Petr Hájek, *On set theories in fuzzy logic.*

We shall shortly survey Zermelo-Fraenkel-like set theories developed in fuzzy predicate logic (BL$\forall$ or stronger) and their BL-valued models in ZF. In the main part of the lecture we shall discuss Cantor-like set theories with full comprehension developed in Łukasiewicz fuzzy predicate logic. It is known to be consistent, as proved by White. It remains consistent if we add a constant $\omega$ (for natural number) satisfying $x \in \omega \equiv (x = \emptyset \vee (\exists y \in \omega)(x = \{y\})$ and a natural induction schema for formulas *not containing the constant* $\omega$. If we assume this induction schema for *all* formulas then we can define addition and multiplication of natural numbers and prove all axioms of Peano arithmetic, but even more: we can define truth for natural numbers which is self-referring and commutes with connectives. We shall show that, this leads to a contradiction, so that this set theory is inconsistent.