Comments on *Have Fuzzy Sets Anything to do with Vagueness?* by Didier Dubois

LLUÍS GODO

The paper by Didier Dubois discusses the controversial relationship between Zadeh's fuzzy sets and vagueness. The author explores the issue from different perspectives and critically examines the role of fuzzy sets in studies of vagueness. Contrary to a common opinion within the fuzzy set community, he claims that fuzziness (in the sense of properties with gradual membership) has little in common with vagueness. The proposed thesis is that vagueness refers to uncertainty or variability in the meaning of predicates, regardless of whether they are Boolean or gradual. In this sense, a fuzzy set, taken as the representation of the meaning of a gradual predicate, is not actually a vague object at all but a very precise one.

Actually the latter view is, in a sense, in accordance with the usual criticism raised by philosophers of vagueness against degree-based approaches, in particular those based on fuzzy sets. The criticism focuses on the artificial precision imposed by a real-valued membership function when chosen as the meaning a gradual vague predicate.[1] Although I basically agree with most of the ideas expressed by the author, still I would like to comment on a few questions.

- One of the main arguments developed by Dubois is to differentiate gradualness from vagueness; the key observation being that vagueness entails some sort of indeterminacy or uncertainty about the meaning of the relevant predicate, while gradualness does not. This is based on the assumption (as far as I understand) that gradual properties can be unambiguously described by a membership function, and hence, once this is accepted, any uncertainty on the meaning of gradual predicates is ruled out. This is justified from an information engineering point of view in Section 2.1 by the empirical fact that in many applications only a fuzzy set approximating the meaning of a predicate is needed. While this may be an acceptable assumption in such a restrictive context, where gradualness in that sense may be felt as different from vagueness, in a more general perspective it seems hardly reasonable to assume a unique one-to-one correspondence between gradual properties and fuzzy sets, and that gradual properties do not suffer from meaning indeterminacy. So, in my opinion, there is no intrinsic difference in general between gradual and vague predicates other than the fact that the former ones are related to measurement variables while the latter may be of a more abstract nature.

[1] See the paper by Nick Smith in this volume for a very clear statement of the problem of what he calls the basic fuzzy theory of vagueness.

- The author proposes some bridges that might reconcile fuzzy sets and theories of vagueness, in particular the idea of extending the epistemic stance to gradual predicates, e.g., by using sets of fuzzy sets as a way to account for a form of epistemic uncertainty about the meaning of gradual predicates: we don't know which is the interpretation (in terms of a membership function) of a gradual property, rather we only know that it belongs to a given set of possible functions. In such a case, this is usually specified with a lower and upper bound of the membership degree for each object to the extension of the predicate. This fuzzy epistemic view, although it may be seen as an improvement, does not completely solve the above mentioned artificial precision problem since it transfers it to the membership functions delimiting the set of acceptable interpretations. Exactly the same problem appears when adopting other, more refined representations than simply sets for modeling partial ignorance about the intended interpretation of a gradual predicate, like higher-order fuzzy sets, probability distributions on families of fuzzy sets, etc.

- I would also like to comment on two questions that arise in Nick Smith's paper in this volume that I think are relevant in connection with this discussion. The first one is that fuzzy sets also appear, as models or interpretations of gradual predicates, in the framework of Mathematical fuzzy logic (MFL) and in the different formal systems of many-valued logics therein. But when reasoning in a given system of MFL one is not committed to choose a particular (precise) interpretation of the predicates, but one rather reasons about a whole set of interpretations of the given theory. In other words, in this logical framework one does not explicitly deal with precise, real-valued degrees of truth: those are only implicit in the models. In this sense, very similar to the spirit of the supervaluationistic approach, the models (of a given theory in a given formal fuzzy logic) can be understood as the set possible precisiations of the meaning of the formulas in your theory, but only what is common to all those interpretations is relevant for making derivations. Of course, the price paid for getting rid of the artificial precision problem in this framework, at least using plain systems of MFL, is the failure of capturing sorites-like inference schemes in full generality.

  The second and final comment, still in connection with the degree-based approach to vagueness by Smith, is on the role played by similarity or closeness as a distinguishing feature of vagueness. Basically, Smith argues that a predicate P is vague if, for any objects a and b, the truth-degrees of $P(a)$ and $P(b)$ are very similar whenever a and b are very similar in all respects relevant to the application of P. This required condition between similarity of objects and similarity of truth-degrees, when P is considered a gradual property in the sense of this paper (i.e. with some associated membership function representing its meaning), it may actually induce some constraints on the kind of acceptable fuzzy set chosen to represent P. Indeed, simplifying much and in the setting of properties related to some measurement variable, that condition is requiring that two values very close on the measurement scale cannot have very different membership degrees. In other words, it is not acceptable for a membership function to have sudden changes within a region of close enough objects. In a MFL setting, if $\approx_P$ denotes a predicate encoding a graded similarity relation on pairs of objects in relation to P, then the above condition is indeed requiring that P must be extensional with respect to $\approx_P$. This can be easily captured by an axiom like $(\forall x, y)(\approx_P(x, y) \to P(x) \leftrightarrow P(y))$, which may be used to further constrain the set of acceptable models for a given theory. Notice that, in semantical terms, that axiom simply expresses that the more true it is that x and y are similar, the closer the truth-value of $P(x)$ is to the the truth-value of $P(y)$.

Lluís Godo
IIIA, Artificial Intelligence Research Institute
CSIC, Spanish Scientific Research Council
Campus de la Universitat Autònoma de Barcelona s/n
08193 Bellaterra, Spain
Email: godo@iiia.csic.es