Comments on *Vagueness Through Definitions* by Michael Freund

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Most theories of vagueness characterize the phenomenon by reference to the extension of our concepts: the vagueness of a predicate is usually described by the existence of borderline cases, namely cases for which the predicate neither clearly applies, nor clearly fails to apply. A lot remains to be said about the relation between the existence of borderline cases and structural aspects of the meaning of vague predicates from which that existence could analytically derive. The first originality of Michael Freund's approach to vagueness in this regard is the connection sought between vagueness and the *intension* or definition of our concepts. What Freund examines in his paper are the defining features of complex concepts, namely the various criteria that enter into the decision of whether or not to apply the concept to a given object.

A second and related originality of Freund's account is the emphasis put on *multidimensionality*. Vagueness is generally looked at from a one-dimensional perspective. For instance, when discussing the vagueness of gradable adjectives such as "bald", "tall" or "young", we generally consider a single dimension of comparison for whether or not an object should fall under the concept. For "bald", membership is typically presented as a function of the number of hairs, for "tall", as a function of height, for "young", as a function of age. However, vagueness is also characteristic of complex concepts expressed by common nouns, such as "car", "bird" or "vegetable". In order to determine whether an object is a car or not, we cannot rely on a unique nor obvious scale of comparison. Rather, as stressed by Freund in his account, the application of such concepts to particular objects involves the consideration of distinct and separable features that can vary independently.

A very important insight of Freund's contribution to this volume is that one source of vagueness lies in the existence of multiple respects of comparison, at least for an important class of predicates, namely definable predicates expressible by common nouns. For instance, when we apply a concept such as "car" to an object, there are several defining features that we need to check for, but also that we need to weigh against each other. From Freund's account, both aspects are responsible for vagueness. An object may instantiate more of the defining features of a concept than some other object. But possibly,

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though an object might instantiate fewer of the defining features of a concept than some other object, it might instantiate features that have more weight for the application of that concept.

I should say at the outset that I find Freund's account of the vagueness of definable concepts both very convincing and very insightful. In this commentary, I would like to question two aspects of his account. The first concerns the grounds for his opposition between qualitative and quantitative vagueness, and the question of whether those really are two heterogenous phenomena. My sense is that the opposition is more adequately phrased in terms of one- vs multi-dimensional vagueness, and that his account suggests a promising unified theory, one that could account for how we deal with vagueness and comparison for distinct lexical classes. In relation to that, the second aspect of his account I wish to discuss concerns Freund's construction of an ordering relation for definable concepts. As I understand Freund's account, a concept will be vague whenever that concept can be applied *partially* to an object, or *to some degree*. A very interesting aspect of his account is that Freund does not take degrees as given, but rather, shows how to construct them from an underlying ordering relation. In so doing, Freund's paper is also a contribution to the general theory of the relation between vagueness and measurement, namely to the study of how numerical scales are obtained for vague predicates. One aspect I shall particularly focus on concerns the link proposed by Freund between multidimensional predicates and the notion of a partially ordered scale.

1 Qualitative and quantitative vagueness

At the beginning of his paper, Freund opposes two kinds of vague concepts: concepts such as "heap", "tall" or "rich", and concepts such as "cause", "beautiful" or "lie". His suggestion is that the former may be grouped under the heading of 'quantitative vagueness', whereas the latter may be grouped under the heading of 'qualitative vagueness'. Because of the apparent heterogeneity between the two classes, Freund suggests that a general theory of vagueness is probably 'doomed to fail'. I believe his own project suggests a more optimistic outlook.

First of all, Freund's account of vagueness remains compatible with the idea that vagueness involves a duality between clear cases and borderline cases. For instance, for the 'qualitatively' vague predicates he considers, a clear case is one that instantiates all of the defining features of a concept, while a borderline case would be one that instantiates only some of the defining features of the concept. Secondly, for Freund the hallmark of vagueness appears to be gradability. To use his example, in the same way in which we can say that 'John is taller than Mary', we can say, even in the nominal domain, that 'a machine-gun is more of a weapon of mass destruction than an arquebus'.

Further remarks can be made about the dichotomy proposed by Freund. Firstly, his distinction is orthogonal to a distinction between lexical categories. For instance, "beautiful" is a gradable adjective, just like "tall", but the latter is viewed as quantitative, and the former qualitative. Likewise, "heap" is a noun, just like "lie", but the latter is called qualitative, and the former quantitative. Reflecting on the grounds for Freund's distinction, the main difference really seems to concern whether the predicate in question comes with a unique salient scale of comparison, or whether it involves several respects of comparison. Consider the case of "beautiful". It is doubtful whether "beautiful" is a definable concept in the sense intended by Freund, simply because there may not be more elementary concepts in terms of which "beautiful" can be defined. However, it is very plausible that "beautiful" comes with the equivalent of what Freund calls "defining features". Consider what we might mean by "a beautiful house". To judge whether a house is beautiful, we may need to assess whether it has a beautiful garden, a beautiful architecture, a beautiful furniture, and so on. Each of those aspects may in turn require the consideration of specific features. Thus, although there may be no analytic definition of "beautiful", we see that the application of the predicate depends on several respects of comparison that constitute an analogue of Freund's defining features.

Freund's central claim, however, is that unlike for "tall' or "heap", for concepts such as "car" or "weapon of mass destruction", we find no obvious numerical scale upon which to order the objects for comparison. This is undeniably correct, but I believe Freund's central thesis, which is that the assignment of numerical degrees from predicates to objects supervenes on a qualitative ordering between them, can be applied across the board.[2] As discussed in several recent theories about vagueness and the grammar of comparison (in particular van Rooij 2011, Sassoon 2010, Solt this volume, Burnett 2011), for a predicate like "tall", comparison depends on a *ratio* scale, one that encodes both information about position in the ordering, but also about differences and about ratios between intervals. For instance, we can say "Sam is taller than Jim" (ordinal information), but also "Sam is 2 cm taller than Jim" (interval information), and finally "Sam is twice as tall as Jim" (ratio information). For some predicates that Freund would call 'quantitative', however, like "heap", the information encoded is not as fine-grained as that provided by a ratio scale. For instance, it seems we cannot say "this is twice as much a heap as that" (even when the number of grains for heap 1 is twice as much the number of grains for heap 2). Or for an adjective like "bald", there is no obvious sense in which one might say "Sam is twice as bald as Jim" (even if Sam has half as much hair on his head). Turning to Freund's definable concepts, note that although one can say "a machine-gun is more of a WMD than an arquebus", without explicit stipulations one could hardly say "a machine-gun is twice as much of a WMD than an arquebus", or even just specify the precise amount to which a machine-gun is more of a WMD than an arquebus. This suggests that for concepts like "WMD" or "car", one might expect comparison to be encoded by an ordinal scale.

Because of that, my sense is that Freund's distinction between 'qualitative' and 'quantitative' vagueness could be cast as follows: the predicates Freund calls qualitative likely include all predicates involving either several defining features (like "car", "vegetable", "blue jacket"), or several respects of comparison (like "beautiful", "intelligent", "healthy"), predicates that can be characterized as multidimensional for that matter. Reciprocally, the predicates Freund calls 'quantitative' can be characterized as one-dimensional, namely predicates for which a unique most salient scale of comparison is relevant. Importantly, however, talk of 'quantitatively' vague predicates obscures the fact that predicates in that class can come with different measurement scales (like "tall", coming with a ratio scale, and "bald", coming with an interval scale). Nevertheless, the difference between 'qualitatively' vague and 'quantitatively' vague predicates may still

[2] The relation between relational scales and numerical scales is the object of representation theorems in measurement theory. See Roberts (1985) for an overview.

have a correlate in terms of the structure of the underlying scales. A possibility might be that all one-dimensional predicates encode information at least about intervals (see Sassoon 2010), and that multidimensional predicates basically encode at most ordinal information.[3] For multidimensional predicates, Freund's own thesis is that the default is a partially ordered scale.

2 Multidimensionality and partially ordered scales

The problem Freund deals with in the second half of his paper is the following: given a multidimensional predicate—such as "car", "bird", "blue jacket"—and a series of objects, to construct an integrated scale of comparison in order to determine how much an object is a car, a bird or a blue jacket.

Note that there are at least three degrees of freedom in this problem, and therefore three potential sources of vagueness in how we actually deal with multidimensional predicates. The first concerns the problem of determining the position of the object relative to each of the concept's dimensions or features. Take a complex concept like "blue jacket": in order to determine whether something is a blue jacket, one needs to determine how blue it is, and how much of a jacket it is. There is room for vagueness within each of the dimensions. The second degree concerns the problem of fixing the relative weight of the dimensions. For instance, a green jacket may, arguably, be more of a blue jacket (or closer to a blue jacket) than a blue sock, if we assume that the dimension of the modifier weighs less than that of the modified concept in this example. There is room for vagueness also at this stage, since in some cases the relative weight of the dimensions may be hard to determine. Finally, even supposing these two steps to be precisely resolved, the third degree concerns the integration of the dimensions into one. As Freund points out in his paper, there are several methods, just as for the aggregation of preferences in social choice theory, and a third source for vagueness concerns the potential indeterminacy of the method itself.

The main interest of Freund's paper is that he offers a canonical scaling method for multidimensional predicates, intended to show how we actually compare objects relative to several dimensions. Freund makes two assumptions to that effect: one is that objects can be completely ordered within each dimension (so vagueness is taken to be resolved at that level), and the other is that dimensions are partially ordered. The output of Freund's integrative construction is a partially ordered scale. Freund's emphasis on partially ordered scales is particularly noteworthy, for it relates to a foundational issue in measurement theory, which concerns the adequacy of totally ordered scales to deal with multidimensional integration. Standard scales since Stevens (1946) are completely ordered scales. However, some psychologists have argued that partially ordered scales could be needed precisely to deal with multidimensionality. This was done in particular by Coombs (1951), who points out that partially ordered scales "fall between nominal and ordinal scales".

[3] Sassoon's view is in fact the following: "Most plausibly, the majority of positive adjectives denote measures with all the properties of interval-scales in the first place, and sometimes, but not always, also properties of ratio-scales". Even for one-dimensional adjectives, however, more fine-grained differences need to be taken into account. See in particular Burnett's 2011 account of scale structure for relative vs. absolute gradable adjectives.

A questionable aspect, however, concerns the link between such partially ordered scales and the derivation of a membership degree for an object relative to the concept. In Freund's Example 1 for how various animals can be ordered relative to the concept 'bird', we see that the bat and tortoise are incomparable in the resulting ordering, basically because both differ on features that are incomparable. Nevertheless, Freund's definition of membership distance assigns a higher degree of 'birdhood' to the tortoise than to the bat. This points to a mismatch between the induced partial order and the complete order intended by the membership distance. The tortoise and dragonfly, for example, though also incomparable, eventually receive the same membership degree.

As I see it, a variant on Freund's construction could consist in first extracting a canonical weak order from the finite partial ordering for features, so that incomparable features with the same rank in the initial partial ordering could be assumed to have equal weight. Under that assumption, features would be completely ordered, so as to directly derive a consistent ordinal scale for what Freund calls membership distance. For example, a compromise between Freund's method and the Condorcet method would be to generalize the notion of *lexicographic ordering* as follows: say that x strictly precedes y (relative to a set of weakly ordered defining features) iff either x has more features of rank 1 than y, or x and y have an identical number of features or rank 1 but x has more features of rank 2, or they have an identical number of features of rank 1 and of rank 2 but x has more features of rank 3, ..., or x and y have an identical number of features of rank 1 to $n-1$, but x has more features of rank n. For instance, in the bat and tortoise case, this method would predict that the tortoise has more birdhood than the bat, directly in agreement with Freund's membership ranking. It would still assign the same position to the tortoise and dragonfly in the ordering, consistently with that ranking.[4]

Ultimately, it is unobvious to me whether, as Coombs and Freund claim on similar grounds, it matters for multidimensional scaling to keep incomparable elements, or whether it is more appropriate to resolve incomparabilities into ties of the appropriate kind to obtain an ordinal scale. This is the difference between considering that two students are incomparable in how good they are in mathematics, because one is very good at geometry, and average in arithmetic, while the other is very good in arithmetic, but

[4] Let us illustrate this. Each animal in Freund's table can be identified by a sequence of 1 and 0, one for each feature, corresponding to whether it has that feature or not. Assume that features are weakly ordered in accordance to their depth in the initial partial order, so that 'animal' has rank 1, 'beak' and 'wings' have equal rank 2, 'lay-eggs' has rank 3, and 'has two-legs' rank 4. Mapping Freund's table from right to left, the bat is representable by the sequence $(1_1, 1_2, 0_2, 0_3, 1_4)$ and the tortoise by $(1_1, 0_2, 1_2, 1_3, 0_4)$.

From our definition, $(1_1, 0_2, 1_2, 1_3, 0_4) < (1_1, 1_2, 0_2, 0_3, 1_4)$, that is tortoise and bat have an identical number of features of rank 1 and of rank 2, but on rank 3 the tortoise has a feature the bat does not have. The tortoise and dragonfly have the same degree of birdhood, since $(1_1, 0_2, 1_2, 1_3, 0_4) = (1_1, 1_2, 0_2, 1_3, 0_4)$ (in all ranks, they instantiate the same number of features). Freund's method would make the same predictions for those two cases under the same assumptions, if we replaced "more salient" by "at least as salient" in his definition on pg. 104. However, that modified definition would make different predictions in general. For instance, it would predict that $(1_1, 1_2, 0_2, 0_2) = (1_1, 0_2, 1_2, 1_2)$, whereas we would predict that $(1_1, 0_2, 1_2, 1_2) < (1_1, 1_2, 0_2, 0_2)$.

Nevertheless, as pointed out to me by Freund, we are back to the original problem that motivates his approach if we try to generalize our definition to cases in which features are satisfied to some degree only. For in the general case "x has more features of rank i than y" may be taken to mean that there are strictly more features γ_i of the same rank i for which $x \leq_{\gamma_i} y$ than such features for which $y \leq_{\gamma_i} x$. This definition gives rise to familiar intransitivities (Condorcet cycles) when features are no longer assumed to be either fully satisfied or fully failed, as they are in the foregoing example.

average in geometry. Likely, we may consider them to be incomparable, but there is also a legitimate sense in which they should be given the same grade at the end of term, by giving arithmetic and geometry equal weights. For Coombs (1951: p. 486), incidentally, the example of course grades is precisely an example of "summative" integration between dimensions, to which he opposes cases in which features 'do not compensate each other'. Coombs however does not entertain lexicographic orderings, which can offer a non-additive way to integrate dimensions into a complete order. Our own suggestion above is actually a mix of compensatory and noncompensatory integration between dimensions, since we assume features of equal rank to compensate each other, but that features of lower rank cannot compensate features of higher rank.

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