Local Existence in Monoidal Logics

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Abstract

Let Ω be a complete Heyting algebra. It is well known that Ω-valued equalities appear naturally in the theory of presheaves on Ω (cf. [2]). In particular every *separated* presheaf on Ω carries a unqiue *separated* Ω-valued equality which is compatible with the underlying presheaf structure.

The aim of this talk is to extend this sutation to the case of *GL*-monoids — i.e. divisible, integral, completely lattice-ordered, commutative monoids with zero. This leads to the introduction of presheaves on *GL*-monoids (cf. [3]). For this framework we develop the corresponding semantics given by the geometric concepts of overlapping and local existence. The concept of overlapping goes back to K. Menger's work on positivistic geometry, while local existence appears in D.S. Scott's work on intuitionistic logic.

As an application we present a sheaf-theoretic treatment of fuzzy groups in the sense of Anthony and Sherwood (cf. [1]).

References

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