Vagueness Through Definitions

MICHAEL FREUND[1]

Overview

At the basis of categorization theory stands the difference between *sharp* and *vague* concepts. Sharp concepts are those for which categorial membership is an all-or-nothing matter: a given object falls or does not fall under a sharp concept, and no intermediate state is conceivable. For vague concepts, on the contrary, intermediate states are possible, and categorial membership becomes a matter of degree. This definition of vagueness as opposed to sharpness conceals the fact that this notion is by no means a uniform one, and that different types of vagueness coexist. The treatment of vague concepts therefore depends of the type of vagueness these concepts instantiate. In this paper, we restrict our attention to the family of concepts that are learnt and known through a list of defining features. Partial membership to the corresponding category then results from partial membership relative to its defining features. In this elementary type of vagueness, we show that the categorization process is fully accounted for by the construction of a membership order among the objects at hand, which, naturally defined for simple concepts, can be easily extended to compound concepts, thus providing an interesting solution to the problem of compositionality.

1 Introduction

In categorization theory, different models have been proposed to account for the notion of concept. The classical model, formalized by Frege [2], identified concepts with functions whose values were truth values. In this model, objects *falling* under a concept, that is, objects the concept term is true of, form a mathematical set, called the *category* or the *extension* of the concept. Membership to this set is in consequence given by a simple two-valued truth function, and set theory is viewed as the adequate tool to deal with categorization. The inadequacy of this rudimentary model to capture phenomena like the *typicality effect* inside a category, or the existence of a *borderline* between membership and non-membership was pointed out by the work of Eleanor Rosch [14] and her followers, who showed in particular the existence of *vague* concepts for which membership cannot be an all-or-nothing matter. This led to several theories proposing different

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models aiming at a correct representation of the major domains of categorization theory like categorial membership, typicality or resemblance.

In order to deal with the problem of vagueness, as early as 1965, Zadeh [18] introduced fuzzy logic as a way to handle concepts for which membership could not be decided on a simple IS-A procedure. Membership functions taking continuous or discrete values in the unit interval were consequently substituted to the two-valued characteristic function primitively associated with a category. Some drawbacks and counterintuitive results pointed out by Osherson-Smith [11] and Kamp-Partee [5] led the researchers to revise the initial fuzzy model and to look for alternative fuzzy logics displaying models more suitable to represent membership relative to vague concepts (see [1] or [7] for a an overview on the most recent work in this area).

For several reasons which will be explained in the next section, we shall depart from the general framework in which categorization theory has progressed in recent years. This paper will only focus on a very specific kind of concepts, and no general theory will be elaborated. We shall circumscribe our work to a very particular area, being quite aware that no generalization of our results is to be expected outside of the domain we choose. This domain is restricted both to a specific family of concepts and to a specific aspect of categorization theory: the class of concepts we are interested in, which we will call *definable*, consists of the concepts that come to the knowledge of an agent through a *definition*, that is with the help of simpler features; the categorization problem we are interested in is restricted to that of *categorial membership* in the concerned class. The solution we shall propose is therefore a particular one: we will show that, for definable concepts, it is possible to account for categorial membership by means of a qualitative ordering that compares the way the same concept may apply to different objects. This ordering can then be extended to compound concepts in a way that fully conforms with the intuition.

2 The notion of vagueness

The treatment of vague concepts suffers from an important drawback, which is its (default) assumption that vagueness is a uniform notion, and therefore bound to receive a uniform treatment. This attitude is quite disputable, though, as the same term covers different phenomena. For instance we may consider on the one hand concepts like *to-be-a-heap*, *to-be-tall*, *to-be-rich*, and on the other hand, concepts like *to-be-a-cause*, *to-be-beautiful*, *to-be-a-lie*. All these are vague concepts, but vagueness in the first group stems from *quantitative* considerations, whereas these considerations are meaningless for concepts of the second kind, which rather deserve to be qualified as *qualitatively* vague. It is thus natural to expect a numerical treatment of membership in the first case while looking for a different model in the second one.

The question then naturally arises whether we could use a criterion to characterize these two kinds of concepts, and whether other distinctions could be made inside the family of vague concepts. We observe however that, at this stage, we do not even have at our disposal a clear way of distinguishing between vague and sharp concepts. Several attempts have been conducted in the last decades to provide a strict definition of vagueness. To say that '*vagueness occurs whenever it is impossible to determine the limit between membership and non-membership to the concerned category*' only leads to the conviction that vagueness itself is a vague notion. In [8], Lupu proposed to define vagueness through membership functions: thus, a concept α would be vague if and only if there existed objects x for which the sentence '*x falls under the concept α*' was not a Boolean proposition. It is interesting to note that this characterization does not imply by itself the impossibility of deciding between membership and non-membership. At any rate, we notice that the proposed definition implicitly assimilates vagueness with fuzziness as it treats this notion as a quantitative magnitude. A slight improvement in this proposal could consist in considering as vague every concept α for which there exists at least three objects to which α applies differently: this simple characterization would avoid implicitly using membership functions that have not been themselves first defined.

These considerations tend to show that a general theory of vagueness is doomed to fail. On the contrary, a study of the different kinds of vagueness together with the search for an appropriate framework for each family of vague concepts constitute at this stage a more promising approach. For this reason, we have chosen to focus our study on a particular family of concepts that can be easily circumscribed, and whose treatment appears possible and effective, without the need of any purpose-built complex formalism.

3 Definable concepts

The family we have chosen to study includes concepts whose meaning—as grasped by a given agent—can be learnt and understood with the help of simpler concepts that are already part of the agent's knowledge. We shall call these concepts '*definable*'. Such are for instance scientific concepts, (a *mammal* is a 'warm-blooded vertebrate that secretes milk', a *Banach space* is a 'complete normed vector space', a *bird* is a 'vertebrate with beak, wings and feathers'...). Note that for each of these examples, it is possible to find more than two objects to which the concept applies differently: being a vertebrate, a fish, for instance, has more *mammalhood* than a worm, and less than a cow; a normed (incomplete) vector space is not quite a Banach space but it is closer to it than, say, the set of odd numbers; a bat, finally, has somehow more *birdhood* than a mouse although it is definitely not a bird. Thus, following the avatar of Lupu's definition proposed in the preceding section, we see that these concepts should be considered as *vague*. Of course, it may seem paradoxical to call *vague* a mathematical or a scientific concept. More generally, we have the feeling that a precise definition should exclude vagueness rather than create it. However, in the absence of any satisfying definition of vagueness, we do not see any contradiction in considering definable concepts as constituting a specific subfamily of vague concepts. In any case, our goal in this paper is not to argue for or against the vagueness of a certain class of concepts, but to provide a suitable framework to study their categorization properties.

Definable concepts may be considered as constituting the heart of the so-called *attributional view* advocated by some authors in the late seventies [16] and [15], which gave rise to the so-called *binary model* [10]. Following this theory, class membership relative to a concept is accounted for by a set of *defining features*, while all questions regarding typicality are taken care of through a (different) set of *characteristic features*. It is only the former set that will retain our attention in the present paper, since our aim is actually to provide a framework for categorial membership.

The attributional view linked categorization problems relative to a concept with categorization relative to its defining features. For example, for a given agent, the concept *to-be-a-bird* may be seen as a definable concept, being defined trough the concepts *to-have-a-beak*, *to-have-feathers*, and *to-have-wings*. For this agent, the *birdhood* of a given item x will be consequently analyzed through its membership relative to the three mentioned defining concepts. Namely, the agent will estimate to what extent x may be considered as having a beak, having feathers and having wings. Similarly, to quote an example of Putnam [13], the *meaning* of the term *tiger* will be captured by reference to the terms *yellow feline, fierce, black stripes,* and *jungle*: the defining features associated with the concept *to-be-a-tiger* then consist of the concepts *to-be-a-yellow-feline, to-have-black-stripes, to-live-in-the jungle* and *to-be-fierce*. The word *tiger* applies to an item in as much as this item is a feline that is yellow with black stripes, fierce and lives in the jungle.

The defining features of a concept constituted a set of (necessary and sufficient) conditions that an item had to fullfil to be considered as a plain exemplar of this concept; at the same time they provided information on items that only partly fell under it, attributing to them an intermediate *membership degree* based on the number of defining features they possessed. However research in this direction was soon abandoned because of the few uncontroversial examples proposed by the theory: concepts cannot be generally learnt through a simple list of key-words. As we shall see, however, some improvement in the treatment of definable concepts generates a technique for membership evaluation that can be carried over to a more general and much representative family of concepts. This justifies our interest for this theory.

An interesting point with the theory of definable concepts is that it implicitly supposes a complexity hierarchy between the target concept and the sources that are used in the defining process. Indeed, a definition is effective only on condition that the terms used in the explanation or the description of a new concept are themselves already part of the agent's knowledge. In this sense, associating with a particular concept a set of features that help understanding it evokes the process of a dictionary or an encyclopedia, which renders theoretically possible the construction of complex concepts from a well-defined set of primitive ones. In principle, we could thus introduce the notion of concept *constructibility*: choosing once and for all a set of *primitive* concepts, the family of constructible concepts can be iteratively enumerated by requiring that

1) primitive concepts are constructible
2) any concept presented with a defining set of constructible features is constructible
3) there are no other constructible concepts than those obtained through 1) or 2).

Such 'constructible' concepts were partly studied in [3] under the additional hypothesis that primitive concepts on which the construction was based were *sharp* concepts. We shall come back later to this notion of conceptual dictionary.

If a definable concept takes all its meaning and properties from its defining features, its associated categorial membership must be inherited from the categorial membership associated with these defining features. This raises the problem of determining whether and how a model could account for this passage from the extensional properties of the defining features to those of the newly defined concept. But before addressing this problem, we first need to explain our position concerning the mathematical formalism that we think is best adequate for the study of categorial membership.

Categorial membership relative to a concept measures how strongly this concept applies to the different objects that an agent has at his disposal. In general, the human mind disposes of no tool to directly evaluate this magnitude: even though differences in membership may be felt—one undoubtedly agrees for instance that a conventional bomb would be more a *weapon of mass destruction* than a machine-gun—it is impossible, except in some limit cases, to assign a precise number that would exactly measure the membership or the non-membership degree of a given item. When directly questioned what membership degree should be attributed to a machine-gun considered as a *weapon of mass destruction*, an agent will be generally unable to provide a sensible answer: what indeed could be the meaning of a sentence like 'a machine-gun is a *weapon of mass destruction* up to degree .35'? Similarly, an agent may be unable to assign a precise membership degree to a sink as a *piece of furniture*, while being fully ready to decide that this sink is 'more' a piece of furniture than a heat-pipe, and 'less' a piece of furniture than a window.

As a matter of fact, the only thing the human mind is capable of concerning membership evaluation is to *compare* two objects and decide which one, if any, falls 'more' under the concerned concept. Thus, the concept *to-be-a-weapon-of mass-destruction* will be generally considered as applying *more* to a machine-gun than to an arquebus, and *less* to a spear than to an arquebus. Clearly, this judgement shows the existence of a basic ordering induced by the concept *to-be-a-weapon-of mass-destruction* in the universe of discourse, but this ordering is by no means a consequence of a supposed degree assignment that the agent has set *a-priori* on the objects at his disposal. Naturally such an assignment may be established by him once the collection of objects of his universe has been displayed before him and comparison has been made between the items of this collection. For instance, a non-decreasing ranking like *bludgeon* $\leq$ *sword* $\leq$ *crossbow* $\leq$ *arquebus* $\leq$ *gun* $\leq$ *machine-gun* $\leq$ *flamethrower* $\leq$ *conventional bomb* $\leq$ *scud* $\leq$ *atomic bomb* may yield *a posteriori* a membership degree of the concerned items, which can be readily visualized on their position on a [0,1] scale: thus, an *atomic bomb* will be considered as being 100% a WMD, a *scud* as 90%, a *conventional bomb* as 80% and so on. The point is that these numerical values will appear as a consequence of a pre-recognized order among the different weapons that are part of the agent's universe: they won't be at the origin of it.

Ordering relations therefore appear to provide the most adequate model to account for categorial membership as perceived by a cognitive agent. Appealing systematically to relations of this type whenever it is possible avoids the drawbacks, shortcomings or counter-intuitive results that may result from the application of more sophisticated theories. It is true that in some cases, order relations may be insufficient to fully treat categorial membership. Such will be for instance the case for *fuzzy concepts*, or for vague concepts of a continuous type: for these concepts, interesting theories have been developed in different domains—fuzzy set theory, geometrical spaces, quantum mechanics. But for the specific class of concepts studied in this paper, that of '*definable concepts*', the family of order relations is sufficiently wide to perfectly model the problem of categorial membership.

With this in mind, the problem of determining how membership is transmitted to a concept from its defining features transforms itself into that of understanding how the different membership orders associated with the source features can melt into a single target membership order.

The construction of such an order has already been proposed in [3] for the family of concepts that can be recursively built out of a set of sharp primitives. We shall propose a simplified construction for elementary definable concepts, that will then be extended to compound concepts as well as to arbitrary definable concepts. But first a remark is necessary.

As we mentioned, the theory of definable concepts suffers from an important drawback, which is its lack of convincing examples. Concepts tend to be not easily definable: *birdhood* may be defined by a certain set of features that will be collectively possessed by birds and only by birds, but this is not so for *fishhood*: naturalists failed to propose a list of features that would apply to and only to fish. One may also think of the term *knowledge*, classically defined as *justified true belief* and thereafter subjected to numerous criticisms. Moreover, definitions rarely boil down to a simple list of words: disjunctions are frequently used, as well as negations, analogies or, even, examples. Thus, natural kind concepts and usual artifacts are seldom, if ever, defined by a simple enumeration of their most representative features, and the latter, when they exist, cannot be systematically considered as simpler than the target concept. Nominal concepts may be introduced through definitions—this is indeed the case for scientific concepts like mathematic definitions or entities classifiers—but the items that intervene in this definition are organized in a dynamical sequential way, allowing a term to act on another, and making use of constructors and modifiers. For this reason, it will be necessary to adapt the apparatus used in the case of elementary definable concepts to a larger class that will include non trivial examples of concept definitions. As we shall see, this task can be achieved by translating the dynamical structure of a definition into an ordered sequence of compound concepts built out through a determination operator, and to which we can apply the construction elaborated in the case of elementary definitions.

4 Categorial membership for definable concepts

Before treating the general case of concepts introduced through a definition, we examine the elementary case where both the structure of the definition and the defining terms used in it are of the simplest form.

4.1 Elementary definable concepts

The family of elementary concepts that are part of the universe of an agent covers the concepts that are brought to his knowledge through the help of several elementary (that is non compound) concepts, which are already part of the agent's knowledge. An elementary concept α is thus present in the agent's mind together with a finite set $\Delta(\alpha)$ of *defining features* that are supposed to be simpler than α. From the point of view of the agent, this set includes all the features that explain or illustrate α, helping to differentiate it from its neighboring concepts. For instance, the meaning of *tent* may be explained to a child through the definition: *tent = shelter made out of cloth*; in the child's mind, the corresponding set $\Delta(\alpha)$ would then consist of the concepts *to-be-a-shelter, to-be-made-of-cloth*. Similarly, the defining feature set associated with the concept *to-be-a-bird* may be the set $\{$*to-be-a-vertebrate*, *to-be-a-oviparous to-have-a-beak*, *to-have-feathers*$\}$, and that associated with the concept *to-be-carnivorous* may consist in the concepts *to-be-an-animal* and *to-eat-meat*. The *definition* of α is of an elementary type, consisting of a simple list, and the elements of $\Delta(\alpha)$ are elementary concepts that are supposed to be *less complex* than the target concept α.

Our basic assumption concerning the concepts of $\Delta(\alpha)$ is that they are part of the agent's knowledge, and that they are sufficient to enable him to acquire full knowledge of the categorial membership associated with α.

From these two requirements, we see that the categorization process of a definable concept results from the categorization process associated of its defining features. For instance, to judge if a given object x is a *bird*, defined as *a vertebrate that is oviparous, has beak, wings and feathers*, we have to evaluate its *being a vertebrate*, its *being oviparous,* its *having a beak* and its *having feathers*.

Note at this point that the terms 'concept' and 'feature' cover different notions. Formally, concepts are most often introduced through the auxiliary *to-be* followed by a noun: *to-be-a-bird, to-be-a-vector-space, to-be-a-democracy*. Features may be presented through a verb (*to-fly*), the auxiliary *to-have* followed by a noun (*to-have-a-beak)*, or the auxiliary *to-be* followed by an adjective (*to-be-tall*). While concepts appear as unary predicates, this condition is no more necessary for features, which may take arbitrary forms. On the ground level, we know that features, like concepts, apply to the objects at hand but, contrary to these latter, they borrow part of their significance from the concept they are attached to. Properties like *to-be-tall*, *to-be-rich*, or *to-be-red* take their full meaning only in a given context, that is when qualifying well-defined entities. Even simple verbal forms like *to-fly*, *to-run*, *to-live-in-water*, *to-be-made-of-metal* need a principal referent concept to fully seize the strength with which they apply to different items. To summarize, we would say that the meaning of a feature depends on the context in which this feature is used, contrary to the meaning of a concept which exists by itself.

It is true that, strictly speaking, features cannot be considered as concepts in the sense of Frege. It is for this reason that they did not call the attention of researchers in such different logical approaches of categorization theory like Fuzzy Logics, Formal Concept Analysis and Description Logics. In these approaches indeed, concepts are implicitly or not, assimilated to unary predicates, that are introduced through a noun (*to-be-a-bird*), a verb (*to-fly*) or an adjective (*to-be-yellow*). In Description Logics binary predicates characterize the *roles* of the language, which are used to express relationship between the concepts [9]. Thus, *to-be-a-tree* will be a concept, expressible by a single symbol A, but *to-have-green-leaves* is a 'role', expressed by a formula of the type '$\forall$ hasLeaves.Green'. This distinction renders impossible the treatment of membership for concepts defined by two-place predicates. In this paper, we shall nevertheless consider that all features that are used to define or characterize a concept can be themselves treated as concepts.

Any feature defining a concept α will therefore be considered as inducing a membership order among the objects at hand that is meant to reflect the strength with which the feature, *taken in the context of* α, applies to the items of the universe of discourse. Note that in most cases, this strength can be measured through a *total* preorder that ranks the objects of the universe on a finite scale. This is clearly true for fuzzy features like *to-be-tall*, *to-be-rich* or *to-be-warm*, since the measure of their applicability is always approximative (to an inch, a cent or a degree); this is even truer in the general process of categorization: in the context of a given concept α, ranking the objects relatively to a feature only yields a small number of non equivalent classes. To determine, for instance, to which extent a flower may be considered as a *poppy*, one evaluates very roughly its redness, its shape and the size of its petals. Comparison with other real or fictitious objects shows only a finite number of ordered non equivalent classes of reds between the color of that particular flower and that of an ideal poppy. Thus, in the context of a *poppy*, there exists only a small number of possible degrees of *redness*.

Let us now come back to the notion of definable concepts. The feature sets associated with this kind of concepts cannot be totally arbitrary, because all the features that define α must have 'something in common'—namely the fact that they qualify a well defined concept. We shall therefore assume as a basic property of the set $\Delta(\alpha)$ that all its elements apply together on at least one object: as we shall see, this will guarantee the existence of objects that fully fall under α.

4.2 A social choice problem for membership inheritance

How precisely does a concept inherit its category membership from that of its defining features ? For the reasons evoked above, the categorial membership induced by a concept α will be best accounted for by a partial preorder, that is a reflexive and transitive relation $\preceq_\alpha$, which can be used as a comparison tool between the objects at hand. The expression '$x \preceq_\alpha y$' will translate the fact that 'the concept α applies at least as much to object y as to object x'. We shall denote by $\prec_\alpha$ the corresponding strict partial order, that is the irreflexive and transitive relation defined by $x \prec_\alpha y$ iff $x \preceq_\alpha y$ and not $y \preceq_\alpha x$. This relation can be used to translate the fact that α applies better (or more) to y than to x.

Our assumption is that the defining features of the concept α are part of the agent's knowledge; this means that the agent knows, for every concept γ of $\Delta(\alpha)$, the structure of the associated membership preorder $\preceq_\gamma$. As we noticed, this latter is supposed to be *connected* (total): given two items x and y, either $x \preceq_\gamma y$, or $y \preceq_\gamma x$. The requirement that the knowledge of the $\preceq_\gamma$'s is sufficient to acquire knowledge of the target order relation $\preceq_\alpha$ shows that $\preceq_\alpha$ should be naturally deduced from the $\preceq_\gamma$'s.

It is interesting to observe that the problem of determining $\preceq_\alpha$ from the orders $\preceq_\gamma$'s is closely related with that encountered in *social choice theory*: there indeed, one tries to aggregate individual votes concerning a certain number of candidates into a general ranking $\leq$ of these candidates that would best approach the individual rankings $\leq_1, \leq_2, \ldots, \leq_n$ proposed by the voters. Our situation is somewhat similar: having to decide if the definable concept α applies more to the item x than to the item y, we may consider each of the defining features γ of α as a *voter*, examine successively if this feature applies more to x than to y, and eventually use a decision procedure to conclude. Some differences however deserve to be pointed out: the first one is that, in social choice theory, the preference relations $\leq_i$ as well as the resulting relation $\leq$ are supposed to be total (connected) relations. On the contrary, from our point of view, and unlike what happens in most categorization theories, we accept that the resulting α-membership of two items may be incomparable. For instance, an agent may consider that the '*birdhood*' of a tortoise cannot be compared with that of a bat, for the reason that tortoises, contrary to bats, share with birds the fact they lay eggs and have a beak, while bats, contrary to tortoises, share with birds their having wings and being warm-blooded. No comparison should therefore be made between these two items, as far as birdhood is concerned. A second difference is that, in categorization theory, one looks for a membership order that best models a known fact, namely the behavior of an intelligent agent while, in social choice, one tries to determine an abstract and general procedure for aggregating votes under some well defined constraints—an impossible task in the general case, as shown by Arrow's famous theorem. Note also that the constraints we are dealing with are different from those encountered in social choice: for instance, the fact that all defining features of a definable concept should apply simultaneously to at least one object would impose that, in all elections, there should be one candidate on which all voters agree.

Of course, the simplest way to build the relation $\preceq_\alpha$ from the relations $\preceq_\gamma$, $\gamma \in \Delta(\alpha)$, would be to take their intersection, simply setting $\preceq_\alpha = \cap_{\gamma\in\Delta(\alpha)} \preceq_\gamma$. This would amount to considering that the concept α applies no more to x than to y if such is the case for every defining feature of α. An alternative to this skeptical approach would consist in 'counting the votes', and set $x \preceq_\alpha y$ if the number of defining features γ such that $x \preceq_\gamma y$ is at least as great as the number of defining features δ such that $y \preceq_\delta x$: this non-transitive relation leads to the so-called Condorcet method. However, the fact that in these approaches, the defining features of α are all given equal importance forbids their adoption, not mentioning the side drawbacks that these procedures may carry. In categorization theory indeed, the defining features of a concept are rarely considered by an agent as equivalent. Thus, each of the sets $\Delta(\alpha)$ is usually presented with a *salience* relation between its elements, which is adequately translated by a strict partial order on $\Delta(\alpha)$. For instance, a particular agent may associate with the concept *to-be-a-bird* the defining set $\{$*to-be-a-vertebrate, to-be-oviparous, to-have-feathers, to-habe-a-beak, to-have-wings*$\}$, considering furthermore that *having wing*s is a more important feature for birdhood than *having a beak*. For this agent consequently, a bat would be given more birdhood than a tortoise.

We therefore have to deal with the presence of voters whose voices have different importance. When this difference is quantifiable, that is, when it can be translated by a natural number that corresponds to an importance rank, it can be accounted for by a simple perequation: the voice of a voter of rank i will weigh i times that of an 'ordinary' voter. Alternatively, it is also possible to attach to each defining feature of α its *cue validity* probability: given a defining feature $\beta \in \Delta(\alpha)$, its cue validity is the probability $P(\alpha/\beta)$ that an object x falls under α, knowing that it has the feature β. Then, the membership degree of an object x, defined as the probability that x falls under α, can be computed as the sum $\Sigma_\beta P(\alpha/\beta)P(\beta)$. However, in our case, there is no reason why the defining features of a concept should be attributed such a numerical rank of importance or such a degree of probability: again, an agent may be quite able to compare the relative salience of two features of a concept without being able to associate a degree to these saliences. Apart from this pragmatic inability, salience may be simply not expressible by a gradation. For instance, taking again the concept *to-be-a-bird* and its defining set $\{$*to-be-a-vertebrate, to-be-oviparous, to-have-feathers, to-have-a-beak, to-have-wings*$\}$, suppose we equip this set with an order that renders *having-wings* more salient than *having-a-beak*, all other features being incomparable. Clearly, any grading function on $\Delta(\alpha)$ consistent with this order would attribute a greater rank to the feature *having-wings* than to the feature *to-be-oviparous*, thus making these two features comparable in $\Delta(\alpha)$, which they were not supposed to be.

Except for some specific cases, we therefore have to work with a completely arbitrary salience relation on $\Delta(\alpha)$. We will only assume that this salience is translated by a strict partial order in $\Delta(\alpha)$. Our task is to build a voting procedure that takes into account the relative importance of the voters, so that, in case of conflict, the voice of a subordinate can be overruled by that of a hierarchical superior. This may be done through the simple following idea which simplifies the construction proposed in [3]: a candidate y will be declared at least as good as a candidate x if for any voter that prefers x to y there exists a more important voter that prefers y to x. Formally this yields the relation:

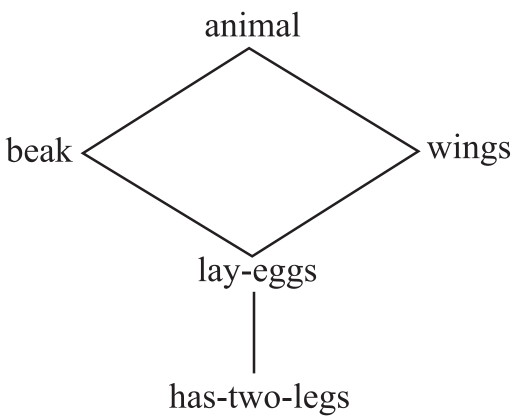
$$x \preceq_\alpha y \text{ iff for each feature } \gamma \in \Delta(\alpha) \text{ such that } y \prec_\gamma x \text{ there exists a feature } \delta \in \Delta(\alpha),\\ \delta \text{ more salient than } \gamma, \text{ such that } x \prec_\delta y.$$

This relation $\preceq_\alpha$ is clearly reflexive; its transitivity follows from the connectedness of the membership preorders $\preceq_\gamma$.

The corresponding strict partial order is then defined by

$$x \prec_\alpha y \text{ if and only if } x \preceq_\alpha y \text{ and there exists a feature } \gamma \in \Delta(\alpha) \text{ such that } x \prec_\gamma y.$$

EXAMPLE 1 Let α be the concept *to-be-a-bird*, with associated defining feature set $\Delta(\alpha) = \{$*to-be-an-animal*, *to-have-two-legs*, *to-lay-eggs*, *to-have-a-beak*, *to-have-wings*$\}$, all features being considered as sharp by the agent. Suppose that the salience order is given by the Hasse diagram, to be read bottom-top:

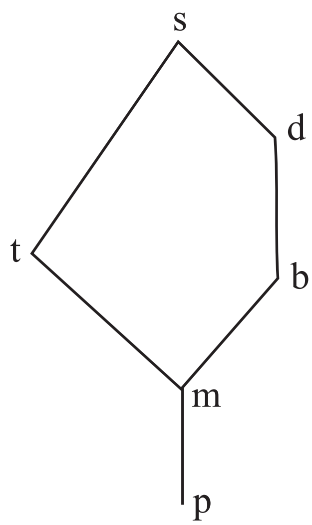


Let us compare the relative birdhood of a *sparrow,* a *mouse*, a *tortoise*, a *bat*, a *dragonfly* and a *plane*, respectively denoted by s, m, t, b, d and p.
In order to determine the induced membership order, we build the following array:

	two-legs	lay-eggs	beak	wings	animal
sparrow	$\star$	$\star$	$\star$	$\star$	$\star$
mouse					$\star$
tortoise		$\star$	$\star$		$\star$
bat	$\star$			$\star$	$\star$
plane				$\star$	
dragonfly		$\star$		$\star$	$\star$

We readily check that $d \prec_\alpha s$, $m \prec_\alpha t$, and $m \prec_\alpha b$. Note that we have $b \preceq_\alpha d$, since the concept *to-have-two-legs* under which the bat falls, contrary to the dragonfly, is dominated by the concept *to-lay-eggs* that applies to the dragonfly and not to the bat. On the other hand, we do not have $d \preceq_\alpha b$, as nothing compensates the fact that the dragonfly lays eggs and the bat does not. This yields $b \prec_\alpha d$. Similarly, we have $p \prec_\alpha m$. We also remark that the tortoise and the bat are incomparable, that is, we have neither $b \preceq_\alpha t$, nor $t \preceq_\alpha b$.

The strict α-membership order induced on these six elements is therefore given by the following diagram:



The construction of the (pre)order $\prec_\alpha$ we presented in this section is simpler and more intuitive than that proposed in [3]. It renders possible a computation of the target membership order directly from the membership orders induced by the defining features. It successfully takes care of the salience order on $\Delta(\alpha)$ even in the cases where this order cannot be expressed by a degree or a rank of importance among the defining features of α. However, in some situations, the results this order leads to may seem disputable: this happens in particular in the case where the concept α is defined by a significant number of features that are all equally salient except for a particular one, which is more salient than all others. Then if this particular attribute applies less to x than to y, we will have $x \prec_\alpha y$, even in the case where every other defining feature δ applies more to x than to y. The situation is analogous to an election where, because of his rank, a single individual could dictate his preferences to everybody.

Although it is rarely the case that $\Delta(\alpha)$ consists of a single salient feature opposed to a bunch of non salient ones, it may be useful to consider an alternative to the above construction so as to take into account the *number* of voters that prefer a candidate to another one. This can be done by requiring that a candidate y cannot be preferred to a candidate x unless, for any group of voters that prefer x to y, there exists an equally numerous group of more salient voters that prefer y to x. In other words we consider now the relation $\leq_\alpha$ defined by:

$x \leq_\alpha y$ if for each sequence of distinct elements $\kappa_1, \kappa_2, \dots \kappa_n$ of $\Delta(\alpha)$ such that $y \prec_{\kappa_i} x$, there exists a sequence of distinct elements $\gamma_1, \gamma_2, \dots \gamma_n$ of $\Delta(\alpha)$, γ_i more salient than κ_i, such that $x \prec_{\gamma_i} y$.

In this situation, each voter may see his decision overruled by a personally attached direct superior, but the voice of a single voter, be it the most important of all, cannot overrule more than one voice.

The relation $\leq_\alpha$ is clearly reflexive. It is not in general transitive. We shall denote by $\preceq^\star_\alpha$ its transitive closure. This latter yields the desired new membership preorder. Note that we have $\preceq^\star_\alpha \subseteq \preceq_\alpha$ and similarly $\prec^\star_\alpha \subseteq \prec_\alpha$. When the salience order on $\Delta(\alpha)$ is empty, both membership orders $\preceq_\alpha$ and $\preceq^\star_\alpha$ boil down to the intersection $\bigcap_{\gamma\in\Delta(\alpha)} \preceq_\gamma$.

4.3 Defining membership through membership orders

It is now possible to precisely define the notion of (full) *membership*: we shall say that an object x 'falls under the concept α' if x is $\prec_\alpha$-maximal among the objects that form the universe of discourse, that is if there exists no object y such that $x \prec_\alpha y$. This is equivalent to saying that x is $\prec_\alpha$-maximal in this set. When this is the case, x will be said to be an *instance* or an *exemplar* of α. This definition by means of maximal membership conforms with the intuition: an object x (fully) falls under a concept α if α cannot apply more to an object y than to x. It has as a consequence that, whatever salience order is set on $\Delta(\alpha)$, an object falls under the definable concept α if and only if it falls under each of its defining features.[1] We find thus again the classical characterization of a defining feature set as a set of features that are 'individually necessary and jointly sufficient to ensure membership relative to α'. We shall denote by $Ext\,\alpha$ the set of all objects that fall under α; this set forms the *extension* or the *category* associated with α.

[1]We use here the assumption made at the end of Section 4.1 that there exists at least one object that falls under all the features of α.

Contrary to full membership, partial membership cannot be directly defined from the membership orders $\prec_\alpha$ or $\prec^\star_\alpha$. However, it is possible to introduce the notion of a *membership distance*, which can be used to measure 'how far' an object x stands from falling under α (see [3] for details). For this purpose, it is enough to consider the number of objects that can be possibly inserted between x and an exemplar of α: more precisely, the $\prec_\alpha$-membership distance μ_α of an object x is defined as the maximal length of a chain $x \prec_\alpha x_1 \prec_\alpha x_2 \prec_\alpha \cdots \prec_\alpha x_n$. Such a chain must necessarily end up with an element of $Ext\,\alpha$ since, given any object x not falling under α, it holds $x \prec_\alpha z$ for any exemplar z of α.

EXAMPLE 2 Let us consider the preceding example and denote by μ_{bird} the membership distance of an object x to the category *birds*. Then we have $\mu_{bird}(t) = 1$ because, except birds themselves, there exists no oviparous animal with beak that would have two legs or that would have wings. Similarly, we have $\mu_{bird}(d) = 1$, since there exists no animal x such that $d \prec_{bird} x \prec_{bird} s$. Since the bat falls under three out of the five elements of Δ_{bird} we have necessarily $\mu_{bird}(b) < 3$, and the inequality $b \prec_{bird} d \prec_{bird} s$ then yields $\mu_{bird}(b) = 2$. As for the mouse, we have $m \prec_{bird} b \prec_{bird} d \prec_{bird} s$, but this is not a chain of maximal length. For instance, noting that men have two legs, we also have the chain $m \prec_{bird} k \prec_{bird} b \prec_{bird} d \prec_{bird} s$, where k denotes a man. This shows that $\mu_{bird}(m) \geq 4$, and thus $\mu_{bird}(m) = 4$ since, among the animals, only four features are sufficient to define a bird (we recall that all these features are supposed to be sharp). As a consequence, we have $\mu_{bird}(p) = 5$.

Note that an object falls under α if and only if its membership distance is null. Clearly, given two objects x and y with $x \prec_\alpha y$, the membership distance of y will be smaller than that of x. The converse is generally not true, though: y may be closer to $Ext\,\alpha$ than x without being comparable with it. In this sense the information provided by the membership distance is not as precise as that provided by membership functions. However, it is interesting to observe that the membership distance provides a *threshold* for categorial membership: if x has membership distance equal to 1, and y falls more than x under α, then it must be the case that y is an exemplar of α. In the above example, any object that has more birdhood than a tortoise is necessarily a bird.

5 The case of compound concepts

Elementary concepts give rise to compound ones through different combinations. The simplest one is the ordinary conjunction, &, which corresponds to a simple juxtaposition of terms, as in (*to-be-green*)&(*to-be-light*). Theoretically, it should be possible to consider also logical operations like concept negation or concept disjunction, but the pseudo-concepts to which these operations lead may end up into meaningless notions, e.g. concepts with no prototypes, or concepts with empty extensions.

We shall therefore only examine concept conjunction, and moreover restrict ourselves to the case where the resulting pseudo concept $\alpha\&\beta$ has a non-empty extension: that is, we suppose given two concepts α and β such that $Ext\,\alpha \cap Ext\,\beta \neq \emptyset$. The membership order on the conjunction $\alpha\&\beta$ is set by considering the (fictitious) associated defining feature set $\Delta(\alpha\&\beta) = \{\alpha, \beta\}$, which we equip with an empty salience order.

We have therefore $x \preceq_{\alpha\&\beta} y$ iff $x \preceq_\alpha y$ and $x \preceq_\beta y$; similarly we define the relation $\preceq^\star_{\alpha\&\beta}$ by $x \preceq^\star_{\alpha\&\beta} y$ iff $x \preceq^\star_\alpha y$ and $x \preceq^\star_\beta y$: thus, the membership order of the conjunction is just the intersection of the membership orders of its components. For instance, given two individuals x and y, x is considered as no more a *physician-and-a-Parisian* than y if x is no more a physician than y and at the same time no more a Parisian than y. Note that we have $\preceq_{\alpha\&\alpha}=\preceq_\alpha=\preceq^\star_{\alpha\&\alpha}$

Let us define the *extension* of $\alpha\&\beta$ as the set of all $\prec_{\alpha\&\beta}$-maximal elements. The hypothesis made on $Ext\,\alpha$ and $Ext\,\beta$ implies that this set is also the set of all $\prec^\star_{\alpha\&\beta}$-maximal elements. We easily check that full membership is compositional in the sense that $Ext\,\alpha \cap Ext\,\beta = Ext\,\alpha\&\beta$.

It is interesting to observe that the relations $\preceq_{\alpha\&\beta}$ and $\preceq^\star_{\alpha\&\beta}$ can be directly recovered by assigning to the concept $\alpha\&\beta$ a defining feature set equal to the disjoint union of $\Delta(\alpha)$ and $\Delta(\beta)$. More precisely, let $\widetilde{\Delta}(\alpha)$ be the set $\{(\gamma,\alpha);\gamma \in \Delta(\alpha)\}$ equipped with the salience order that makes (γ,α) more salient than (δ,α) if and only if γ is more salient than δ. Similarly set $\widetilde{\Delta}(\beta) = \{(\delta,\beta);\delta \in \Delta(\beta)\}$ with the same corresponding salience order. The structure of these sets emphasizes the fact that features are in general dependent of the concept they apply to. Consider now the set $\widetilde{\Delta}(\alpha\&\beta) = \widetilde{\Delta}(\alpha) \cup \widetilde{\Delta}(\beta)$ with the salience order that extends those of $\widetilde{\Delta}(\alpha)$ and $\widetilde{\Delta}(\beta)$ and is empty elsewhere. The membership order induced by the concept $\alpha\&\beta$ with associated defining feature set $\widetilde{\Delta}(\alpha\&\beta)$ is then exactly the order $\preceq_{\alpha\&\beta}$.

More interesting than the simple conjunction of two concepts is, in the framework of categorization theory, the modification or the determination of a concept by another one. In [3] we have introduced a specific connective called the *determination operator*, that can be used to account for the *modification* of a principal concept α by a modifier β. This determination, denoted by $\beta \star \alpha$, is most often translated by the combination of an adjective or an adjectived verb with a noun (e.g. the concepts *to-be-a-carnivorous-animal, to-be-a-flying-bird, to-be-a french-student, to-be-a-red-apple*), but it can also be rendered by a noun-noun combination (eg. *to-be-a-pet-fish, to-be-a-barnyard-bird*). Typically, in the compound concept $\beta \star \alpha$, the main concept α is defined through a predicate of the type *to-be-x*, while the accessory concept β is of the form *to-have-the-property-y*.

It is important to keep in mind that we consider only the conceptual combinations that are *intersective*: the objects that fall under the composed concept $\beta \star \alpha$ are exactly those that both fall under α and under β. Thus, and to mention the best known examples, the determination connective cannot be used to form complex concepts like *to-be-a-brick-factory*, *to-be-a-criminal lawyer* or *to-be-a-topless-district*: indeed, a brick factory need not be a factory that is made out of bricks, a criminal lawyer not a lawyer that is a criminal, and a topless district not a district that is topless (see [5] for the distinction between intersective and non-intersective modifiers). Note also that the intersection condition forbids us to consider a concept like *to-have-green-leaves* as the determination of the concept *to-have-leaves* by the concept *to-be-green*.

Some determinations that are not properly intersective may nevertheless benefit from our treatment of concept determination. Consider for instance a concept like *to-be-a-good-violinist*. We do not have intersection properly here, since a good violinist is not somebody that is good and that is violinist. However, we have to remember that features take part of their meaning from the concepts they are used with. Here the feature used as a modifier, *good*, takes the meaning *to-play-well*, and a good violinist is a violinist that plays well. Thus, setting α for the principal concept *to-be-a-violinist* and β for the modifier *to-play-well-the-violin* we can consider that the meaning of the concept *to-be-a-good-violinist* is correctly translated by the composition $\beta \star \alpha$.

Modified concepts are generally not definable, and they are not brought to the agent's knowledge with the help of a defining features set. It is not difficult however to extend the order $\preceq_\alpha$ defined in the preceding paragraph to this family of concepts. The construction of the membership order induced by $\beta \star \alpha$ is obtained by attaching to this concept the (fictitious) set of features $\Delta(\beta \star \alpha) = \{\beta, \alpha\}$ equipped with an order that makes α more salient than β. We have thus $x \preceq_{\beta\star\alpha} y$ iff $x \preceq_\alpha y$ and either $x \prec_\alpha y$, or $x \preceq_\beta y$. This construction yields a preorder that takes into account the predominance of the principal concept α. In this model, the concept *to-be-a-flying-bird* will be considered as applying more to a penguin than to a bat. Note that we have again $\preceq_{\alpha\star\alpha} = \preceq_\alpha = \preceq^\star_{\alpha\star\alpha}$.

As in the case of the conjunction, we observe that the relation $\preceq_{\beta\star\alpha}$ may be d through the disjoint union of $\Delta(\alpha)$ and $\Delta(\beta)$, equipped with an order that extends their respective salience orders, making furthermore any element of $\Delta(\alpha)$ more salient than any element of $\Delta(\beta)$.

5.1 Structured defining sets

As we already mentioned, the defining set associated with a (definable) concept cannot be simply described by means of a list of features. Rather, these features are most often articulated through some operators or constructors. The description of a term can also make use of locutions like *on which, through which*, etc. The key words that intervene in the definition of a concept are at any case constituted by a certain number of nouns, verbs, and modifiers, to which an apparatus consisting of auxiliary verbs, pronouns, locutions and ingredient markers provides the final *Gestalt*.

Similarly, a conceptual dictionary cannot be expressive if it does not propose, together with its set of primitive concepts, a structure or a *grammar* that helps extending the meaning of an enumeration of defining features. This fact has already been underlined by Ray Jackendoff [4] and Anna Wierzbicka [17]. In particular, in her research on a *Natural Semantic Metalanguage* (NSM), Wierzbicka and her followers proposed, together with a list of conceptual primitives, a list of conceptual elementary structures that constitute the syntax of this (meta)language (For an introduction to Wierzbicka work, see [12] or [6]).

Categorial membership clearly becomes more difficult to evaluate when the definition by which is introduced the target concept rests on a non-trivial subjacent grammar. As an example of the problems encountered in this situation, let us consider the following (structured) definition of *maple*:

'*a tall tree growing in northern countries, whose leaves have five points, and the resin of which is used to produce a syrup*'.

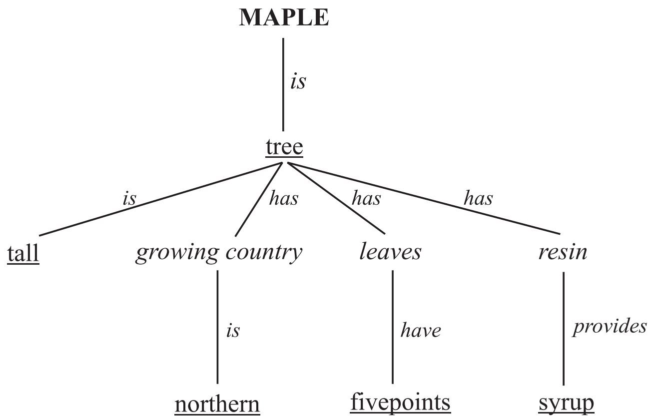


Figure 1. Definition tree for *maple*.

Among the features listed in this definition, only that of *tree* directly applies to the target concept *maple*. All the other ones are linked with some secondary concepts: thus, *tall* refers to *tree*, *to-have-fivepoints* refers to *leaves*, *northern* refers to *country*, and *syrup* refers to *resin*. The key features that intervene in this definition are the concepts *tree, tall, northern, fivepoints, syrup*. The apparatus is encapsulated by the sequence: is a + *key-feature*, that is + *key-feature*, whose growing country is + *key-feature*, whose shape of leaves is + *key-feature*, whose resin produces + *key-feature*. We can formalize the whole definition by a tree in which the edges translate the verbs used in the definition process, and where the nodes underlined in roman letters stand for the key features and those in italics indicate the auxiliary concepts used in the *Gestalt* (Figure 1).

It is clear that the membership of an object *x* relatively to the concept *to-be-a-maple* not only depends on its own membership relative to the concept *to-be-a-tree*, but also depends on the membership of other objects (the leaves of *x*, the resin of *x*) relative to auxiliary concepts (*to-have-five-points, to-provide-a-syrup*): it is not the object *x* itself that may be qualified as *having fivepoints* but the auxiliary object ‘leaves of *x*’. We see in this example an important difference with the simple defining features sets used in the preceding section, where the membership of an object relative to the target concept was directly evaluated through the membership of *this same object* relative to the defining features.

The problem is now that the auxiliary objects that correspond to the auxiliary concepts may simply not exist for the chosen item *x* whose membership is to be evaluated: thus, it may be the case that *x* is not a tree, or that *x* is a pine-tree and has no leaves, in which case it is meaningless to pretend to evaluate the membership of *its* leaves relative to the concept *to-have-fivepoints*. This observation seems to ruin any attempt to evaluate an item’s membership through the elementary key-features that intervene in its definition when the defining feature set is not presented as a simple list.

Fortunately, in a great number of cases, we can still use the (complex) auxiliary features, provided we do not systematically try to break them down into elementary components. Let us take again the *maple* example: we may interpret its definition by saying that an object x is a maple if *it is a tree* that *is tall*, if *it has a growing country* that *is northern* and if *it has resin* that *produces syrup*. This translation enables us to consider the membership of the same single object, x, relative to the four compound concepts *(to-be-tall)⋆(to-be-a-tree)*, *to-have-a-((northern)⋆(growing-country))*, *to-have((five points) ⋆(leaves))*, *to-have-a-((producing syrup)⋆(resin))*. The first concept—*(to-be-tall)⋆(to-be-a-tree)*—is obtained through a determination, and its application to any object will be evaluated from that of the concepts *to-be-a-tree* and *to-be-tall*, as shown in Section 5. As for the other ones, we have to consider them as complex indecomposable concepts: there exists indeed no way to compute their associated membership through that of their constituents: similarly, the concept *to-eat-a-red-apple* cannot be simply formed through the concepts *to-eat*, *to-be-an-apple* and *to-be-red*. We find here again the difference of treatment between one-place and two-place predicates.

In order to compare the respective *maplehood* of two items, we will therefore have to compare their respective memberships relative to each of these four concepts. Note that this again amounts to associating with the target concept *to-be-a-maple* a set of defining features: this set simply consists of the concepts: *to-have-a-(producing syrup)⋆(resin)*, *(to-be-tall)⋆(to-be-a-tree)*, *to-have-a-(northern)⋆(growing-country), to-have(five points) ⋆(leaves)*, on which a salience order may be set as in the elementary case.

These observations show that, in the general case, it is possible to account for the categorial membership associated with any definable concept whose structured definition can be modelled by an ordered set of simple or compound concepts. For such a concept α we will simply define the membership order $\prec_\alpha$ as one of the orders defined in paragraph 4.2.

As we see, the evaluation of the categorial membership order relative to a much larger family of concepts than those that can be elementarily definable is now possible.

5.2 Conceptual dictionaries

In this final section, we examine the theoretical case of concepts that can be recursively defined from a fixed set of primitive concepts. Formally, we suppose given a fixed set of concepts $\mathscr{C}$ together with a *defining function* Δ from $\mathscr{C}$ into the set $\wp_0(\mathscr{C})$ of all finite subsets of $\mathscr{C}$: this function associates with every concept α of $\mathscr{C}$ a subset $\Delta(\alpha)$ that, when non empty, gathers the defining features of α. The elements of $\mathscr{C}$ whose image by Δ is the empty set are the *primitive* concepts of the *dictionary* constituted by the pair $(\mathscr{C},\Delta)$. Primitive concepts are characterized by the fact that they cannot be learnt through other concepts: they have no defining features, they are presented as a whole. The defining function Δ is supposed to be effective and non-redundant, in the sense that a finite number of operations should be sufficient to define any concept of $\mathscr{C}$ from the primitive ones. To be more precise, we require the following condition of *Finite Delta Sequences* to be satisfied by the dictionary:

(FDS): any sequence $\alpha_0, \alpha_1, \ldots, \alpha_{n-1}, \alpha_n, \ldots$ with $\alpha_i \in \Delta(\alpha_{i-1})$ is finite.

Let $\alpha <_\Delta \beta$ be the relation on $\mathscr{C}$ defined as:

$\alpha <_\Delta \beta$ if there exists a sequence $\alpha = \alpha_0, \alpha_1, \ldots, \alpha_{n-1}, \alpha_n = \beta$ such that for all $i < n-1$, $\alpha_i \in \Delta(\alpha_{i+1})$.

The relation '$\alpha <_\Delta \beta$' may be read as 'α is *simpler* than β': indeed, it translates the fact that α intervenes in the definition of β, and thus must be more accessible to the agent.

The (FDS) condition readily implies the following properties:

- The graph determined by the function Δ is acyclic
- $<_\Delta$ is a strict partial order
- Every $<_\Delta$-descending chain is finite
- The $<_\Delta$-minimal elements are the primitive elements of $\mathscr{C}$.

Given a concept α of $\mathscr{C}$, a *defining chain* for α is a descending chain from α that has maximal length. By condition (FDS) such a chain necessarily ends up with a primitive concept, which can be considered as a *root* of α. Its length measures the *complexity* of the concept α.

The membership order associated with a concept α of the dictionary depends on the membership orders associated with all the elements β that intervene in its definition, that is all the elements β such that $\beta <_\Delta \alpha$. Using the construction proposed in the preceding paragraph, it is theoretically possible to compute the α-membership order from the orders associated with all the primitive elements γ such that $\gamma <_\Delta \alpha$. However, the procedure is complex because it requires to take into account the salience orders of all the sets $\Delta(\beta)$, $\beta <_\Delta \alpha$. It is only in the particular case where these salience orders are empty that the membership order associated with α can be directly computed from its roots.

Things are simpler if we restrict ourselves to the problem of determining the *extension* of α, that is the set of all objects that fall under α: we have indeed the equality $Ext\,\alpha = \bigcap Ext\,\gamma$, where the intersection is taken over all the roots of α. An object therefore falls under a concept of the dictionary if and only if it falls under all its roots. Full membership can be thus totally evaluated at the ground level.

6 Conclusion

Concept definitions through defining features provide an interesting and effective tool for the study of problems linked with categorization theory. They render possible the construction of a purely qualitative membership order that enables to compare the membership of two objects relative to a given concept. This order, that takes into account the relative salience of the different features by which a concept is defined, can be extended to complex concepts built through juxtaposition or determination. It enjoys compositional properties and yields the extension of a composed concept as the intersection of the extensions of its components. These results can be extended to concepts whose definition requires the use of a simple subjacent structure, and provide an interesting insight on the extensional properties of conceptual dictionaries. However, one has to be aware that this study only concerns the family of concepts for which exists a definition, which is not the case for all concepts. Also, categorial membership in itself is not sufficient to account for problems that go beyond the extensional treatment of concepts: in particular, problems linked with prototype or resemblance theory need more sophisticated tools than those provided in this work.

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Michael Freund
Institut D'Histoire et de Philosophie des Sciences et des Techniques
13 rue du Four
75006 Paris, France
Email: micfreund@gmail.com