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## Comments on *Fuzzy Logic and Higher-Order Vagueness* by Nicholas J.J. Smith

FRANCESCO PAOLI

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This interesting and well-written paper is an attempt to provide a solution to the long-standing problem of higher-order vagueness, or artificial precision, which has so far undermined the plausibility of the fuzzy perspective on vagueness. After reviewing other approaches to the problem, the author contends that the proper way out of this quandary is what he terms *fuzzy plurivaluationism*. The arguments advanced by the author in favour of his viewpoint are clear and well-articulated, while the paper contains a survey of the state of the art on this ongoing debate that stands out for its commendable completeness. In what follows, therefore, I will not dwell on the undeniable merits of the paper, but I will rather attempt to explain why I remain unconvinced by Smith's reasoning to the effect that fuzzy plurivaluationism is really the way to go.

Smith is right when he claims that any serious bid to tackle the issue of vagueness should offer not only a recipe to solve the sorites paradox and the like, but also a general view on the nature of the phenomenon itself. We must distinguish "between a *surface characterization* of  $P$ —a set of manifest conditions, possession of which marks out the  $P$ 's from the non- $P$ 's—and a *fundamental definition* of  $P$ —a statement of the fundamental underlying nature of the  $P$ 's, which explains why they have such-and-such surface characteristics" (pp. 12–13). Features like the presence of borderline cases or blurry boundaries, or sorites susceptibility, may constitute a surface characterization of vagueness (of predicates), but none of these marks counts, according to Smith, as a fundamental definition of the concept. The latter is given instead by what he calls the *Closeness principle*: "If  $a$  and  $b$  are very close/similar in respects relevant to the application of  $F$ , then ' $Fa$ ' and ' $Fb$ ' are very close/similar in respect of truth".

This is all well and good insofar as we confine ourselves to *dimensional* predicates ('tall', 'old'), which come with a naturally associated metrics. It is crystal clear what it means for  $a$  and  $b$  to be very close/similar in respects relevant to the application of 'tall', for we can conveniently rephrase our statement in terms of the *measures* of  $a$ 's and  $b$ 's heights. But there is more to vagueness (of predicates) than dimensional predicates have to offer, although their sorites susceptibility (and the circumstance that the sorites paradox is ubiquitous in discussions about vagueness) has turned them by and large into the prototypical examples of the concept. Evaluative predicates ('beautiful', 'clever') are more difficult to get a hold of, given that they lack an obvious metrics (as Max Cresswell [4, p. 281] vividly puts it: "Must we postulate the kalon as a degree of beauty or the andron as a degree of manliness?") and are typically multidimensional. Still, they are uncontentiously considered as vague, given their possession of several traits collectively associated with vagueness—they are intrinsically gradable; they are compatible with

hedges; they admit a nontrivial comparative; they have borderline cases of application; they normally come in antonymous pairs. It remains to be seen whether the Closeness principle makes sense when  $F$  is an evaluative predicate. What does it mean for  $a$  and  $b$  to be very close/similar in respects relevant to the application of ‘clever’?

I may agree with Smith that the presence of borderline cases does not provide a fundamental definition of vagueness—perhaps borderline cases are just the surface manifestation of some more basic property still waiting to be unveiled. Yet, I am not completely persuaded by his claim that it fails to be a necessary and sufficient condition for vagueness of predicates. Smith cites the example of a predicate, ‘schort’, having the same associated metrics as ‘short’, but whose domains of application and non-application (relative to the given metrics) are separated by sharp cut-off points from a no man’s land where ‘ $a$  is schort’ is neither true nor false. The example is admittedly artificial, and I believe that its artificiality could divert from its controversial character. It makes perfectly sense to say that ‘ $a$  is short’ in case  $a$  is a borderline case for a short person. More than that: a necessary condition for  $a$  to be a borderline case for  $F$  is that  $F$  is actually *defined* for  $a$  (no one would say that clothes-pegs make borderline cases for flirtatious girls simply because the corresponding sentence is nonsensical, hence neither true nor false). Here, on the other hand, the definition reads as though ‘schort’ makes sense exactly of those people who are either less than four feet or more than six feet tall. Maybe I am wrong on that score, but what I would like to see is an example of a natural language predicate (not an artificial one) that exhibits the behaviour alluded to by the author.

Fuzzy plurivaluationism is, in my opinion, plagued by a further problem. One of the central questions about vagueness is whether it is due e.g. to the way we use language, to our lack of knowledge regarding certain aspects of reality, to some intrinsic features of the world around us. All the main competing theories take a stand about this issue. For instance, vagueness can be seen either as a matter of ignorance (epistemicism), or a semantic phenomenon due to ambiguity (super- or plurivaluationism), or an ontological phenomenon intrinsic to the nature of vague properties (this view seems to be at least implicit in some basic fuzzy theories). It is unclear to me how we should assess fuzzy plurivaluationism under this respect. On p. 8, when discussing classical plurivaluationism and its denial of a unique intended model, the source of vagueness is ascribed to “semantic *indeterminacy*—or equally, semantic *plurality*”, but in view of the remarks on p. 9 (“Fuzzy plurivaluationism is just like classical plurivaluationism except that its models are fuzzy”) it looks like we can safely assume that the picture also holds for its fuzzy variant. That stands to reason, given the fact that fuzzy plurivaluationism is, indeed, a form of plurivaluationism. However, it does not seem to sit well with the adoption of a fuzzy semantics of *any* sort. Fuzzy semantics assigns a truth value in between 0 and 1 to ‘Bob is tall’ whenever Bob has the property to some intermediate degree. In the standard fuzzy perspective, we do not say many things at once: we say just one thing about a property which neither definitely applies nor definitely fails to apply. On this hybrid approach, on the other hand, vagueness would seem to depend from the latter phenomenon as much as from semantic indeterminacy. On the basis of what Smith says, I do not have sufficient evidence to assess whether this is the correct interpretation. If so, perhaps this concurrence calls for more substantial justification.

Smith carefully distinguishes between the problem of artificial precision and other related, but different, meanings that the phrase ‘higher-order vagueness’ has in the literature. On one of such readings, you have this problem if your semantics provides for a sharp drop-off between the region of definitely positive (or negative) instances of a predicate and its borderline cases. Traditional classical plurivaluationism and basic fuzzy semantics are generally believed to suffer from this problem. If fuzzy plurivaluationism has to establish itself as one of the prominent options about vagueness in the next future (and I believe it can), such concerns should be addressed. One of these is due to Williamson [5, p. 156 ff.]. Take a sorites series from orange to red. It is hopeless, Williamson contends, to look for the first shade which is definitely red—i.e. that it is red in every acceptable model. Here we have fuzzy models rather than classical models; but what prevents Williamson’s objection from going into effect? More precisely: what is the first shade which has the property ‘red’ to degree 1 in every acceptable model?

Finally, I have the impression that the fuzzy metalanguage approach has been dismissed too quickly. Fuzzy class theory ([2, 3]), briefly mentioned in the paper, has been used [1] in an attempt to frame this position into the precise language of fuzzy mathematics, rather than classical mathematics. This would seem to defuse Goguen’s objection quoted on p. 10.

The fact that I disagree with its main thesis, of course, does not imply that I did not enjoy this paper, or that I did not value its worth. I hope that giving a thought to these small observations will help its author to polish a perspective that is likely to gain increasing resonance in the next years.

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Francesco Paoli  
Department of Philosophy  
University of Cagliari, Italy  
Via Is Mirrionis 1  
09123 Cagliari, Italy  
Email: paoli@unica.it