Comments on *Comparison of Complex Predicates: 'and', 'or' and 'more'* by Galit W. Sassoon

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In "Comparison of complex predicates: *and*, *or*, and *more*", Galit W. Sassoon reports the results of a survey involving complex predicates such as 'fat and tall' and 'fat or bald'. Respondents were asked to compare a variety of characters with regards to these complex predicates, for example to say whether or not one is more *fat and tall* than another.

The experimental exploration of compound predicates is in its infancy, and Sassoon has significantly expanded the range of data we now have available. This is important and valuable work.

Sassoon interprets her data as posing a difficulty for certain fuzzy-logic-based theories of vague predication. In this note, I want to suggest that her data, if robust, would cause difficulty for a wider range of fuzzy- and fuzzyish-logic-based theories than those she considers. I also want to urge caution about some of the conclusions Sassoon draws from her data.

1 Linear order and partial order

One crucial piece of data Sassoon appeals to in her argument against fuzzy approaches to these comparisons of complexes can be seen in §2.2.2, where she discusses responses to questions 4c ('Is any of them more *fat and bald* than the other?') and 4e ('Are they equally *fat and bald*?'). These questions are asked about the characters Aharon and Danny, where Aharon weighs 100 kg and is not bald, and Danny weighs 70 kg and is bald. To both questions, the answer 'No' predominates. Sassoon suggests that degree functions cannot account for this pattern of judgments.

Indeed, no assignment of degrees from a *linearly ordered* set could account for this data. If Aharon and Danny bear degrees of *fat and bald* from a linearly ordered scale, then either Aharon's degree is greater than Danny's, Danny's degree is greater than Aharon's, or the two degrees are equal—this is exactly the linear order condition. Sassoon's introduction to degree theories in §1.2 suggests that she assumes that the degree theorist is committed to this linear order condition: "Fuzzy logic is a form of multi-valued logic, whereby propositions may have as a truth value any number in the real interval [0, 1]". Note that the real interval [0, 1] is linearly ordered: given any two numbers x and y in the interval, either x is greater than y, y is greater than x, or $x = y$.

However, a number of degree theorists have advanced theories on which the degrees are *not* linearly ordered, but rather only partially ordered. For example, the theories advanced in [Slaney, 1988, Paoli, 1999, Weatherson, 2005] all have this feature. NB: I'm not too fussed about terminology here: maybe these approaches count as 'fuzzy' and maybe they don't. What's interesting, I think, is to explore the ways in which these theories can and cannot account for Sassoon's data. As it turns out, I think these theories all face difficulties here, suggesting that Sassoon may well be able to use her data to rule out, or at least cause trouble for, a wider range of logical approaches than she considers.

1.1 Partially ordered degrees

The theories advanced in [Slaney, 1988, Paoli, 1999, Weatherson, 2005] are importantly different from each other in a number of respects, which I hereby declare my intention to ignore. The important features for our purposes here are shared by all three approaches. I use $\leq$ for the partial order on degrees.

First, all three approaches allow for degrees c and d that violate linear order; that is, such that $c \not\leq d$ and $d \not\leq c$. This feature seems to allow them to accommodate Sassoon's data in 4c and 4e: if respondents assign Aharon and Danny such degrees for *fat and bald*, then their responses are just what we should expect. Neither is greater than the other, nor are they equal.

1.2 Conjunction and monotonicity

The trouble threatens, rather, when we look to section 3, particularly questions 3a ('Is Moshe more *fat* than Danny?') and 3e ('Is Moshe more *fat and tall* than Danny?'). In this section, Moshe weighs 100 kg and is 195 cm tall, and Danny weighs 70 kg and is also 195 cm tall. Respondents overwhelmingly chose 'Yes' as a response to question 3a, and 'No' as a response to question 3e.

In what follows, I need to make two assumptions about responses to questions that Sassoon, unfortunately, did not ask participants. Neither is terribly risky, I don't think. The first is that respondents would overwhelmingly answer 'Yes' to the question whether Moshe and Danny are *equally* tall, or the question whether they are *as tall as* each other. If we assume linear order, respondents' 'No' answers to 3c ('Is one of them more *tall* than the other?') would settle this, but here we are precisely *not* assuming linear order, so this becomes a separate question. However, since both Moshe and Danny are 195 cm tall, it would be very surprising if respondents did not judge them to be equally tall. The second assumption is perhaps more controversial, but I think also relatively secure: we need to know what respondents think of the question whether Moshe and Danny are *equally* fat and bald. I assume the answer is 'No'; this seems quite plausible, but is not supported by any data.

With these two assumptions in hand, we have enough to put pressure even on the partially-ordered approaches to degrees cited above. All three take conjunction (which I'll write $\wedge$) to be *monotone*.[1] ([Paoli, 1999] offers a logical system with two distinct conjunctions, but both are monotone.) But let f_M and f_D be Moshe's and Danny's respective degrees of fatness, and let t_M and t_D be their respective degrees of tallness (in all cases, as attributed by respondents). By the first assumption, $t_D = t_M$. By the responses to 3a, $f_D \leq f_M$. Thus, by the monotonicity of conjunction, $f_D \wedge t_D \leq f_M \wedge t_M$. However, responses to 3e rule out Moshe's being *more* fat and tall than Danny, and the second

[1] Where $\leq$ is the partial order on degrees, a binary connective $\cdot$ is *monotone* iff $c \leq c'$ and $d \leq d'$ imply $c \cdot d \leq c' \cdot d'$.

assumption rules out their being *equally* fat and tall. Something, then, has gone wrong. If the assumptions and the data are all sound, then the trouble is with the assumption that conjunctions are monotone. Even partially-ordered approaches, then, may well face trouble in the area.

This trouble is not exclusive to partially-ordered theories, though. All t-norms are monotone; thus, all fuzzy logics that interpret conjunction as a t-norm—including the vast majority of linearly-ordered fuzzy logics—will face this difficulty as well.

2 Disjunction

It seems a good bet that this trouble with the conjunctive data might have an echo in the disjunctive data. After all, all three above theories predict disjunction to be monotone as well, as does any fuzzy theory that analyzes disjunction with a t-conorm (again, the vast majority of linearly-ordered fuzzy logics). If the disjunctive data reproduce the pattern of the conjunctive data, the same problem ought to appear.

Unfortunately, things here are much less clear. The most basic data about comparative judgments of disjunctions comes from questions 4g ('Is one of them [ie Aharon or Daniel] more *fat or bald* than the other?') and 4h ('Who is more *fat or bald*?'). Here, most respondents (74%) answered 'Yes' to 4g, but only 46% of respondents answered 4h at all. (Of these, 63% said 'Danny'.)

Sassoon interprets this as follows: since Aharon is more fat than Danny and Danny is more bald than Aharon, each of them is either more fat or more bald than the other. If respondents interpret 'more *fat or bald*' as 'more fat or more bald', then we should expect them to judge that each of Aharon and Danny is more fat or bald than the other, leading to a 'Yes' response to 4g. However, since each is more fat or bald than the other, when respondents are asked to identify which one is more fat or bald in 4h, they cannot, and as a result do not answer the question.

This is plausible enough as far as it goes, but I think it cannot yet be taken as established. When Sassoon says 'the 19 subjects (54%) not providing an answer to [4h] indicate that they interpreted 'more *fat or bald*' as 'more fat *or* more bald', classifying both characters as such', she overstates the case. There are any number of reasons why a respondent might fail to answer any particular question. If the reason is indeed the one Sassoon hypothesizes, this should be easy enough to get better evidence for. First and foremost, one could simply expand the answer space, allowing respondents to answer 'Both' to 4h. Sassoon's explanation for the non-responses would predict respondents to be strongly drawn to the 'Both' answer; presumably competing explanations would not.

Sassoon goes on to interpret the responses of the 16 respondents who do answer 4h, I think far too hastily. She says: '[O]f the 16 answers, the majority (10, which make 63%) selected Danny as their candidate, explaining that Danny is clearly bald and hence *fat or bald*, while Aharon is not clearly fat, thus not clearly *fat or bald*. Thus, these subjects were using a Boolean union rule for classification under disjunctions'. But this is a nonsequitur, even if all 10 respondents provided this exact reasoning, because the very same reasoning would be predicted by a wide variety of fuzzy and other theories as well. Any logic of disjunction validating the inference from A to $A \vee B$ will do. Thus, the reasoning in favor of 'Danny' offered by Sassoon's respondents to 4h does very little to support her Boolean hypothesis.

Sassoon seems to realize this, and briefly considers the possibility that those respondents that answered 4h were able to do so via their use of a fuzzy disjunction. She argues against this as follows: '[H]ad this been the case, these 16 subjects would have agreed to consider one of the characters as *more fat and bald* in the second conjunctive condition, but they did not … Thus, these results do not indicate fuzzy reasoning'. This assumes that conjunction and disjunction will be *uniform* in their fuzziness: either both fuzzy or neither. This does not seem like a warranted assumption. Moreover, if Sassoon is willing to make such an assumption, it's unclear why she considers disjunctive responses at all. The argument she offers that conjunctions are not interpreted fuzzily ought to have settled the issue.

Once we allow for the possibility that conjunctions are not interpreted fuzzily but disjunctions are, however, we see that some separate argument is needed for the claim that disjunctions are not interpreted fuzzily. I should flag: Sassoon may well be right that they are not. In fact, I see no better explanation for the lack of responses to 4h than the one she offers. But if her explanation is right, it should not be hard to find better evidence for it than this.

In sum, I take Sassoon's data to show, in some respects, more than she gives it credit for. Linear order is not the only trouble with fuzzy logics; theories taking conjunction to be monontone will have trouble as well. This covers not only the usual linearly-ordered fuzzy theories, but also their best-known partially-ordered relatives. On the other hand, her data regarding judgments of disjunctions is not as compelling as she takes it to be. More caution, and more data, is required.

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