague adjectives are treated as boolean predicates whose extension crucially is context dependent. Scale based models are nowadays more popular in linguistics; there is much work going on in exploring connections between the scale structure of gradable adjectives and their use in natural language. Delineation models, on the other hand, have their roots in the philosophy of vagueness and typically make use of supervaluation [5].

The general aim of this contribution is to compare these two kinds of approaches in terms of their ability to model the notions of context and context update and to elucidate where their particular strengths and weaknesses lie in this respect. For this purpose, two concrete models are picked, namely Barker [3] as an example for a scale based approach and Kyburg and Morreau [15] as an example for a delineation based approach. Finally also Shapiro's [19] account of vagueness in context is presented and we point out how to combine it with Kyburg and Morreau's approach. Shapiro himself is not a linguist, but his model explicitly refers to conversationalists taking part in a conversation during which vague standards are shifted. The main result is to show in which situations Barker's and Kyburg and Morreau's approaches make the same predictions and which aspects cannot be modeled properly in either model. As an example Kyburg and Morreau's model is clearly superior when the hearer is confronted with new information which requires the shifting of vague standards such that previously established information has to be retracted. Barker however does not consider such situations; once vague information is added to the current context it is never invalidated. On the other hand, Barker's model provides more flexibility when it comes to predicate modifiers such as

[1] Supported by Eurocores-ESF/FWF grant I143-G15 (LogICCC-LoMoReVI)

very, *definitely*, or *clearly* and can straightforwardly handle more complex comparisons including degrees, such as '*Jack is 2 cm taller than John*".

The paper is structured as follows: in Section 2 we give a short overview highlighting some examples of how contexts are modeled in linguistics for other purposes than vagueness. In Section 3 we present Barker's, Kyburg and Morreau's, and Shapiro's approaches in more detail. Finally, Section 4 first points out how to combine Shapiro's approach with Kyburg and Morreau's and then explores the mentioned connections between Barker's and Kyburg and Morreau's approaches. This done by first defining a simple intermediate representation for contexts such that both types of contexts as defined by Barker and by Kyburg and Morreau can be translated into this representation. Then we show that, if a situation is modeled twice, one time as defined by Barker's scale based approach and one time as defined by Kyburg and Morreau's delineation approach, the same intermediate representation is reached in both models.

2 Contexts in Linguistics

Vagueness is not the only motivation for explicitly modeling context in linguistic formal semantics. Contexts are also most prominently employed for resolving anaphoric relations (see e.g. [7, 6]) or for modeling (existential) presuppositions [8] in a compositional way. In these cases contexts are identified with (sets of) possible assignments of objects to variables. The main point of these so-called *dynamic semantics* is that the meaning of a proposition is identified with its ability to update the context, its *context change potential*. Typically contexts are modeled as sets of possible worlds. Deciding if a proposition is *true* in a given context C then amounts to updating C accordingly and checking if the resulting context is empty or not.

As an example consider the text "*A man walks. He whistles.*" Assume, for the sake of simplicity, that the domain of all men under consideration consists of just three ones, men $\sharp 1$, $\sharp 2$, and $\sharp 3$, the predicate *walks* is true for both men $\sharp 1$ and $\sharp 2$, and the predicate *whistles* is true only for man $\sharp 2$. Moreover, assume that previous analysis has already yielded that *He* refers to *man*, therefore the text can be annotated as "*A* man_1 *walks.* He_1 *whistles.*" The semantic analysis of "*A* man_1 *walks*" first introduces a new discourse referent—Heim uses file cards as a metaphor—such that the individual referred by 1 is a man and walks. Thus the intermediary context is $\{\{1 \mapsto \sharp 1\}, \{1 \mapsto \sharp 2\}\}$. Similarly, "$He_1$ *whistles.*" filters out the first assignment since man $\sharp 1$ does not whistle in our little model resulting in the final context $\{1 \mapsto \sharp 2\}$. More complex ways of updating the context arise when taking into account relative clauses, quantifiers, or negation.

3 Contexts and Vagueness

When dealing with vagueness richer structures than just possible assignments from variables to individuals are required. In this section we outline three different approaches to that purpose: Barker [3] as an example of a scale based[2] account of vagueness, Kyburg

[2] In linguistics scale based approaches are also called *degree based*. In order to avoid confusion with other degree based approaches to vagueness such as fuzzy logic, we use the name *scale based* here. This is to stress that degrees only occur on certain scales corresponding to vague adjectives (such as degrees of height, of weight, or of color) but not as degrees of truth.

and Morreau's approach as an account of a delineation based one, and Shapiro's approach which is also delineation based.

Whereas scale-based accounts of vagueness map vague predicates to degrees on an adequate scale, delineation accounts treat predicates like *tall* as boolean predicates whose extension is context dependent. Specific scales can then be modeled by non-vague propositions constraining the space of possible precifications.

Barker's and Kyburg and Morreau's approaches may not be the most prominent examples of scale and delineation based linguistic accounts of vagueness. For example, other more popular approaches are Kennedy's [11, 10] and Klein's [12, 13, 14]. However, in this contribution the focus is at comparing how the context, and therefore the interpretations of vague predicates, are updated dynamically during a conversation, and in this respect both Barker's and Kyburg and Morreau's approaches are more explicit than e.g. Klein's and Kennedy's. Barker himself also gives this as a motivation for his approach [2]: "*However, what is still missing [...] is an explicit account of precisely how the context change potential of an expression depends on the compositional meaning of the expression.*" Finally, Shapiro motivates his contextual approach to vagueness very elaborately, but more from a philosophical point of view than from a linguistic one. Nevertheless, as described below, his model lines up very nicely Kyburg and Morreau's.

3.1 Barker's approach

As an example of a scale based approach, Chris Barker [3] defines a context C as a set of possible worlds w where in each world $w \in C$ a vague predicate has a precise threshold value and each relevant object a precise (maximal) degree to which it possesses the predicate in question. Barker's running example is the vague predicate *tall*; the meaning of this predicate is denoted as $[\![tall]\!]$. Although many vague predicates such as *tall* or *heavy* intuitively correspond to a set of linearly ordered degrees, this is not the case for predicates like *stupid* or *clever*. Barker therefore only assumes that the set of degrees is partially ordered and obeys monotonicity, i.e. if *John* is *stupid* to degree d, he is also stupid to each degree smaller than d.

Formally there is a delineation function $\mathbf{d}$ which maps gradable adjective meanings to degrees locally for each possible world; for example $\mathbf{d}(w)([\![tall]\!])$ denotes the (vague) standard of tallness in the world w. For a degree d and a vague predicate α, let $w[d/\alpha]$ denote a possible world which is exactly like w except for setting $\mathbf{d}(w)(\alpha)$ to d. Moreover, for each vague predicate there exists a corresponding constant relation in the object language, e.g. **tall**, where $\mathbf{tall}(d,\mathbf{j})$ gives all possible worlds where the individual denoted by $\mathbf{j}$ is tall *at least* to the degree d. Other scale based approaches, e.g. that of Kennedy [11], favor another more transparent formalization instead, where each individual is assigned a single degree of height. Deciding if *John* is *tall* at least to degree d then amounts to comparing d with John's degree of height. However, this is not sufficient without further restrictions on the scale structure, namely if the scale is not linearly ordered. For linear scales Kennedy notes that such a formalization is equivalent to Barker's one. In this case, *John*'s degree of height in the possible world w can be expressed as $\max\{d : w \in \mathbf{tall}(d,\mathbf{j})\}$ using Barker's definitions.

As argued by Klein [12] vague adjectives always have to be interpreted relative to a comparison class: while John may be tall in some context, he may not be a tall basketball player. Even, if the adjective occurs without the noun (i.e. basketball player) this comparison class is seen as contextually determined. This can easily be incorporated into Barker's model by defining different threshold values for each relevant comparison class, effectively treating e.g. "*tall for a basketball player*" and "*tall for a man*" as different vague predicates with the all possible worlds w satisfying that $\mathbf{d}(w)([\![tall_{basketball player}]\!])$ $\geq \mathbf{d}(w)([\![tall_{man}]\!])$.

Initially a large context C stands for ignorance.[3] If, e.g., only the individuals *John* and *Eve* and the vague predicate *tall* are under consideration (and we do not yet know anything about *John*'s and *Jack*'s tallness), C consists of all potential combinations of threshold values for *tall* as well as John's and Eve's degrees of height. If we then hear the utterance "*John is tall*", all possible worlds in C are filtered out where John's degree of height is higher than the threshold for tallness resulting in a new context C'. Thus, the evaluation of such simple sentences is made locally in each possible world under consideration. The meaning of *tall* therefore is defined by Barker as[4]

$$[\![tall]\!](x)(C) = \{w \in C : w \in \mathbf{tall}(\mathbf{d}(w)([\![tall]\!]), x)\}.$$

So, $[\![tall]\!](x)$ denotes a function which takes a context C as argument and returns all possible worlds in C, where the individual denoted by x exceeds the (local) standard of tallness denoted by $\mathbf{d}(w)([\![tall]\!])$. The new context C' is then obtained by applying $[\![tall]\!]$ to the original context C:

$$C' = [\![tall]\!](\mathbf{j})(C)$$

with $\mathbf{j}$ referring to *John*. If we then hear "*Jack is taller than John*", again worlds in C' are filtered out by comparing Jack's and John's degrees of height there. In this case, automatically only possible worlds are left in the final context where Eve is regarded as tall as well (due to monotonicity of the scale structure).

Barker's approach allows one to model so-called *predicate modifiers* such as *very*, *definitely*, or *clearly*. Like for simple predicates such as *tall*, *very tall* operates separately on each possible world under consideration: An individual is considered as *very* tall, if she is not only *tall*, but *tall* by some margin. The exact size of this margin is again subject to vagueness and may differ between possible worlds (note that the first argument α is the meaning of a predicate, e.g. $[\![tall]\!]$):

$$\begin{aligned}[\![very]\!](\alpha)(x)(C) =_{\mathrm{DEF}} \{w \in \alpha(x)(C) : \exists d.(&w[d/\alpha] \in \alpha(x)(C) \\ &\wedge \mathbf{very}(\mathbf{d}(w)([\![very]\!]), \mathbf{d}(w)(\alpha), d)\}\end{aligned}$$

where $\mathbf{d}(w)([\![very]\!])$ denotes the (local) margin imposed by *very* in world w; and **very** is a relation over degrees such that $\mathbf{very}(s, d, d')$ holds exactly if the difference between d' and d is larger than s.

[3] It seems reasonable to assume the context to be finite due to some level of granularity enforced by our perceptional and cognitive capabilities.

[4] Barker, in common linguistic tradition, gives all his definitions in lambda-notation, e.g. as $[\![tall]\!] = \lambda x \lambda C.\{w \in C : w \in \mathbf{tall}(\mathbf{d}(w)([\![tall]\!]), x)\}$. Here, we chose a syntax more familiar to non-linguists. Moreover, note the use of $[\![tall]\!]$ inside its own definition. This circularity is of a harmless type as the usage of $[\![tall]\!]$ inside of $\mathbf{d}(w)([\![tall]\!])$ is only to refer to the function's name as a marker.

Other more complicated predicates like *definitely tall* do not operate on each world locally, but depend on the overall structure of the context. Let C be a context and $w \in C$ a possible world. Then "*John is definitely tall*" is true in w if and only if John is tall in all worlds u where John is as tall as in w, i.e. the local degree of John's height in u is compared to all possible standards for tallness adequate for this specific height. Thus, the predicate modifier $[\![definitely]\!]$ can be defined as follows:

$$[\![\mathit{definitely}]\!](\alpha)(x)(C) =_{\text{DEF}} \{w \in \alpha(x)(C) : \forall d.(w[d/\alpha] \in C) \to w[d/\alpha] \in \alpha(x)(C)\}.$$

Similarly to the predicate modifiers *definitely* and *very* Barker also gives precise truth conditions for *clearly*, which is a combination of both very and definitely. The comparative form *taller* can also be formalized using Barker's framework as a predicate modifier: $[\![\text{-}er]\!]$ takes as first argument the meaning of a predicate such as $[\![tall]\!]$ and returns a function for comparing two individuals with respect to *tallness*. Barker proposes different different ways of defining the comparative based on different treatments of the comparative in linguistic literature. One of them is:

$$[\![\text{-}er]\!](\alpha)(x)(y)(C) =_{\text{DEF}} \{w \in C : \exists d.(w[d/\alpha] \in \alpha(y)) \wedge (\neg w[d/\alpha] \in \alpha(x))\}.$$

Using this formulation an individual **a** is *taller* than an individual **b** if and only if there is a degree d such that **a** is at least that tall, but **b** is not. The important point here is that no matter which formulation is used the comparative does not depend on the (local) standard of *tallness*; and neither does an utterance of the form "**a** *is taller than* **b**" have any sharpening update effect on this standard.

Although not considered by Barker, also negation can be easily formulated as a predicate modifier. The possible worlds where an individual **a** is *not tall* are exactly those filtered out when updating with the information that **a** is *tall*:

$$[\![not]\!](\alpha)(x)(C) =_{\text{DEF}} C \backslash [\![\alpha]\!](x)(C).$$

As Barker explains, context update can account for two different kinds of statements, namely descriptive and meta-linguistic ones. Consider as an example the sentence "*For an American Feynman is tall.*": in a descriptive reading this statement can be uttered to inform a hearer (vaguely) about Feynman's height. But if the exact height of Feynman is known to both conversationalists (e.g. because he is standing next to them), the sentence can be used to tell the hearer about one's usage of the word *tall*, i.e. about the general tallness of people in America.

In general, the further the conversation proceeds and the more sentences are uttered, the smaller and the more precise the context gets. According to Barker, possible worlds are never added to the context during the conversation. This seems consistent with other dynamic linguistic approaches modeling e.g. anaphoric relations or existential presuppositions as explained in Section 2. There, possible worlds consist of assignments of variables to individuals instead of containing threshold values and so on. However the contexts do not necessarily get smaller the more the conversation proceeds, but this is due only to the fact that new variables may be introduced during a conversation. As Barker requires that all relevant individuals and vague adjectives are a priori present in the initial context, this situation does not occur here.

3.2 Kyburg and Morreau's approach

An alternative approach has been introduced by Kyburg and Morreau [15]. In contrast to many other linguistic theories of vagueness this one does not employ degrees and scales nor threshold values for vague standards. For vague predicates such as *tall* Barker's approach is straightforward, since for deciding whether a person is tall we take into account only her height, which can be measured in a natural way. There are, however, other predicates like *clever* for which there exists no such natural scale. For example, we can say that some person is *clever* in some respect but not in some other respect.

According to Kyburg and Morreau therefore a context is modeled as a set of sentences over a "vague language". They employ the metaphor of a blackboard where the common knowledge and shared presuppositions are written visible to all participants. The vague language is standard first order predicate logic augmented with a sentential operator D. In contrast to classical logic, however, predicates may be decided only partially for vague predicates.

Thus a model is a partial interpretation of the vague predicates together with possible (consistent) ways of making them precise; the so-called *precification space*. At each precification point in such a space, an object can belong to the extension of a vague predicate or to the anti-extension or to none of these. Consistency of precification points requires that no object is both in the extension and in the anti-extension of a vague predicate at the same time.

This is formalized as follows: a model is defined as a quadruple $\langle \mathscr{U}, \mathscr{P}, \leq, l \rangle$ where $\mathscr{U}$ denotes the domain of relevant individuals, $\mathscr{P}$ denotes a set of abstract precification points, and l denotes a partial interpretation function. Therefore, l assigns to each predicate symbol R and each precification point $p \in \mathscr{P}$ two disjoint subsets of $\mathscr{U}$, namely $l^+(R,p)$ and $l^-(R,p)$, i.e. the extension and anti-extension of R at p.[5] The interpretation l must respect the partial ordering $\leq$ with minimal point @ in the sense that for all points $p,q \in \mathscr{P}$, if $p \leq q$ then $l^+(R,p) \subseteq l^+(R,q)$ and $l^-(R,p) \subseteq l^-(R,q)$.

Not every consistent precification does make sense in a conversation. If we have two people, *John* and *Jack*, where *John* is taller than *Jack*, we must not judge *Jack* as *tall* and *John* as *not tall*. Such restrictions on the structure of the precification space are called *penumbral connections*.

Another requirement on the precification space is that for each precification point p there is a further complete refinement q of p. A precification point is complete if each object is either in the extension or the anti-extension of the vague predicates in question. This requirement directly refers to supervaluationist theories of vagueness [5]. As in other (linguistic) theories, e.g. [16] and [9] supervaluation then is used for evaluating truth at a precification point. A formula is true or false at a given precification point, if it is true or false, respectively, for all possible complete precifications; evaluation at a complete point is classical. Truth in a model is identified with truth at the root @.

The notions of *determinate truth* and *borderline* cases are defined as follows: A proposition ϕ is called *determinately true* or *determinately false*, denoted as $D(\phi)$ and $D(\neg\phi)$, if it is true or false, respectively, in all possible precification points in the whole context space $\mathscr{P}$, i.e. possible contexts. If there exist both such precification points

[5] For n-place relation symbols the extension and anti-extension consist of subsets of $\mathscr{U}^n$.

in $\mathscr{P}$, then ϕ is called *indeterminate*, denoted as $I(\phi)$. Note that a proposition of the form $D(\phi)$ or $I(\phi)$ is not vague anymore since it is evaluated on the whole space of contexts no matter which one is the current one. If α is a vague predicate and x an object then x is called a borderline case of α if $I(\alpha(x))$ holds. Therefore, in distinction to Barker, it is possible to have a precification point where both $I(tall(x))$ and $tall(x)$ are available signifying that x is a borderline case of *tallness*, but the current conversation has established that x is to be regarded as *tall*.

For example, the penumbral connection indicated above can be formalized as

$$D((\text{taller-than}(John, Jack) \wedge \text{tall}(Jack)) \rightarrow \text{tall}(John))).$$

Kyburg and Morreau use *belief revision theory* [1] to describe context updates. Belief revision is a well studied topic in knowledge representation. It is based on the definition of a *revision operator* $*$ for adding new information to a knowledge base (and thus possibly invalidating old one). A knowledge base is a set of sentences closed under deduction. The idea is to view the context as a knowledge base and to perform a context update by revising the current context with the new information. While belief revision operates on the set of sentences known to be true at a point in a conversation, an update with consistent information can be regarded as moving alongside a branch in the precification space in an adequate model. If vague information is invalidated, i.e. if the new information is inconsistent with previously established information, then this can be regarded as a jump to another branch in the precification space.

When performing a context update with a set of propositions, it seems reasonable to treat vague and non-vague propositions differently: Take the new information as a starting point. Then simply add the non-vague sentences of the prior context. Thus, non-vague sentences thus are never invalidated, so the resulting context may be inconsistent if the new information contradicts some non-vague sentence in the old knowledge base. Kyburg and Morreau argue that in this case inconsistency appropriately models the resulting context, because even a human agent is at least puzzled when faced with non-vague contradictory information. In the following we assume that new non-vague information is never inconsistent. For the vague sentences Kyburg and Morreau give examples, such as *accommodation*, where human agents do invalidate former sentences in order to maintain consistency. As an example a pig may be referred to as "*the fat one*" in one situation if it stands next to another skinny pig. In the next situation, where it stands next to an even fatter one, "*the fat one*" designates the other pig. This process is called *accommodation*, the shifting of vague standards. Therefore, after adding the non-vague sentences, just add as many sentences from the original context while retaining consistency. By this procedure a new precification point in the space $\mathscr{P}$ is reached, since the penumbral connections belong to the non-vague sentences which are not invalidated.

Kyburg and Morreau's way of updating ensures that the new information will be present in the resulting context, only old vague sentences are invalidated. More complex situations arise if there are several distinct ways to form the new context. Consider, e.g., a context C includes two vague propositions ϕ and ψ. When updating with the proposition $\neg\phi \vee \neg\psi$, there are several possibilities: either invalidate ϕ, or ψ, or invalidate both. The last one may seem as the only canonical solution in this situation, but both other possibilities are superior in the sense that less information is invalidated.

The solution offered by belief revision (in short) is the following: one defines an operator $*$ for revising a knowledge base with new information, where a knowledge base is identified with a set of sentences closed under modus ponens. Such an operator is characterized by the so-called AGM postulates.

Let C be a consistent, deductively closed knowledge base and s a sentence. Then $*$ is a belief revision operator if it satisfies the following rules:

Closure $C * s$ is a deductively closed set of formulas,

Success $s \in C * s$,

Inclusion $C * s \subseteq C + s$, where $C + s$ is the deductive closure of $C \cup \{s\}$,

Vacuity if $\neg s \notin C$ then $C * s = C + s$,

Consistency $C * s$ is consistent, if s is consistent, and

Extensionality if s and t are logically equivalent then $C * s = C * t$.

These postulates ensure that a revision operator "behaves well" in the sense that the new information is guaranteed to be present in the resulting context C (Success), no superfluous unrelated information is added to C (Inclusion), enough information is invalidated in order to retain consistency (Consistent), and the update is independent of the syntactic representation of s (Extensionality). Whatever can be proved using the AGM-postulates will then hold for *any* way of updating the context. The theory on belief revision is well developed and there are alternative characterizations of the revision operator giving more concrete choices for devising such operators.

Following Kyburg and Morreau, suitable operators discard as little information as possible when confronted with new inconsistent data. Otherwise, a trivial revision operator would be to discard the whole knowledge base C when confronted with new inconsistent information s. As Kyburg and Morreau argue, such a behavior is not desirable, as only as little information as possible inconsistent with s will invalidated by a human. However, this is not always an unambiguous prescription as illustrated by the following example. It can be shown that the definition of a suitable revision operator boils down to the definition of a so-called *selection function* which chooses one knowledge base out of a set of (maximal consistent) ones.

Consider two men, John and Jack standing next to each other, where Jack is taller and has more hair than John and consider "*John is not tall*", and "*Jack is not bald*" as vague propositions which have already been uttered during the conversation. Now assume that a speaker mentions "*the tall and bald guy*". The resulting context then has to designate exactly one of these two men as tall and bald, but without revoking either "*John is not tall*" or "*Jack is not bald*" this will not happen. In the first case John's status of tallness is not added to the resulting context. We then can infer that John is the man referred to and hence also is regarded as tall. On the other hand we can as well revoke Jack's status of baldness, ultimately picking him as referent.

A corresponding selection function thus has to choose one out of the two (maximal) information sets, where either John is not regarded as tall or Jack is not regarded as bald. Which one of these will be taken is beyond Kyburg and Morreau's account of vagueness.

3.3 Shapiro's approach

Shapiro's approach to vagueness [19, 20] shares many features with supervaluation, but also some fundamental differences. A common criticism of supervaluation focuses on the completability requirement [4]. Arguably, in some cases such complete precifications may not exist, as for example in typical Sorites situations; instead they are artifacts of the model.

As a running example Shapiro uses a Sorites situation as follows: assume, 10000 men are lined up ordered according to their amount and arrangement of hair where the first man has no hair at all and the last one has full hair. A group of conversationalists is asked to judge whether man $\sharp 1$ is bald, then man $\sharp 2$ and so on, where all conversationalist must establish one judgment for each man $\sharp i$. The principle of *tolerance* then dictates that we cannot judge a man $\sharp i$ to be bald, when at the same time judging man $\sharp i+1$ not to be bald, considering that the amount and arrangement of hair of these men differ only marginally. However, this implies that there exist no complete precifications consistent with the facts that the first and the last men are bald and not bald, respectively, and therefore the principle of tolerance cannot be seen as a penumbral connection with the completability requirement in force.

On the technical side, Shapiro's approach is obtained by dropping the completability requirement. Instead it is allowed at each precification point to leave the baldness of some men unjudged. This way, by distinguishing between ϕ *is false at a further precification point* and ϕ *is not true at all further precification points* he obtains more fine-grained notions of truth than in a supervaluationist framework, where these two propositions coincide. Most notably, the tolerance principle can be formulated for Sorites situations as a penumbral connection requiring that there are no complete precifications.

Shapiro introduces the notion of *forcing*. A proposition ϕ is forced at a precification point P if for each refinement Q of P there is a further refinement Q' of P such that ϕ is true at Q'. He argues that a formula ϕ being forced at each precification point is an adequate characterization of determinate truth as described above for supervaluationist approaches. Note that both these notions coincide if we enforce the completability requirement.

There is also a further, more conceptual, difference between supervaluation and Shapiro's approach: supervaluation entertains the slogan '*Truth is supertruth*' implying that truth is not to be interpreted locally with respect to a single precification point but globally for the whole precification space. Accordingly, precification points themselves only serve as technical vehicle for determining truth. According to Shapiro, however, truth is always relative to some precification point and as a conversation proceeds, the current point may and will change. In the next section we will explain why this point of view is arguably more adequate for linguistic contextual models of vagueness than the supervaluation approach, because precification points do indeed correspond to (intermediary) contexts; both notions, *truth in a context* and *determinate truth* are of interest.

In the beginning of a conversation only the externally determined non-vague facts are known to the hearer. As the conversation proceeds and sentences are uttered more and more judgments are made resulting in new contexts which are refinements of the initial one. This means that typically propositions which are true or false remain true or false, respectively, only (some) yet unsettled cases are decided. Shapiro therefore models the context space as a tree structure with the initial context at the root and demonstrates that context update basically consists of moving alongside this tree's branches. However, when it comes to Sorites situations, Shapiro carefully argues that at some point a jump to another branch in the tree will occur, invalidating propositions which have already been true in a former context. So, when going through a Sorites series as above, at some point j, the speaker will eventually switch from *bald* to *not bald*. Then not only man $\sharp j$ is added to the anti-extension of *bald*, instead the last few men judged previously are removed from the extension of *bald*. This is necessary in order for tolerance to remain in force. Shapiro does not, however, give any clue on how actually to determine how many men are affected from this context jump.

4 Comparison of contextual approaches to vagueness

In this section we compare the three different accounts of vagueness presented above. We point out how to combine Shapiro's and Kyburg and Morreau's approaches and why this indeed makes sense.

For Barker's and Kyburg and Morreau's approaches we show how a conversational context can be translated from one approach into the other one and to what respect predictions made differ between these approaches. Moreover we elucidate differences in the expressiveness of the approaches, i.e. in which situations which approach is superior to the other one.

4.1 Shapiro - Kyburg/Morreau

Shapiro's and Kyburg and Morreau's accounts of vagueness are both delineation-based, but whereas the latter one makes use of supervaluation, Shapiro presents his approach as an alternative to supervaluation.

A context according to both approaches amounts to a partial interpretation of the (vague) predicates in question. Therefore a context as in Kyburg and Morreau has essentially the same structure as a context according to Shapiro. However, the space of possible contexts, and therefore the way how connectives (and modifiers such as *definitely*) are interpreted in a context may differ. A model according to Kyburg and Morreau is also a model according to Shapiro, but not the other way round as both are partial interpretations, but Shapiro does not require that the completability requirement is in force, i.e. that each partial interpretation can be extended to a complete interpretation; there it is possible for penumbral connections to constrain the context space further than with that requirement in force. Therefore, it may be the case that a complete assignment in a model according to Kyburg and Morreau is not available there. This occurs especially in Sorites situations as explained in the previous section where the penumbral connections forbid complete sharpenings:

Using supervaluation as suggested by Kyburg and Morreau, the sentence "*Man* $\sharp i$ *is bald, but man* $\sharp i+1$ *is not*" is not predicted to be (determinately) false (for $1 \leq i < 10000$), instead it is indeterminate which, arguably, goes against intuition. Moreover, the sentence "*There exists an i such that man* $\sharp i$ *is bald and man* $\sharp i+1$ *is not*" turns out to be determinately true! Shapiro, however, argues that by the principle of tolerance there exist no complete interpretations in such situations and therefore predicts both sentences as expected. He also notes that his formal definitions of definite truth coincide with the ones given by supervaluation in situations where all partial interpretations can indeed be extended to compete ones.

Shapiro's account of vagueness may seem incompatible with Kyburg and Morreau's one when it comes to context update at the first glance. Shapiro does not explicitly write about context update, but rather of moving from one point in the precification space to another one.

Since, however, in Shapiro's approach the non-vague facts—including also penumbral connections—are guaranteed to hold at all precification points, we can interpret a context update as described by Kyburg and Morreau simply as a jump to another precification point. This way, Kyburg and Morreau's way of updating contexts can be regarded as a refinement of Shapiro's, describing the update more precisely by means of belief revision theory.

As noted in the last section Shapiro's notion of truth at a context arguably fits Kyburg and Morreau's model more than supervaluation. Whereas the latter one promotes (super)truth relative to the whole context space, Kyburg and Morreau very much agree with Shapiro in regarding partial interpretations as concrete contexts in conversations which change over time, and with truth being relative to such a context. Therefore a version of Kyburg and Morreau's approach with Shapiro's account of vagueness instead of supervaluation seems fruitful. Technically this allows one to disregard the completability requirement when modeling context spaces.

4.2 Kyburg/Morreau - Barker

Barker's and Kyburg and Morreau's accounts of vagueness may seem completely different and incompatible to each other at the first glance, as the first one treats vague predicates by assigning degrees to individuals and comparing these degrees with some threshold value locally for a possible world. Nevertheless, we show that for both accounts contexts (or models of context, respectively) can be identified with sets of classical worlds:

DEFINITION 1 *Let $\mathscr{U}$ be the set of individuals and $\mathscr{R}$ the set of vague predicates under consideration. Then a classical world s is a complete interpretation of all the predicates in $\mathscr{R}$ formalized as a set of literals such that for all $R \in \mathscr{R}$ and for all $u \in \mathscr{U}$ either $R(u) \in s$ or $\neg R(u) \in s$.*

DEFINITION 2 *Let $s \in S$ be a complete interpretation, $R \in \mathscr{R}$ a predicate, and $\mathscr{U}$ be the universe of discourse. Then the positive and negative extensions of R at s are defined as $l^+(R,s) =_{DEF} \{u \in \mathscr{U} : R(u) \in s\}$ and $l^-(R,s) =_{DEF} \mathscr{U} \backslash l^+(R,s)$, respectively.*

There is a subtle difference between Kyburg and Morreau's and Barker's approaches to vagueness. Whereas Kyburg and Morreau explicitly model the precification space $\mathscr{P}$ and points $p \in \mathscr{P}$ in this space, Barker does not directly make this distinction. For him the initial context C_0 also determines all other possible contexts, the context space there consists of all subsets of C_0. Penumbral connections are only implicitly given by the contents of the possible worlds and the scale structures for the vague adjectives in question and not modeled explicitly.

More than that, Kyburg and Morreau's notion of context as a set of sentences does not directly correspond to a point in a precification space; instead several models may be adequate for a context. As we will see below a context as in Barker directly corresponds to a set of complete interpretations, therefore Barker's contexts are less general than Kyburg and Morreau's. Consider e.g. the proposition $D(P(a)) \vee D(\neg P(a))$, adequate models in Kyburg and Morreau's framework may either have a in the extension of P at all precification points or in the anti-extension, a situation which is not straightforwardly expressible in Barker's approach. When comparing both accounts of vagueness we therefore contrast Barker's notion of a context with Kyburg and Morreau's notion of a context's model. It is however crucial to keep in mind that a context as in Kyburg and Morreau may have several distinct models and thus this framework is more powerful in the sense that contexts are seen as something more general than by Barker.

In the following we will describe how both Kyburg and Morreau's and Barker's notions of contexts can be translated to the intermediate representation defined above as well as the inverse; given an intermediate representation obtained from some context we need to be able to reconstruct the corresponding contexts in either approach. For Kyburg and Morreau's approach there are two different ways to interpret a model as a set of classical worlds: Initially, one might be tempted to identify a partial interpretation with all its complete extensions. This method, however, is not fruitful as seen in the following example: assume two vague predicates *tall* and *heavy* with the penumbral connections stating that nobody can be both tall and not heavy (or not tall but heavy) at the same time (but can be tall with his state of heaviness left undecided, nevertheless), formalized as

$D(\forall x \neg(\mathbf{tall}(x) \wedge \neg\mathbf{heavy}(x))$ and
$D(\forall x \neg(\neg\mathbf{tall}(x) \wedge \mathbf{heavy}(x))$.

Furthermore, assume that one individual denoted by **a** is under consideration. Then the according precification space $\mathscr{P}$has the following structure:

$$\begin{tikzcd}
 & p_0 : \{\} \arrow[dl] \arrow[d] \arrow[dr] \arrow[drr] & & \\
p_1 : \{\mathbf{tall}(\mathbf{a})\} \arrow[d] & p_2 : \{\mathbf{heavy}(\mathbf{a})\} \arrow[dl] & p_3 : \{\neg\mathbf{tall}(\mathbf{a})\} \arrow[dr] & p_4 : \{\neg\mathbf{heavy}(\mathbf{a})\} \arrow[d] \\
p_5 : \{\mathbf{tall}(\mathbf{a}), \mathbf{heavy}(\mathbf{a})\} & & & p_6 : \{\neg\mathbf{tall}(\mathbf{a}), \neg\mathbf{heavy}(\mathbf{a})\}
\end{tikzcd}$$

Now, if we identify a partial interpretation with all of its complete extensions, p_1 and p_2 are both identified with p_5 (and the same for p_3, p_4, and p_6), thus losing information about the original partial interpretation. Because of this loss of information we cannot distinguish between situations where e.g. **a** is tall, but his status of heaviness is undecided, or **a** is heavy, but his status of tallness is undecided, or **a** is both tall and heavy. However, as it is our aim to directly compare the expressiveness of Barker's and Kyburg and Morreau's approaches, it is required that each representation of a context as a set of classical worlds corresponds to exactly one context as in Kyburg and Morreau.

Therefore we take another, even simpler, road: identify a partial interpretation with *all* classical extensions, not necessarily present in $\mathscr{P}$. If, e.g. somebody's status of heaviness is undecided at p_1, ensure that p_1 translation to complete interpretations includes one where she is heavy and one where she is not, regardless of any penumbral connections:

DEFINITION 3 *Let $\mathscr{P}$ be a precification space, $p \in \mathscr{P}$ a partial interpretation, and $\mathscr{U}$ the (finite) universe of discourse. Then the translation from p to a set of classical worlds, denoted as $T_{km}p$, is defined as*[6]

$$T_{km}p =_{DEF} \bigcirc_{R\in\mathscr{R}} \bigcirc_{u\in\mathscr{U}} \phi(R,u,p) \text{ where}$$
$$\phi(R,u,p) = \begin{cases} \{R(u)\} & \text{iff } u \in l^+(R,p) \\ \{\neg R(u)\} & \text{iff } u \in l^-(R,p) \\ \{R(u), \neg R(u)\} & \text{otherwise} \end{cases}$$

for all $R \in \mathscr{R}$.

This definition ensures that, if $R(u)$ is true (or false) at p then $R(u)$ will be true (or false, respectively) at all classical worlds in $T_{km}p$ and otherwise there will be worlds where $R(u)$ is true and worlds in $T_{km}p$ where $R(u)$ is false.

The transformation also works the other way round: given a set of complete interpretations the according partial one can be reconstructed. For each vague predicate, let its extension and anti-extension be the intersections of its extensions (and anti-extensions, respectively) in all complete interpretations under consideration.

PROPOSITION 4 *Let $\mathscr{P}$ be a precification space and S a set of complete interpretations. Then a partial interpretation $p \in \mathscr{P}$ is identified with S, denoted $p = T_{km}^{-1}S$, if and only if for all $R \in \mathscr{R}$ both*

$$l^+(R,p) = \bigcap_{s\in S} l^+(R,s) \qquad \text{and} \qquad l^-(R,p) = \bigcap_{s\in S} l^-(R,s)$$

hold, thus $T_{km}(T_{km}^{-1}S) = S$.

Proof For any $p \in \mathscr{P}$, $R \in \mathscr{R}$, and $u \in \mathscr{U}$: if $u \in l^+(R,p)$ then $\phi(R,u,p) = \{R(u)\}$, thus $R(u) \in s$ for all $s \in T_{km}p$ and therefore $l^+(R,p) \subseteq \bigcap_{s\in S} l^+(R,s)$. If $u \notin l^+(R,p)$ then $\phi(R,u,p) = \{\neg R(u)\}$ or $\phi(R,u,p) = \{R(u), \neg R(u)\}$ and, thus, there exists $s \in T_{km}p$ such that $R(u) \notin s$. Therefore $l^+(R,p) = \bigcap_{s\in S} l^+(R,s)$ and completely analogously for $l^-(R,p)$. $\square$

[6]We use the operator $\bigcirc$ to denote the concatenation of sets: let $A = A_1,\ldots,A_n$ be a family of sets. Then $\bigcirc_A$ is defined as

$$\bigcirc_A =_{\text{DEF}} \{\{a_1,a_2,\ldots,a_n\} : a_1 \in A_1, a_2 \in A_2, \ldots, a_n \in A_n\}.$$

Mote that in general for an arbitrary S the partial interpretation $T_{km}^{-1}S$ need not exist in $\mathscr{P}$, if it is excluded by some penumbral connection. Likewise, it may be the case that $T_{km}p = S$, and thus $p = T_{km}^{-1}S$, but also $p = T_{km}^{-1}S$‘ with $S \neq S$‘. Consider the vague predicates *tall*, two individuals denoted by **a** and **b**, and a partial interpretation p where both **a**'s and **b**'s states of tallness is undecided: $l^{+}(\mathbf{tall}, p) = l^{-}(\mathbf{tall}, p) = \{\}$. Then $T_{km}p = S = \{\{\mathbf{tall}(\mathbf{a}), \mathbf{tall}(\mathbf{b})\}, \{\mathbf{tall}(\mathbf{a}), \neg\mathbf{tall}(\mathbf{b})\}, \{\neg\mathbf{tall}(\mathbf{a}), \mathbf{tall}(\mathbf{b})\}, \{\neg\mathbf{tall}(\mathbf{a}), \neg\mathbf{tall}(\mathbf{b})\}\}$ and $T_{km}^{-1}S = p$, but also $T_{km}^{-1}\{\{\mathbf{tall}(\mathbf{a}), \neg\mathbf{tall}(\mathbf{b})\}, \{\neg\mathbf{tall}(\mathbf{a}), \mathbf{tall}(\mathbf{b})\}\} = p$.

On the other hand, a context as in Barker can be easily seen as a set of classical worlds: consider a set C of possible worlds according to Barker and a possible world $w \in C$. Thus, e.g. for the vague predicate *tall* the expression $\mathbf{d}(w)([\![tall]\!])$ denotes the local standard of tallness in world w, and an individual **a** is *tall* in w if and only if $w \in \mathbf{tall}(\mathbf{d}(w)([\![tall]\!]), \mathbf{a})$. Each world w can then straightforwardly be identified with a complete interpretation by evaluating the relevant predicates:

DEFINITION 5 *Let C be a context according to Barker. Then the translation of C to complete interpretations T_bC is defined as*

$$T_bC = \{s(w) \mid w \in C\} \text{ where } s(w) \text{ is the smallest set such that}$$
$$R(u) \in s(w) \text{ if } w \in \mathbf{R}(\mathbf{d}(w)(R), u) \text{ and}$$
$$\neg R(u) \in s(w) \text{ if } w \notin \mathbf{R}(\mathbf{d}(w)(R), u)$$

for all individuals $u \in \mathscr{U}$ and $R \in \mathscr{R}$.

Translating a set of complete interpretations back to a model as in Barker without any further information, however, is not feasible: one would have to fix arbitrary degrees for the cutoff points of the vague predicates, for the degrees to which individuals possess these predicates and, moreover, the scale of these degrees. However, if we assume a fixed initial context C_0 as the "most indefinite" context, i.e. the context at the beginning of a conversation, we can view the power set of C_0 as a context space. We can then find the appropriate subset C of C_0, i.e. a "less indefinite" context, given a set of complete interpretations S just by taking those possible worlds in C_0 which are sanctioned by some interpretation $s \in S$:

PROPOSITION 6 *Let S be a complete interpretation, C_0 a fixed context as defined by Barker. Then a context $C \subseteq C_0$ can be determined such that $T_bC = S$ by setting*

$$T_b^{-1}S = \{w \in C_0 : \exists s \in S. \forall u \in \mathscr{U}. \forall R \in \mathscr{R}. w \in \mathbf{R}(\mathbf{d}(w)(R), \mathbf{u}) \leftrightarrow R(u) \in s\}.$$

This way, given a precification space $\mathscr{P}$ of partial interpretations and an initial context C_0 one can switch between Barker's and Kyburg and Morreau's notions of context: let $p \in \mathscr{P}$ be a partial interpretation. Then $C = T_b^{-1}T_{km}p$ denotes the corresponding context according to Barker and, vice versa, given C we can find p as $p = T_{km}^{-1}T_bC$ as described by Definitions 3 and 4 and Propositions 1 and 2. Moreover, this ability to switch back and forth between both notions of context ensures that no information is lost during this process which is crucial for our aim of comparing the expressiveness of both approaches. At this point the question naturally arises, to which extent these two models are equivalent in the sense that they make the same predictions given such contexts C and p.

One subtle difference between both approaches is the treatment of scale structure: Barker simply stipulates that degrees range over an appropriate scale. There are few restrictions on the scale—it must be a partial order obeying monotonicity—but its exact type is not described *inside* the model. For all of his examples Barker uses linear scales, like people's height. Contrarily, for Kyburg and Morreau there is a priori no such externally defined scale structure, instead it can be defined inside the model itself by the means of penumbral connections. Let us use *tall* as an example for an adjective denoting a degree on a linear scale. Other weaker scale structures, however, can be modeled completely analogously. The binary predicates **taller_than** and **as_tall_as** and the unary vague one **tall** can be can be characterized as:

$$\begin{array}{rl}
(NV_1) & D(\forall x \forall y.(\mathbf{as_tall_as}(x,y) \vee \neg \mathbf{as_tall_as}(x,y)) \\
(RE_1) & D(\forall x.\mathbf{as_tall_as}(x,x)) \\
(TR_1) & D(\forall x \forall y \forall z.(\mathbf{as_tall_as}(x,y) \wedge \mathbf{as_tall_as}(y,z)) \\
 & \rightarrow \mathbf{as_tall_as}(x,z)) \\
(SY_1) & D(\forall x \forall y.(\mathbf{as_tall_as}(x,y) \rightarrow \mathbf{as_tall_as}(y,x)) \\
(NV_2) & D(\forall x \forall y.(\mathbf{taller_than}(x,y) \vee \neg \mathbf{taller_than}(x,y)) \\
(TR_2) & D(\forall x \forall y \forall z.(\mathbf{taller_than}(x,y) \wedge \mathbf{taller_than}(y,z)) \\
 & \rightarrow \mathbf{taller_than}(x,z)) \\
(TI_2) & D(\forall x \forall y.\mathbf{taller_than}(x,y) \vee \mathbf{taller_than}(y,x) \vee \mathbf{as_tall_as}(x,y)) \\
(i) & D(\forall x \forall y.(\mathbf{tall}(x) \wedge \mathbf{taller_than}(y,x)) \rightarrow \mathbf{tall}(y)) \\
(ii) & D(\forall x \forall y.(\neg \mathbf{tall}(x) \wedge \mathbf{taller_than}(x,y)) \rightarrow \neg \mathbf{tall}(y))
\end{array}$$

Properties (NV_1) and (NV_2) here ensure that both **as_tall_as** and **taller_than** are non-vague; at each partial interpretation any pair of individuals is either in their extension or anti-extension. This goes in accordance with Barker [3] arguing that comparative clauses like *taller than* do not involve any vagueness. Properties (RE_1), (SY_1), and (TR_1) ensure that **as_tall_as** is indeed an equivalence relation by postulating reflexivity, symmetry, and transitivity. Similarly, properties (TR_2),(TI_2) ensure that **taller_than** is a strict total order by postulating that it is transitive and trichotomous. Finally, properties (i) and (ii) ensure that the vague predicate **tall** respects this total ordering. Rooij [18] also gives similar axiomatizations for other orderings motivated by taking the positive adjective, e.g. *tall*, as a starting point and observing how *tall* behaves with respect to comparison classes. These orderings can be characterized by penumbral connections completely analogously as described here for a linear ordering.

Assume now that a situation is modeled twice: one time using a scale based model in the sense of Barker and another time using a delineation model in the sense of Kyburg and Morreau where the scales used in Barker's model are accordingly encoded as penumbral connections. Our aim is to elaborate under which circumstances predictions made by these models coincide.

DEFINITION 7 *Let C_0 be a context as defined by Barker and $\mathscr{P}$ be a precification space in the sense of Kyburg and Morreau. C_0 and $\mathscr{P}$ are called* corresponding models *if the following conditions are met:*

- *for each vague predicate p in C_0 there is predicate P in $\mathscr{R}$ and vice versa,*
- *for each individual* **a** *in C_0 there is an object a in $\mathscr{U}$ and vice versa,*
- *for each $m \in \mathscr{P}$ there exists $C = T_b^{-1} T_{km} m \subseteq C_0$ with $C \neq \emptyset$, and*
- *for each $C \subseteq C_0$ there exists $m = T_{km}^{-1} T_b C \in \mathscr{P}$.*

The notion of *corresponding* models of the two different approaches intuitively means that the same situation is modeled both using Barker's approach and using Kyburg and Morreau's one making the same assumptions. Therefore the last two requirements ensure that neither the penumbral connections on the partial interpretations are too strong, i.e. they forbid situations which are present in the other model, nor the initial set of possible worlds C_0 implicitly contains restrictions which are not reflected in the penumbral connections. These two requirements also imply that for each vague predicate under consideration the scale implicitly given in Barker's model is expressed in $\mathscr{P}$ via penumbral connections: if according to Barker, some individual is regarded as *tall* in all possible worlds, another individual with a higher degree of height is automatically also regarded as tall. If a precification space $\mathscr{P}$ according to Kyburg and Morreau fails to sanction such relationships via penumbral connections, then there will be precification points which cannot be translated to an according context as in Barker. Also note that, on the other hand, there are penumbral connections for Kyburg and Morreau's contexts which cannot—even implicitly—be expressed in Barker's approach. Therefore such penumbral connections have to be omitted when looking for a corresponding model following Barker. Take $D(P(a) \to P(b) \vee \neg P(b))$ as an example. Suppose, initially both $P(a)$ and $P(n)$ are undecided. Then, as soon as we add the new information $P(a)$ we also have to decide whether $P(b)$ or $\neg P(b)$. Using Barker this requires that among the possible worlds where **a** is P there are such worlds where also **b** is P and such worlds where **b** is not P. But when translating such a context to a partial interpretation of Kyburg and Morreau's model as described, this amounts to a precification point where a is in the extension of P while b's status of P-ness is left undecided. Therefore a context space with such a penumbral connection cannot correspond to one of Barker's contexts. Excluding such penumbral connections does, arguably, cause no harm since such a situation seems highly artificial and the ambiguity concerning the next precification point when updating with $P(a)$—whether $P(b)$ or $\neg P(b)$—would lead to confusion between the speaker and hearer in a real-life situation.

PROPOSITION 8 *Let C_0 and $\mathscr{P}$ be two corresponding models as in Definition 7 and let s be a proposition of the form "***a** *is p", "***a** *is not p", or "***a** *is more p than* **b***" such that $[\![s]\!](C_0) \neq \emptyset$.*

Then there exists a unique, most general partial interpretation $m \in \mathscr{P}$ such that $P(a)$ is true *at m, $P(a)$ is* false *at m, or $Per\_than(a,b)$ is* true *at m, respectively, and the translations to sets of complete interpretations $T_b([\![s]\!](C_0))$ and $T_{km}m$ coincide.*

Proof Assume, no such unique, most general partial interpretation $m \in \mathscr{P}$ exists as described. This means that when adding s to the current knowledge base the penumbral connections enforce that also some other information is added but there are several ways to do so. Such a situation is, however, ruled out by the last two requirements of Definition 7 since it can not be equivalently modeled in a context as defined by Barker as exemplified above.

Let m_0 be the root of $\mathscr{P}$; by Definition 7 both $T_b^{-1}T_{km}m_0$ and $T_{km}^{-1}T_bC_0$ coincide and $c = T_{km}m_0 = T_bC_0$. When performing the update with s as described by Barker, the possible worlds of C_0 where s does not hold are filtered out to form the new resulting context: $C' = [\![s]\!](C_0) \subseteq C_0$. The according translation to sets of classical worlds $c' = T_bC'$ can therefore be equivalently obtained by removing those sets from c where s does not hold. Observe that because of Definition 7 there exists $m' = T_{km}^{-1}c' \in \mathscr{P}$. Since s holds at all classical worlds in c', it is also true at m'; e.g. if s is of the form "**a** *is p*" then $a \in l^+(p, m')$. Therefore, $m \leq m'$ and if m and m' are different, there exists a literal t such that t is true at m' but indefinite at m, hence $c_0 \supset T_{km}m \supset c$. However, as c denotes exactly the worlds in c_0 where s is true, $T_{km}m$ cannot lie between these two while satisfying s. Therefore m and m' coincide and thus $T_{km}m$ and T_bC coincide. □

Note the requirement that $[\![s]\!](C_0) \neq \emptyset$. This means that the proposition s is not inconsistent with the context C_0. If it is, Barker's and Kyburg and Morreau's approaches differ significantly: While, using Barker, the empty context is returned for signaling a false or inconsistent proposition, Kyburg and Morreau describe how to update the context and still retaining as much information as possible as described in Section 3.2.

THEOREM 9 *Let C_0 and $\mathscr{P}$ with root m_0 be two corresponding models as in Definition 7 and consider a sequence $s_1, \ldots, s_n$ of propositions of the form "**a** is p", "**a** is not p", and "**a** is more p than **b**". Let the context and precification point obtained after updating C_0 and m_0 with all of s_1 to s_n be denoted as C and m, respectively. If $C \neq \emptyset$, then an additional proposition s is true at C if and only if s is true at m.*

Proof The theorem follows immediately from Proposition 8 by noting that after each update the resulting contexts can be translated into the same set of complete interpretations. Therefore also C and m can be translated to the same such set, and thus both approaches make the same predictions. □

Informally, Theorem 9 tells us that both Barker's and Kyburg and Morreau's approaches are capable of making exactly the same predictions under three conditions: first, the assumptions made about the modeled situation must be such that they can be expressed in both models. Second, the uttered propositions are of the simple form as in the theorem. This is largely due to the fact that Kyburg and Morreau do not deal with measure phrases in their model. Third, all information must be consistent as described above. This is shown by describing how a context in either model can be translated to some intermediate representation (i.e. a set of classical worlds) without losing any information and vice versa how a context in either approach can be constructed from this intermediate representation. Finally, the important observation is that an utterance of a simple proposition has the same update effect on that intermediate representation of the context, no matter if we are using Barker's or Kyburg and Morreau's notion of context.

Conclusion and further work

Although scale and delineation based models seem fundamentally different at the first glance, we have seen that under certain preconditions they are capable of making the same predictions with regard to simple propositions. Barker's approach allows one to formulate predicate modifiers such as *very*, *definitely*, or *clearly* in a more fine grained manner than Kyburg and Morreau's since it explicitly models (possible) standards for vague predicates. Moreover, it straightforwardly allows for more complex propositions e.g. such as "*John is 2 cm taller than Jack*". On the other hand delineation models, and in particular Kyburg and Morreau's model, are clearly superior when it comes to inconsistent information and the shifting of vague standards. Their notion of context update builds on the well known and well elaborated theory of belief revision. However, there seems to be no fundamental reason why something similar should not be possible to be modeled for a scale based account; only the relationship to the AGM theory of belief revision will most probably be less direct than for Kyburg and Morreau's approach. Therefore such a treatment of inconsistent vague information within a scale based approach is left as a promising task for future research.

An advantage of the delineation approach other than accommodation of vague standards is that it seems computationally more feasible than scale based approaches. As we have seen, Barker initially presupposes a context space containing all possible standards of the vague predicates in question and all possible assignments from the relevant individuals to degrees. Constraints on these values such as relations between the vague standards of different predicates or the knowledge that some individual possesses some property to a higher degree than another individual are only implicit in the data and not modeled explicitly. Also scales for the vague gradable predicates under consideration are not modeled explicitly. For a delineation approach, on the other hand, initially only a partial interpretation together with a set of penumbral connections is sufficient and no degree values are needed.

One point where scale based and delineation approaches can be combined is Kyburg and Morreau's notion of context update. Remember the example in Section 3.2 where one out of the two individuals Jack and John was selected by the definite description "*the tall and bald guy*". There, we have to either revoke John's status of baldness or Jack's status of tallness. The only criterion that at as little information as possible shall be invalidated does not suffice in this situation to pick one of the two referents. It seems as in order to make (realistic) predictions about how humans handle such situations, we need more information, namely their degrees of height and tallness, respectively. Then the referent will be preferred for whom the standard has to be shifted by a lesser amount. It seems easier to remove the status of tallness from someone who is 1.70 m tall than from someone who is 1.90 m tall, although in some scenario both may not be described as tall at all. Kyburg and Morreau describe the use of *selection functions* for selecting one out of several ways to perform an update as described above. Such a selection function may be defined by means of Barker's model: perform an update in Barker's model for each way of updating and then make a choice such that as few possible worlds are filtered out as possible.

Another point of future research is the connection between Kyburg and Morreau's approach and Shapiro's account of vagueness in context. As we have seen, by dropping the completability requirement and using Shapiro's model instead of supervaluation allows for the formulation of more fine-grained penumbral connections especially when it comes to Sorites situations. Moreover, Shapiro's interpretation of partial precifications as model for contexts during a conversation and not just a technical vehicle arguably fits Kyburg and Morreau's purpose more closely.

BIBLIOGRAPHY

[1] C.E. Alchourrón, P. Gärdenfors, and D. Makinson. On the logic of theory change: Partial meet contraction and revision functions. *Journal of symbolic logic*, 50(2):510–530, 1985.
[2] C. Barker. On the dynamics of second-order vagueness. preliminary report.
[3] C. Barker. The dynamics of vagueness. *Linguistics and Philosophy*, 25(1):1–36, 2002.
[4] M. Dummett. Wang's paradox. *Synthese*, 30(3):301–324, 1975.
[5] K. Fine. Vagueness, truth and logic. *Synthese*, 30(3):265–300, 1975.
[6] J. Groenendijk and M. Stokhof. Dynamic predicate logic. *Linguistics and Philosophy*, 14(1):39–100, 1991.
[7] I. Heim. File change semantics and the familiarity theory of definiteness. In *Formal semantics: The essential readings*, pages 223–248. Blackwell Publishers, 2008.
[8] I. Heim. On the projection problem for presuppositions. In *Formal semantics: The essential readings*, pages 249–260. Blackwell Publishers, 2008.
[9] H. Kamp. Two theories about adjectives. *Formal semantics of natural language*, pages 123–155, 1975.
[10] C. Kennedy. Projecting the adjective: The syntax and semantics of gradability and comparison. *The number sense*, 4(4):10, 1999.
[11] C. Kennedy. Vagueness and grammar: The semantics of relative and absolute gradable adjectives. *Linguistics and Philosophy*, 30(1):1–45, 2007.
[12] E. Klein. A semantics for positive and comparative adjectives. *Linguistics and Philosophy*, 4(1):1–45, 1980.
[13] E. Klein. The interpretation of adjectival comparatives. *Journal of Linguistics*, 18(01):113–136, 1982.
[14] E. Klein. Comparatives. In A. von Stechow and D. Wunderlich, editors, *Semantics*, pages 673–691. Berlin: de Gruyter, 1991.
[15] A. Kyburg and M. Morreau. Fitting words: Vague language in context. *Linguistics and Philosophy*, 23(6):577–597, 2000.
[16] D. Lewis. General semantics. *Synthese*, 22(1):18–67, 1970.
[17] D. Lewis. Scorekeeping in a language game. *Journal of philosophical logic*, 8(1):339–359, 1979.
[18] R. Rooij. Vagueness and linguistics. *The Vagueness Handbook, to appear*.
[19] S. Shapiro. *Vagueness in context*. Oxford University Press, 2006.
[20] S. Shapiro. Reasoning with slippery predicates. *Studia Logica*, 90(3):313–336, 2008.

Christoph Roschger
Vienna University of Technology
Favoritenstr. 9-11/E1852
1040 Wien, Austria
Email: roschger@logic.at