Semantic frameworks for lattice-based logics

Ewa Orlowska

We discuss frameworks for developing a relational (Kripke-style) semantics for lattice-based logics. The background is provided by the Urquhart representation theorem for not necessarily distributive lattices (Urquhart 1978) and Allwein and Dunn developments on Kripke semantics for linear logic (Allwein and Dunn 1993). We present a generalisation of Urquhart-Allwein-Dunn method to a duality between classes of lattice-based algebras and some classes of abstract frames. A general scheme of our methodology can be summarized in the following steps:
Step 1. A class of algebras is given. It consists of signature and/or axiomatic extensions of not necessarily distributive lattices.
Step 2. We define a class of relational structures (frames) such that the class provides a Kripke-style semantics for a logic whose algebraic semantics is determined by the class of algebras in question.
Step 3. For any algebra W of the given class we define its canonical frame. The universe X(W) of this frame consists of the pairs (x_1,x_2) such that x_1 is a maximal filter and x_2 is a maximal ideal of the lattice reduct of W and, moreover, they satisfy some additional conditions. The relations of the canonical frame correspond in an appropriate way to the operations of the algebra.
Step 4. For any frame X of the class defined in step 2 we define its complex algebra. The universe of the complex algebra is a family $C(X) \subseteq 2^X$,
Step 5. We prove a representation theorem saying that every algebra form the class in question is isomorphic to a subalgebra of the complex algebra of its canonical frame.

It can be observed that the representation theorems of that form provide a relational representability. Indeed, set X(W) of step 3 is a binary relation on 2^W, and the universe of the representation algebra, the set C(X(W)), consists of subrelations of relation X(W).

Assumptions on filter-ideal pairs in step 3 and on a family C(X) in step 4 lead to different semantic frameworks and different representation theorems.

Within a framework of that kind the lattice-based modal logics has been investigated in Orlowska and Vakarelov (2003). In Düntsch, Orlowska, and Radzikowska (2003) the method has been applied to lattice-based relation algebras.
In this paper we present a framework for substructural logics as an example.

References

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