
Reply to Christian Fermüller's Comments on *Vagueness: A Mathematician's Perspective*

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Once, mathematicians discussed intensely the foundations of their field, but awareness of the underlying problems has largely disappeared. Today hardly any mathematician spends much time contemplating what mathematics actually is; we are satisfied with the belief that we examine things in great generality and with great precision. In our daily work, the question of what it means to model something by formal means is in fact rarely important. Sometimes, however, the lack of insight into the process of mathematical modelling leads to problems. Philosophers have traditionally dealt with all sorts of fundamental questions. However, if they are unable to provide satisfactory advice, we (mathematicians) must ourselves take the initiative, though we then risk compromising ourselves. We may easily come into conflict with achievements established in philosophy or elsewhere.

My concern has been the formal treatment of vague properties. In search of a reasonable basis for my work on vagueness, I learned that the path I chose ignores substantial recent achievements in philosophy and linguistics. Who would have expected that the harmless-sounding topic "vagueness" would lead to conflicts of this magnitude?

I appreciate that Chris Fermüller's comments are formulated in a way that indicates tolerance. Still, he rejects my argument almost entirely. Fermüller suggests that there is no hope of basing a reasonable theory of natural language upon the approach that I have called perceptualism a conviction he shares with the vast majority of contemporary philosophers. Thus, I must accept the fact that I am not in line with the experts in fundamental matters. Moreover, Fermüller points out that my arguments also conflict with principles upheld in linguistics.

I am open to new ideas. From these sobering conclusions, however, hardly anything follows. Philosophers have not provided a convincing and commonly understandable solution to the problem addressed, let alone a solution with practical impact. Moreover, the friction between linguists' work and my standpoint concerns, presumably, natural language processing technologies, a very productive field, but one in which, I dare say, achievements lag behind original expectations. For instance, it is at present not possible to infer automatically and reliably from a medical report whether a given diagnosis is confirmed or excluded.

Let me review the matter of content. If Fermüller conjectures that I reject the possibility of finding a uniform and all-encompassing formal approach to natural language such as a universal calculus that is at our disposal to formalize any kind of linguistic situation, he is right. In particular, I do not value first-order theories referring to a fixed

model of the world, allowing, for instance, quantification over “all” existing objects of a certain kind. Work in this style might be of some particular use, but its applicability is restricted for reasons of principle. By means of a first-order theory, we can explore only a specific abstract structure; mathematics does not really have a wider scope. In natural language, in contrast, we deal with dynamically changing structures; as we speak, models are continuously created, modified, revised, rejected and replaced. This is the relevant topic. In mathematics, we must—and we can—systematically check all relevant possible behaviours of a property, and we are then capable of making general statements. Language instead deals with the typical circumstances in which a property holds; we make statements in terms of similarities and differences, and to check systematically all related circumstances is neither possible nor required. Fermüller hints at a critical attitude of mine towards the syntax-semantics-pragmatics triangle. Separation of these three areas is, without doubt, possible; priorities, however, are often chosen in a somewhat peculiar way. To assign the main role to syntax blurs the fundamental distinction between natural and formal language.

Fermüller’s criticism culminates in the claim that a theory of natural language based on the pure notion of perception would be untenable. All I can set against these doubts is an explanation of how a formalization of natural language based on perceptualism would look. Note that my actual contribution focussed on the formal treatment of vagueness in the case when new mathematical methods are required, and, as usual, I considered the case in which a coarse and the finest possible level of granularity are to be combined. Formalizing natural language in general is a different matter; we then deal with coarse levels only. To explain in a single paragraph the implications of the perception-based viewpoint is, however, impossible; I restrict myself to some key points.

The crucial role is not played by perceptions in isolation; such an approach might indeed be untenable; the link between perceptions made at different times would be missing. The crucial role is played by the models which systematize these perceptions.

Accordingly, perceptualism puts emphasis on models rather than on formal statements. A natural-language utterance (where “natural language” is understood in the same restricted way as in the main text) evokes a picture in a person’s mind, and it is this picture, against the background of pictures imagined before, that is to be modelled. A model is a finite first-order structure that captures selected aspects of the situation in question in comparison with other situations. This idea contrasts sharply with the common procedure of directly creating formulas according to the syntactical structure of the sentence. Moreover, a picture can be more or less detailed. By default, we do not choose the finest available model associated with a situation, but one that includes just enough details to represent the situation adequately. The finest possible model is the limit of a sequence of increasingly fine-grained models and is generally not relevant; the converse standpoint that a coarse model arises from blurring a fine model is rejected. Finally, the role of the “world” as the totality of what we observe around us is identified with the systematics according to which we endow pictures with structure. The “world model” offers generally applicable structural features including, in particular, our everyday experiences.

Fermüller argues that we can easily distinguish between an object and its perception. A theory of language that does not account for this difference would indeed be of limited

use. To see how this aspect can be included in our considerations, we must clarify what the distinction actually means. When referring to the object itself, we associate with it all its actual and possible properties known or predictable from experience; when referring to its perception, we mean merely the fact that we observe it, even if it is a mirage. Thus, in one case we deal with all the consequences implied by integrating the object into the associated model; in the other case, we deal with one observation.

It is my own firm conviction that questions refer ultimately to perceptions or else they are meaningless; problems may just *appear* to be problems. To ask whether objects are mind-independent is to leave the realm of perceptions; the question is meaningless. Accordingly, we cannot associate with an object anything other than actual and possible experiences involving it. We see that this restriction does not conflict with the requirement to treat objects and their perceptions in different formal ways.

However, this restriction does conflict with Fermüller's demand to model sentences such as "Peter is tall" independently of perceptions. This utterance is made when, among the people the speaker has in mind, Peter sticks out due to his height. To ask for independent criteria of "tallness" does not make sense.

Fermüller claims that a realist frame is a prerequisite for natural language modelling. This claim is intended to oppose my standpoint that the realism-based viewpoint cannot reasonably account for vagueness. However, I was referring to an extreme form of realism, on the basis of which the notion of vagueness cannot even be properly characterized. The word "real" is part of our language and thus has a role. We can ask what "real" means within the model that describes circumstances specified in natural language: it means conforming to the model. Consequently, a frame for natural language must indeed be realist, namely in the sense that it can deal with the statement that something is in line with generally accepted expectations.

Understanding reality relative to a model in fact brings us close to a methodology for which I have indicated tolerance: "realism-based ontology". Its proponents might strongly deny any parallels with my approach. However, I consider that, when it comes to practical work, the main principles are shared. In particular, it is essential for both approaches that statements can be made from different perspectives, in particular at different levels of granularity, and that no perspective is *a priori* superior to others. Furthermore, in both approaches, theories are never considered as fixed and definite, but as subject to revision whenever necessary.

Fermüller evaluates the idea of shifting levels of granularity positively, and he predicts "provocatively"—that an elaboration of this idea will bring us close to a realist frame. I have indicated the sense in which he is, in fact, right. Still, the prediction is surprising. Realists seem to think in terms of precise facts; all coarse notions are assumed to possess an interpretation at the one and only finest possible level. If we rely on perceptualism, there simply will not be any fine level: unless somebody uses a ruler, lengths will not be modelled in centimetres. The fact that shifting levels of granularity involves ambiguity will, for the realists, remain an unsolved problem.

Personally, I take the—no less provocative—opposite viewpoint: whatever can be achieved within a framework based on naive realism can *a fortiori* be achieved within a framework based on perceptualism.