Reply to Lluís Godo's Comments on *Have Fuzzy Sets Anything to Do with Vagueness?*

DIDIER DUBOIS

In the following I briefly reply to the points made by Lluís Godo in his comment. Some of them reflect familiar objections made to the representation of vague categories by precise membership functions.

- Outside the need for a simple practical representation of verbal pieces of gradual information in information engineering areas, it is clear that I did not claim the possibility of having a one-to-one correspondence between gradual properties and membership functions in the absolute. For one, if the consensus on the meaning of words across people is often sufficient for words to be useful in communication between them, it is far from a perfect consensus. Worse, even for a single person the meaning of words is not fully stable across circumstances of their usage. Finally, membership functions are clearly an idealisation of some complex reality that is quite hard to grasp, and that is called vagueness. To be fair, membership functions are especially useful for predicates referring to measurable scales. Beyond this framework (which is nevertheless instrumental in applications) it is not clear they can be easily defined at all.

- The point whereby replacing membership functions by set-valued ones that capture imprecision on the actual membership values is paradoxical, since it transfers artificial precision to the membership functions delimitating the set of acceptable interpretations, is questionable. It makes sense if one admits that a membership grade is then replaced by a set that represents a new kind of membership grade more adapted to the vagueness situation. Then indeed the difficulty to be precise about the original membership function becomes the difficulty to describe a set-valued one precisely, and so forth ad infinitum with higher order representations. However the set-valued function used does not represent a membership function, it is not a substitute to it. It only represents what we know of the membership grades that are assumed to be precise (in the adopted model, if not in reality). So the precision on upper and lower bounds is not crucial: they are just upper and lower bounds. It is always possible to widen the sets if the set-valued function is viewed too narrow to be trusted. This is the same situation as when guessing the value of some quantity x which is incompletely known. Then we write $x \in [a,b]$ where a,b are precise. In case of doubt we can always decrease a and increase b, instead of trying to figure out precise values of a and b, and modeling them in turn by intervals. The point is that if the actual objective value of x

exists, there is no real value for the upper and lower bounds. Likewise, as already we can question the existence of membership functions (we have argued elsewhere they are just convenient fictions [1]) we can definitely claim that the existence of membership functions delimitating the set of acceptable representatives is even more elusive, and thus the question of their precision is arguably meaningless.

- I agree that in mathematical fuzzy logic, degrees of truth are internalized and thus the issue of measuring membership or truth-values is irrelevant in that setting. On the contrary in more explicit use of fuzzy sets the question of obtaining membership functions in a simple and relevant manner remains an important issue.

- I am also sympathetic to the idea that some extensionality property seems to be at work when modeling gradual concepts by membership functions. Especially if the measurement scale is continuous, it comes down to avoiding thresholding effects. Gradualness should come along with a form of continuity of membership functions enabling a graceful degradation when moving away from prototypes of a gradual concept. Requiring more than mere continuity, namely some kind of smoothness, by constraining the derivatives of the membership function, or exploiting similarity conditions (the more x and y are similar, the more the truth-value of $P(x)$ is close to the truth-value of $P(y)$) then becomes a natural issue to study.

BIBLIOGRAPHY

[1] D. Dubois, H. Prade: Fuzzy sets—A convenient fiction for modelling vagueness and possibility, comments on a paper by Laviolette and Seaman. *IEEE Transactions on Fuzzy Systems*, 2(1):16–21, 1994.