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**Comments on *Handling Borderline Cases Using Degrees: An Information Processing Perspective***  
**by Henri Prade and Steven Schockaert**

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The paper discusses, in an information processing setting, different situations where explicit graded representation of borderline cases may be advantageous rather than a problem. In the first part of the paper, the authors consider the use of fuzzy labels in three different cognitive tasks (specification of preferences, categorization, representation of incomplete knowledge) which lead to three different semantics for membership degrees in fuzzy sets, namely in terms of preference, similarity, and uncertainty, respectively. Then the authors focus on two further applications of using degrees as a flexible representation tool for borderline cases. The first one is in the field of formal concept analysis, where one can attach typicality degrees to objects. It is shown that the typicality ordering on objects induces in turn a gradation of the properties describing concepts involving typical objects in terms of the importance of these properties. Finally, they show how an approximate understanding of a non-vague property may help in dealing with possible inconsistencies in an information exchange process. In this short note, rather than formulating critical comments I just want to offer a few additional, complementary remarks to some aspects addressed in the paper.

The authors make an interesting distinction between dealing with non-vague understanding of vague terms on the one hand and vague understanding of non-vague terms on the other. The use of fuzzy labels, as described in Section 2, falls into the first scenario, since fuzzy labels (as considered by the authors) are explicitly specified by corresponding fuzzy sets, i.e. by precise real-valued functions.<sup>1</sup> Lawry's label semantics<sup>2</sup> offers an alternative model where, instead of associating a fuzzy set to a fuzzy label, one attaches to each object  $x$  a probability distribution  $P_x$  over a set  $\mathcal{L}$  of fuzzy labels covering the domain of some variable. Moreover, the probability  $P_x(L)$  measures the appropriateness with that a label  $L \in \mathcal{L}$  applies to  $x$ . One could discuss advantages and problems of such an approach, but in any case it in fact amounts to an even sharper example of a non-vague, degree-based understanding of fuzzy labels. Notice that this approach has also been extended to include elements of prototype theory into the picture.<sup>3</sup>

About the extension of formal concept analysis described in Section 3, it is indeed interesting to further refine the model in order to allow restricting the extension of a

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<sup>1</sup>This point is further discussed in the paper by Didier Dubois in this volume.

<sup>2</sup>J. Lawry, A framework for linguistic modelling, *Artificial Intelligence* 155 (2004) 1–39.

<sup>3</sup>J. Lawry and Y. Tang, Uncertainty modeling for vague concepts: A prototype theory approach. *Artificial Intelligence* 173 (2009), 1539–1558.

concept to those elements that are more typical. A small remark here is that a typicality ordering on objects is something that depends in turn on the target formal concept to be characterized. For instance a penguin is a less typical bird than a sparrow, but it is also a more typical animal-living-in-cold-areas than a sparrow. This concept dependent ordering is at variance with the traditional fuzzy approach to formal concept analysis, where the relation  $R$  in a formal context expresses the degrees with which a set of objects satisfy a set of properties, independently of the concept being characterized. However these two kinds of orderings are not incompatible, one could easily imagine an approach allowing degrees in both the satisfaction of properties by objects and in the context dependent typicality of objects.

In the final part of the paper, the authors claim that artificially introducing vagueness, and thus borderline cases, in some situations may indeed help to resolve inconsistencies related to different understandings of the meaning of the same expression by two agents. The idea is to assume similar but more imprecise meanings, in order to provide some room for making them compatible to some extent. Although the basic notion at work in this approach seems to be that of closeness or similarity, in the communication example developed in the paper the weakening of the proposition asserted by the speaker is represented by a set of propositions standing for the possible meanings of the original assertion, ranked according to the degree of certainty with which they are satisfied. This may be viewed as a set of graded borderline propositions. So, in a sense, the underlying similarity on propositions is turned into an (ordinal) uncertainty mapping. Actually, from a technical point of view, given the proposition asserted by the speaker, a similarity relation on the set of possible worlds allows one to define the (fuzzy) set of worlds that are close or similar to those satisfying the proposition. Taking this as the available (incomplete) information, a possibility distribution is induced, as explained in the section on fuzzy labels expressing uncertainty. From that one can infer the degree of certainty with which different propositions of interest are believed to hold. Therefore, although similarity-based reasoning and truthlikeness have different motivations regarding uncertain reasoning, there are situations where concrete bridges can be established.

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