

## 1 Active Clauses

- (C9)  $\vdash m_0 + (((k * (l_0 + (\mathbf{1} + \mathbf{1}))) + (l_0 * (m_0 + \mathbf{1}))) + \mathbf{1}) = k + ((k + (m_0 + \mathbf{1})) * (l_0 + \mathbf{1}))$
- (C51)  $m_0 = k_0 + (r_0 * ((t_0 + \mathbf{1}) * (t_1 + \mathbf{1}))) \vdash m_0 = k_0 + ((r_0 * (t_0 + \mathbf{1})) * (t_1 + \mathbf{1}))$   
 (subsumes 1 clause)
- (C53)  $m_0 = k_0 + (r_0 * ((t_0 + \mathbf{1}) * (t_1 + \mathbf{1}))) \vdash m_0 = k_0 + ((r_0 * (t_1 + \mathbf{1})) * (t_0 + \mathbf{1}))$   
 (subsumes 1 clause)
- (C54)  $\vdash (((t_0 + \mathbf{1}) * t_1) + t_0) + \mathbf{1} = (t_0 + \mathbf{1}) * (t_1 + \mathbf{1})$  (subsumes 1 clause)
- (C70)  $\mathbf{0} < \mathbf{p}_0 \vdash \mathbf{p}_0 = s_7(\mathbf{p}_0) + \mathbf{1}$
- (C71)  $\mathbf{0} < \mathbf{p}_0 \vdash t_0 = s_5(\mathbf{p}_0, t_0) + (s_6(\mathbf{p}_0, t_0) * \mathbf{p}_0)$
- (C72)  $\mathbf{0} < \mathbf{p}_0, s_5(\mathbf{p}_0, t_0) = \mathbf{0} \vdash t_0 = \mathbf{0} + (s_6(\mathbf{p}_0, t_0) * \mathbf{p}_0)$
- (C73)  $\mathbf{0} < \mathbf{p}_0 \vdash s_5(\mathbf{p}_0, t_0) < \mathbf{p}_0$
- (C87)  $\mathbf{0} < \mathbf{p}_1 \vdash \mathbf{p}_1 = s_7(\mathbf{p}_1) + \mathbf{1}$
- (C88)  $\mathbf{0} < \mathbf{p}_1 \vdash t_0 = s_5(\mathbf{p}_1, t_0) + (s_6(\mathbf{p}_1, t_0) * \mathbf{p}_1)$
- (C89)  $\mathbf{0} < \mathbf{p}_1, s_5(\mathbf{p}_1, t_0) = \mathbf{0} \vdash t_0 = \mathbf{0} + (s_6(\mathbf{p}_1, t_0) * \mathbf{p}_1)$
- (C90)  $\mathbf{0} < \mathbf{p}_1 \vdash s_5(\mathbf{p}_1, t_0) < \mathbf{p}_1$
- (C104)  $\mathbf{0} < \mathbf{p}_2 \vdash \mathbf{p}_2 = s_7(\mathbf{p}_2) + \mathbf{1}$
- (C105)  $\mathbf{0} < \mathbf{p}_2 \vdash t_0 = s_5(\mathbf{p}_2, t_0) + (s_6(\mathbf{p}_2, t_0) * \mathbf{p}_2)$
- (C106)  $\mathbf{0} < \mathbf{p}_2, s_5(\mathbf{p}_2, t_0) = \mathbf{0} \vdash t_0 = \mathbf{0} + (s_6(\mathbf{p}_2, t_0) * \mathbf{p}_2)$
- (C107)  $\mathbf{0} < \mathbf{p}_2 \vdash s_5(\mathbf{p}_2, t_0) < \mathbf{p}_2$
- (C121)  $\mathbf{0} < \mathbf{p}_3 \vdash \mathbf{p}_3 = s_7(\mathbf{p}_3) + \mathbf{1}$
- (C122)  $\mathbf{0} < \mathbf{p}_3 \vdash t_0 = s_5(\mathbf{p}_3, t_0) + (s_6(\mathbf{p}_3, t_0) * \mathbf{p}_3)$
- (C123)  $\mathbf{0} < \mathbf{p}_3, s_5(\mathbf{p}_3, t_0) = \mathbf{0} \vdash t_0 = \mathbf{0} + (s_6(\mathbf{p}_3, t_0) * \mathbf{p}_3)$
- (C124)  $\mathbf{0} < \mathbf{p}_3 \vdash s_5(\mathbf{p}_3, t_0) < \mathbf{p}_3$
- (C170)  $t_0 = \mathbf{p}_3, nat_{26} * nat_{27} = t_0 \vdash nat_{26} = \mathbf{1}, nat_{26} = t_0$
- (C171)  $t_0 = \mathbf{p}_3 \vdash \mathbf{1} < t_0$
- (C172)  $t_0 = \mathbf{p}_2, nat_{26} * nat_{27} = t_0 \vdash nat_{26} = \mathbf{1}, nat_{26} = t_0$
- (C173)  $t_0 = \mathbf{p}_2 \vdash \mathbf{1} < t_0$
- (C174)  $t_0 = \mathbf{p}_1, nat_{26} * nat_{27} = t_0 \vdash nat_{26} = \mathbf{1}, nat_{26} = t_0$
- (C175)  $t_0 = \mathbf{p}_1 \vdash \mathbf{1} < t_0$
- (C176)  $t_0 = \mathbf{p}_0, nat_{26} * nat_{27} = t_0 \vdash nat_{26} = \mathbf{1}, nat_{26} = t_0$
- (C177)  $t_0 = \mathbf{p}_0 \vdash \mathbf{1} < t_0$

- (C187)  $\vdash m_0 = \mathbf{1}, s_1(m_0) * s_4(m_0) = m_0$
- (C190)  $\vdash m_0 = \mathbf{1}, s_1(m_0) = \mathbf{p}_0, s_1(m_0) = \mathbf{p}_1, s_1(m_0) = \mathbf{p}_2, s_1(m_0) = \mathbf{p}_3$
- (C193)  $t_0 = \mathbf{p}_3, \mathbf{1} = n_0 * t_0 \vdash$
- (C196)  $t_0 = \mathbf{p}_2, \mathbf{1} = n_0 * t_0 \vdash$
- (C199)  $t_0 = \mathbf{p}_1, \mathbf{1} = n_0 * t_0 \vdash$
- (C202)  $t_0 = \mathbf{p}_0, \mathbf{1} = n_0 * t_0 \vdash$

## 2 Subsumed

- (C1)  $\mathbf{1} + (l_0 + \mathbf{1}) = \mathbf{1} \vdash$  by (A14)
- (C17)  $m_0 = k_0 + (r_0 * ((t_0 + \mathbf{1}) * (t_1 + \mathbf{1}))) \vdash m_0 = k_0 + ((r_0 * (t_0 + \mathbf{1})) * (t_1 + \mathbf{1}))$   
by (C36)
- (C21)  $m_0 = k_0 + (r_0 * ((t_0 + \mathbf{1}) * (t_1 + \mathbf{1}))) \vdash m_0 = k_0 + ((r_0 * (t_1 + \mathbf{1})) * (t_0 + \mathbf{1}))$   
by (C39)
- (C22)  $\vdash (((t_0 + \mathbf{1}) * t_1) + t_0) + \mathbf{1} = (t_0 + \mathbf{1}) * (t_1 + \mathbf{1})$  by (C40)
- (C36)  $m_0 = k_0 + (r_0 * ((t_0 + \mathbf{1}) * (t_1 + \mathbf{1}))) \vdash m_0 = k_0 + ((r_0 * (t_0 + \mathbf{1})) * (t_1 + \mathbf{1}))$   
by (C51)
- (C39)  $m_0 = k_0 + (r_0 * ((t_0 + \mathbf{1}) * (t_1 + \mathbf{1}))) \vdash m_0 = k_0 + ((r_0 * (t_1 + \mathbf{1})) * (t_0 + \mathbf{1}))$   
by (C53)
- (C40)  $\vdash (((t_0 + \mathbf{1}) * t_1) + t_0) + \mathbf{1} = (t_0 + \mathbf{1}) * (t_1 + \mathbf{1})$  by (C54)
- (C65)  $\mathbf{0} < \mathbf{p}_0 \vdash (q_0 * \mathbf{p}_0) + (r_0 * \mathbf{p}_0) = (q_0 + r_0) * \mathbf{p}_0$  by (A24)
- (C66)  $\mathbf{0} < \mathbf{p}_0 \vdash (i_0 + (q_0 * \mathbf{p}_0)) + (r_0 * \mathbf{p}_0) = i_0 + ((q_0 * \mathbf{p}_0) + (r_0 * \mathbf{p}_0))$  by (A4)
- (C74)  $\mathbf{0} < i_0, \mathbf{0} < \mathbf{p}_0, i_0 < \mathbf{p}_0, i_0 + (q_0 * \mathbf{p}_0) = \mathbf{0} + (r_0 * \mathbf{p}_0) \vdash$  by (A30)
- (C75)  $i_0 < \mathbf{0}, \mathbf{0} < \mathbf{p}_0, i_0 < \mathbf{p}_0, i_0 + (q_0 * \mathbf{p}_0) = \mathbf{0} + (r_0 * \mathbf{p}_0) \vdash$  by (A29)
- (C76)  $\vdash i_0 = \mathbf{0}, i_0 < \mathbf{0}, \mathbf{0} < i_0$  by (A32)
- (C78)  $\mathbf{0} + \mathbf{1} < \mathbf{p}_0 \vdash \mathbf{0} < \mathbf{p}_0$  by (A1)
- (C79)  $\vdash \mathbf{0} + \mathbf{1} = \mathbf{1}$  by (A7)
- (C82)  $\mathbf{0} < \mathbf{p}_1 \vdash (q_0 * \mathbf{p}_1) + (r_0 * \mathbf{p}_1) = (q_0 + r_0) * \mathbf{p}_1$  by (A24)
- (C83)  $\mathbf{0} < \mathbf{p}_1 \vdash (i_0 + (q_0 * \mathbf{p}_1)) + (r_0 * \mathbf{p}_1) = i_0 + ((q_0 * \mathbf{p}_1) + (r_0 * \mathbf{p}_1))$  by (A4)
- (C91)  $\mathbf{0} < i_0, \mathbf{0} < \mathbf{p}_1, i_0 < \mathbf{p}_1, i_0 + (q_0 * \mathbf{p}_1) = \mathbf{0} + (r_0 * \mathbf{p}_1) \vdash$  by (A30)
- (C92)  $i_0 < \mathbf{0}, \mathbf{0} < \mathbf{p}_1, i_0 < \mathbf{p}_1, i_0 + (q_0 * \mathbf{p}_1) = \mathbf{0} + (r_0 * \mathbf{p}_1) \vdash$  by (A29)
- (C93)  $\vdash i_0 = \mathbf{0}, i_0 < \mathbf{0}, \mathbf{0} < i_0$  by (A32)
- (C95)  $\mathbf{0} + \mathbf{1} < \mathbf{p}_1 \vdash \mathbf{0} < \mathbf{p}_1$  by (A1)

- (C96)  $\vdash \mathbf{0} + \mathbf{1} = \mathbf{1}$  by (A7)
- (C99)  $\mathbf{0} < \mathbf{p}_2 \vdash (q_0 * \mathbf{p}_2) + (r_0 * \mathbf{p}_2) = (q_0 + r_0) * \mathbf{p}_2$  by (A24)
- (C100)  $\mathbf{0} < \mathbf{p}_2 \vdash (i_0 + (q_0 * \mathbf{p}_2)) + (r_0 * \mathbf{p}_2) = i_0 + ((q_0 * \mathbf{p}_2) + (r_0 * \mathbf{p}_2))$  by (A4)
- (C108)  $\mathbf{0} < i_0, \mathbf{0} < \mathbf{p}_2, i_0 < \mathbf{p}_2, i_0 + (q_0 * \mathbf{p}_2) = \mathbf{0} + (r_0 * \mathbf{p}_2) \vdash$  by (A30)
- (C109)  $i_0 < \mathbf{0}, \mathbf{0} < \mathbf{p}_2, i_0 < \mathbf{p}_2, i_0 + (q_0 * \mathbf{p}_2) = \mathbf{0} + (r_0 * \mathbf{p}_2) \vdash$  by (A29)
- (C110)  $\vdash i_0 = \mathbf{0}, i_0 < \mathbf{0}, \mathbf{0} < i_0$  by (A32)
- (C112)  $\mathbf{0} + \mathbf{1} < \mathbf{p}_2 \vdash \mathbf{0} < \mathbf{p}_2$  by (A1)
- (C113)  $\vdash \mathbf{0} + \mathbf{1} = \mathbf{1}$  by (A7)
- (C116)  $\mathbf{0} < \mathbf{p}_3 \vdash (q_0 * \mathbf{p}_3) + (r_0 * \mathbf{p}_3) = (q_0 + r_0) * \mathbf{p}_3$  by (A24)
- (C117)  $\mathbf{0} < \mathbf{p}_3 \vdash (i_0 + (q_0 * \mathbf{p}_3)) + (r_0 * \mathbf{p}_3) = i_0 + ((q_0 * \mathbf{p}_3) + (r_0 * \mathbf{p}_3))$  by (A4)
- (C125)  $\mathbf{0} < i_0, \mathbf{0} < \mathbf{p}_3, i_0 < \mathbf{p}_3, i_0 + (q_0 * \mathbf{p}_3) = \mathbf{0} + (r_0 * \mathbf{p}_3) \vdash$  by (A30)
- (C126)  $i_0 < \mathbf{0}, \mathbf{0} < \mathbf{p}_3, i_0 < \mathbf{p}_3, i_0 + (q_0 * \mathbf{p}_3) = \mathbf{0} + (r_0 * \mathbf{p}_3) \vdash$  by (A29)
- (C127)  $\vdash i_0 = \mathbf{0}, i_0 < \mathbf{0}, \mathbf{0} < i_0$  by (A32)
- (C129)  $\mathbf{0} + \mathbf{1} < \mathbf{p}_3 \vdash \mathbf{0} < \mathbf{p}_3$  by (A1)
- (C130)  $\vdash \mathbf{0} + \mathbf{1} = \mathbf{1}$  by (A7)
- (C188)  $\vdash m_0 = \mathbf{1}, s_4(m_0) * s_1(m_0) = s_1(m_0) * s_4(m_0)$  by (A15)
- (C189)  $\vdash m_0 = \mathbf{1}, \mathbf{0} + (s_4(m_0) * s_1(m_0)) = s_4(m_0) * s_1(m_0)$  by (A7)
- (C195)  $t_0 = \mathbf{p}_3 \vdash \mathbf{0} + (n_0 * t_0) = n_0 * t_0$  by (A7)
- (C198)  $t_0 = \mathbf{p}_2 \vdash \mathbf{0} + (n_0 * t_0) = n_0 * t_0$  by (A7)
- (C201)  $t_0 = \mathbf{p}_1 \vdash \mathbf{0} + (n_0 * t_0) = n_0 * t_0$  by (A7)
- (C204)  $t_0 = \mathbf{p}_0 \vdash \mathbf{0} + (n_0 * t_0) = n_0 * t_0$  by (A7)

### 3 Clauses with Tautologies

- (C2)  $nat_2 + (nat_3 + \mathbf{1}) = \mathbf{1} \vdash nat_2 + (nat_3 + \mathbf{1}) = \mathbf{1}$
- (C3)  $nat_4 = \mathbf{1} \vdash nat_4 = \mathbf{1}$
- (C4)  $nat_7 = \mathbf{1} \vdash nat_7 = \mathbf{1}$
- (C5)  $nat_7 = nat_5 + (nat_8 * (nat_6 + \mathbf{1})) \vdash nat_7 = nat_5 + (nat_8 * (nat_6 + \mathbf{1}))$
- (C6)  $nat_5 = \mathbf{1} \vdash nat_5 = \mathbf{1}$
- (C7)  $m_0 + (n_0 + \mathbf{1}) \in x \vdash m_0 + (n_0 + \mathbf{1}) \in x$
- (C8)  $m_0 + (n_0 + \mathbf{1}) = k + (nat_1 * (l_0 + \mathbf{1})) \vdash m_0 + (n_0 + \mathbf{1}) = k + (nat_1 * (l_0 + \mathbf{1}))$
- (C10)  $k \in x \vdash k \in x$

- (C11)  $nat_{11} = \mathbf{1} \vdash nat_{11} = \mathbf{1}$
- (C12)  $nat_{11} = nat_9 + (nat_{12} * (nat_{10} + \mathbf{1})) \vdash nat_{11} = nat_9 + (nat_{12} * (nat_{10} + \mathbf{1}))$
- (C13)  $nat_9 = \mathbf{1} \vdash nat_9 = \mathbf{1}$
- (C14)  $n_0 = \mathbf{1} \vdash n_0 = \mathbf{1}$
- (C15)  $n_0 = \mathbf{1} \vdash n_0 = \mathbf{1}$
- (C16)  $m_0 = \mathbf{0} + (nat_{91} * \mathbf{p}_3) \vdash m_0 = \mathbf{0} + (nat_{91} * \mathbf{p}_3)$
- (C18)  $m_0 = \mathbf{0} + (nat_{90} * \mathbf{p}_2) \vdash m_0 = \mathbf{0} + (nat_{90} * \mathbf{p}_2)$
- (C19)  $m_0 = \mathbf{0} + (nat_{89} * \mathbf{p}_1) \vdash m_0 = \mathbf{0} + (nat_{89} * \mathbf{p}_1)$
- (C20)  $m_0 = \mathbf{0} + (nat_{88} * \mathbf{p}_0) \vdash m_0 = \mathbf{0} + (nat_{88} * \mathbf{p}_0)$
- (C23)  $k_0 = \mathbf{0} + (nat_{87} * \mathbf{p}_3) \vdash k_0 = \mathbf{0} + (nat_{87} * \mathbf{p}_3)$
- (C24)  $k_0 = \mathbf{0} + (nat_{86} * \mathbf{p}_2) \vdash k_0 = \mathbf{0} + (nat_{86} * \mathbf{p}_2)$
- (C25)  $k_0 = \mathbf{0} + (nat_{85} * \mathbf{p}_1) \vdash k_0 = \mathbf{0} + (nat_{85} * \mathbf{p}_1)$
- (C26)  $k_0 = \mathbf{0} + (nat_{84} * \mathbf{p}_0) \vdash k_0 = \mathbf{0} + (nat_{84} * \mathbf{p}_0)$
- (C27)  $n_0 = \mathbf{0} + (nat_{83} * \mathbf{p}_3) \vdash n_0 = \mathbf{0} + (nat_{83} * \mathbf{p}_3)$
- (C28)  $n_0 = \mathbf{0} + (nat_{82} * \mathbf{p}_2) \vdash n_0 = \mathbf{0} + (nat_{82} * \mathbf{p}_2)$
- (C29)  $n_0 = \mathbf{0} + (nat_{81} * \mathbf{p}_1) \vdash n_0 = \mathbf{0} + (nat_{81} * \mathbf{p}_1)$
- (C30)  $n_0 = \mathbf{0} + (nat_{80} * \mathbf{p}_0) \vdash n_0 = \mathbf{0} + (nat_{80} * \mathbf{p}_0)$
- (C31)  $n_0 = \mathbf{0} + (nat_{76} * \mathbf{p}_3) \vdash n_0 = \mathbf{0} + (nat_{76} * \mathbf{p}_3)$
- (C32)  $n_0 = \mathbf{0} + (nat_{79} * \mathbf{p}_2) \vdash n_0 = \mathbf{0} + (nat_{79} * \mathbf{p}_2)$
- (C33)  $n_0 = \mathbf{0} + (nat_{78} * \mathbf{p}_1) \vdash n_0 = \mathbf{0} + (nat_{78} * \mathbf{p}_1)$
- (C34)  $n_0 = \mathbf{0} + (nat_{77} * \mathbf{p}_0) \vdash n_0 = \mathbf{0} + (nat_{77} * \mathbf{p}_0)$
- (C35)  $m_0 = \mathbf{0} + (nat_{75} * \mathbf{p}_2) \vdash m_0 = \mathbf{0} + (nat_{75} * \mathbf{p}_2)$
- (C37)  $m_0 = \mathbf{0} + (nat_{74} * \mathbf{p}_1) \vdash m_0 = \mathbf{0} + (nat_{74} * \mathbf{p}_1)$
- (C38)  $m_0 = \mathbf{0} + (nat_{73} * \mathbf{p}_0) \vdash m_0 = \mathbf{0} + (nat_{73} * \mathbf{p}_0)$
- (C41)  $k_0 = \mathbf{0} + (nat_{72} * \mathbf{p}_2) \vdash k_0 = \mathbf{0} + (nat_{72} * \mathbf{p}_2)$
- (C42)  $k_0 = \mathbf{0} + (nat_{71} * \mathbf{p}_1) \vdash k_0 = \mathbf{0} + (nat_{71} * \mathbf{p}_1)$
- (C43)  $k_0 = \mathbf{0} + (nat_{70} * \mathbf{p}_0) \vdash k_0 = \mathbf{0} + (nat_{70} * \mathbf{p}_0)$
- (C44)  $n_0 = \mathbf{0} + (nat_{67} * \mathbf{p}_2) \vdash n_0 = \mathbf{0} + (nat_{67} * \mathbf{p}_2)$
- (C45)  $n_0 = \mathbf{0} + (nat_{69} * \mathbf{p}_1) \vdash n_0 = \mathbf{0} + (nat_{69} * \mathbf{p}_1)$
- (C46)  $n_0 = \mathbf{0} + (nat_{68} * \mathbf{p}_0) \vdash n_0 = \mathbf{0} + (nat_{68} * \mathbf{p}_0)$

- (C47)  $n_0 = \mathbf{0} + (nat_{64} * \mathbf{p}_2) \vdash n_0 = \mathbf{0} + (nat_{64} * \mathbf{p}_2)$
- (C48)  $n_0 = \mathbf{0} + (nat_{66} * \mathbf{p}_1) \vdash n_0 = \mathbf{0} + (nat_{66} * \mathbf{p}_1)$
- (C49)  $n_0 = \mathbf{0} + (nat_{65} * \mathbf{p}_0) \vdash n_0 = \mathbf{0} + (nat_{65} * \mathbf{p}_0)$
- (C50)  $m_0 = \mathbf{0} + (nat_{63} * \mathbf{p}_1) \vdash m_0 = \mathbf{0} + (nat_{63} * \mathbf{p}_1)$
- (C52)  $m_0 = \mathbf{0} + (nat_{62} * \mathbf{p}_0) \vdash m_0 = \mathbf{0} + (nat_{62} * \mathbf{p}_0)$
- (C55)  $k_0 = \mathbf{0} + (nat_{61} * \mathbf{p}_1) \vdash k_0 = \mathbf{0} + (nat_{61} * \mathbf{p}_1)$
- (C56)  $k_0 = \mathbf{0} + (nat_{60} * \mathbf{p}_0) \vdash k_0 = \mathbf{0} + (nat_{60} * \mathbf{p}_0)$
- (C57)  $n_0 = \mathbf{0} + (nat_{59} * \mathbf{p}_1) \vdash n_0 = \mathbf{0} + (nat_{59} * \mathbf{p}_1)$
- (C58)  $n_0 = \mathbf{0} + (nat_{58} * \mathbf{p}_0) \vdash n_0 = \mathbf{0} + (nat_{58} * \mathbf{p}_0)$
- (C59)  $n_0 = \mathbf{0} + (nat_{56} * \mathbf{p}_1) \vdash n_0 = \mathbf{0} + (nat_{56} * \mathbf{p}_1)$
- (C60)  $n_0 = \mathbf{0} + (nat_{57} * \mathbf{p}_0) \vdash n_0 = \mathbf{0} + (nat_{57} * \mathbf{p}_0)$
- (C61)  $nat_{53} = \mathbf{0} + (nat_{55} * \mathbf{p}_0) \vdash nat_{53} = \mathbf{0} + (nat_{55} * \mathbf{p}_0)$
- (C62)  $nat_{53} = nat_{50} + (nat_{54} * (nat_{52} + \mathbf{1})) \vdash nat_{53} = nat_{50} + (nat_{54} * (nat_{52} + \mathbf{1}))$
- (C63)  $nat_{50} = \mathbf{0} + (nat_{51} * \mathbf{p}_0) \vdash nat_{50} = \mathbf{0} + (nat_{51} * \mathbf{p}_0)$
- (C64)  $\mathbf{0} < \mathbf{p}_0, t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_0) \vdash t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_0)$
- (C67)  $\mathbf{0} < \mathbf{p}_0, m_0 = i_0 + (q_0 * \mathbf{p}_0) \vdash m_0 = i_0 + (q_0 * \mathbf{p}_0)$
- (C68)  $\mathbf{0} < \mathbf{p}_0, i_0 = \mathbf{0} \vdash i_0 = \mathbf{0}$
- (C69)  $\mathbf{0} < \mathbf{p}_0, i_0 < \mathbf{p}_0 \vdash i_0 < \mathbf{p}_0$
- (C77)  $t_0 = i_0 + (q_0 * \mathbf{p}_0) \vdash t_0 = i_0 + (q_0 * \mathbf{p}_0)$
- (C80)  $\mathbf{1} < \mathbf{p}_0 \vdash \mathbf{1} < \mathbf{p}_0$
- (C81)  $\mathbf{0} < \mathbf{p}_1, t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_1) \vdash t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_1)$
- (C84)  $\mathbf{0} < \mathbf{p}_1, m_0 = i_0 + (q_0 * \mathbf{p}_1) \vdash m_0 = i_0 + (q_0 * \mathbf{p}_1)$
- (C85)  $\mathbf{0} < \mathbf{p}_1, i_0 = \mathbf{0} \vdash i_0 = \mathbf{0}$
- (C86)  $\mathbf{0} < \mathbf{p}_1, i_0 < \mathbf{p}_1 \vdash i_0 < \mathbf{p}_1$
- (C94)  $t_0 = i_0 + (q_0 * \mathbf{p}_1) \vdash t_0 = i_0 + (q_0 * \mathbf{p}_1)$
- (C97)  $\mathbf{1} < \mathbf{p}_1 \vdash \mathbf{1} < \mathbf{p}_1$
- (C98)  $\mathbf{0} < \mathbf{p}_2, t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_2) \vdash t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_2)$
- (C101)  $\mathbf{0} < \mathbf{p}_2, m_0 = i_0 + (q_0 * \mathbf{p}_2) \vdash m_0 = i_0 + (q_0 * \mathbf{p}_2)$
- (C102)  $\mathbf{0} < \mathbf{p}_2, i_0 = \mathbf{0} \vdash i_0 = \mathbf{0}$
- (C103)  $\mathbf{0} < \mathbf{p}_2, i_0 < \mathbf{p}_2 \vdash i_0 < \mathbf{p}_2$

- (C111)  $t_0 = i_0 + (q_0 * \mathbf{p}_2) \vdash t_0 = i_0 + (q_0 * \mathbf{p}_2)$
- (C114)  $\mathbf{1} < \mathbf{p}_2 \vdash \mathbf{1} < \mathbf{p}_2$
- (C115)  $\mathbf{0} < \mathbf{p}_3, t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_3) \vdash t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_3)$
- (C118)  $\mathbf{0} < \mathbf{p}_3, m_0 = i_0 + (q_0 * \mathbf{p}_3) \vdash m_0 = i_0 + (q_0 * \mathbf{p}_3)$
- (C119)  $\mathbf{0} < \mathbf{p}_3, i_0 = \mathbf{0} \vdash i_0 = \mathbf{0}$
- (C120)  $\mathbf{0} < \mathbf{p}_3, i_0 < \mathbf{p}_3 \vdash i_0 < \mathbf{p}_3$
- (C128)  $t_0 = i_0 + (q_0 * \mathbf{p}_3) \vdash t_0 = i_0 + (q_0 * \mathbf{p}_3)$
- (C131)  $\mathbf{1} < \mathbf{p}_3 \vdash \mathbf{1} < \mathbf{p}_3$
- (C132)  $nat_{40} = \mathbf{p}_3 \vdash nat_{40} = \mathbf{p}_3$
- (C133)  $nat_{40} = \mathbf{1} \vdash nat_{40} = \mathbf{1}$
- (C134)  $nat_{40} * nat_{41} = \mathbf{p}_3 \vdash nat_{40} * nat_{41} = \mathbf{p}_3$
- (C135)  $\mathbf{1} < \mathbf{p}_3 \vdash \mathbf{1} < \mathbf{p}_3$
- (C137)  $nat_{38} = \mathbf{p}_2 \vdash nat_{38} = \mathbf{p}_2$
- (C138)  $nat_{38} = \mathbf{1} \vdash nat_{38} = \mathbf{1}$
- (C139)  $nat_{38} * nat_{39} = \mathbf{p}_2 \vdash nat_{38} * nat_{39} = \mathbf{p}_2$
- (C140)  $\mathbf{1} < \mathbf{p}_2 \vdash \mathbf{1} < \mathbf{p}_2$
- (C142)  $nat_{36} = \mathbf{p}_1 \vdash nat_{36} = \mathbf{p}_1$
- (C143)  $nat_{36} = \mathbf{1} \vdash nat_{36} = \mathbf{1}$
- (C144)  $nat_{36} * nat_{37} = \mathbf{p}_1 \vdash nat_{36} * nat_{37} = \mathbf{p}_1$
- (C145)  $\mathbf{1} < \mathbf{p}_1 \vdash \mathbf{1} < \mathbf{p}_1$
- (C147)  $nat_{34} = \mathbf{p}_0 \vdash nat_{34} = \mathbf{p}_0$
- (C148)  $nat_{34} = \mathbf{1} \vdash nat_{34} = \mathbf{1}$
- (C149)  $nat_{34} * nat_{35} = \mathbf{p}_0 \vdash nat_{34} * nat_{35} = \mathbf{p}_0$
- (C150)  $\mathbf{1} < \mathbf{p}_0 \vdash \mathbf{1} < \mathbf{p}_0$
- (C152)  $nat_{32} = t_0 \vdash nat_{32} = t_0$
- (C153)  $nat_{32} = \mathbf{1} \vdash nat_{32} = \mathbf{1}$
- (C154)  $nat_{32} * nat_{33} = t_0 \vdash nat_{32} * nat_{33} = t_0$
- (C155)  $\mathbf{1} < t_0 \vdash \mathbf{1} < t_0$
- (C156)  $t_0 = \mathbf{p}_0 \vdash t_0 = \mathbf{p}_0$
- (C157)  $nat_{30} = t_0 \vdash nat_{30} = t_0$

- (C158)  $nat_{30} = \mathbf{1} \vdash nat_{30} = \mathbf{1}$
- (C159)  $nat_{30} * nat_{31} = t_0 \vdash nat_{30} * nat_{31} = t_0$
- (C160)  $\mathbf{1} < t_0 \vdash \mathbf{1} < t_0$
- (C161)  $t_0 = \mathbf{p_1} \vdash t_0 = \mathbf{p_1}$
- (C162)  $t_0 = \mathbf{p_0} \vdash t_0 = \mathbf{p_0}$
- (C163)  $nat_{28} = t_0 \vdash nat_{28} = t_0$
- (C164)  $nat_{28} = \mathbf{1} \vdash nat_{28} = \mathbf{1}$
- (C165)  $nat_{28} * nat_{29} = t_0 \vdash nat_{28} * nat_{29} = t_0$
- (C166)  $\mathbf{1} < t_0 \vdash \mathbf{1} < t_0$
- (C167)  $t_0 = \mathbf{p_2} \vdash t_0 = \mathbf{p_2}$
- (C168)  $t_0 = \mathbf{p_1} \vdash t_0 = \mathbf{p_1}$
- (C169)  $t_0 = \mathbf{p_0} \vdash t_0 = \mathbf{p_0}$
- (C178)  $m_0 = \mathbf{0} + (nat_{16} * t_0) \vdash m_0 = \mathbf{0} + (nat_{16} * t_0)$
- (C179)  $t_0 = \mathbf{p_3} \vdash t_0 = \mathbf{p_3}$
- (C180)  $m_0 = \mathbf{0} + (nat_{15} * t_0) \vdash m_0 = \mathbf{0} + (nat_{15} * t_0)$
- (C181)  $t_0 = \mathbf{p_2} \vdash t_0 = \mathbf{p_2}$
- (C182)  $m_0 = \mathbf{0} + (nat_{14} * t_0) \vdash m_0 = \mathbf{0} + (nat_{14} * t_0)$
- (C183)  $t_0 = \mathbf{p_1} \vdash t_0 = \mathbf{p_1}$
- (C184)  $m_0 = \mathbf{0} + (nat_{13} * t_0) \vdash m_0 = \mathbf{0} + (nat_{13} * t_0)$
- (C185)  $t_0 = \mathbf{p_0} \vdash t_0 = \mathbf{p_0}$
- (C191)  $m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C192)  $m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C194)  $t_0 = \mathbf{p_3}, m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C197)  $t_0 = \mathbf{p_2}, m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C200)  $t_0 = \mathbf{p_1}, m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C203)  $t_0 = \mathbf{p_0}, m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C205)  $m_0 = \mathbf{0} + (nat_{19} * \mathbf{p_3}) \vdash m_0 = \mathbf{0} + (nat_{19} * \mathbf{p_3})$
- (C207)  $m_0 = \mathbf{0} + (nat_{25} * t_0) \vdash m_0 = \mathbf{0} + (nat_{25} * t_0)$
- (C208)  $t_0 = \mathbf{p_2} \vdash t_0 = \mathbf{p_2}$
- (C209)  $t_0 = \mathbf{p_1} \vdash t_0 = \mathbf{p_1}$

- (C210)  $t_0 = \mathbf{p}_0 \vdash t_0 = \mathbf{p}_0$
- (C211)  $m_0 = \mathbf{0} + (nat_{20} * \mathbf{p}_2) \vdash m_0 = \mathbf{0} + (nat_{20} * \mathbf{p}_2)$
- (C213)  $m_0 = \mathbf{0} + (nat_{24} * t_0) \vdash m_0 = \mathbf{0} + (nat_{24} * t_0)$
- (C214)  $t_0 = \mathbf{p}_1 \vdash t_0 = \mathbf{p}_1$
- (C215)  $t_0 = \mathbf{p}_0 \vdash t_0 = \mathbf{p}_0$
- (C216)  $m_0 = \mathbf{0} + (nat_{21} * \mathbf{p}_1) \vdash m_0 = \mathbf{0} + (nat_{21} * \mathbf{p}_1)$
- (C218)  $m_0 = \mathbf{0} + (nat_{23} * t_0) \vdash m_0 = \mathbf{0} + (nat_{23} * t_0)$
- (C219)  $t_0 = \mathbf{p}_0 \vdash t_0 = \mathbf{p}_0$
- (C220)  $m_0 = \mathbf{0} + (nat_{22} * \mathbf{p}_0) \vdash m_0 = \mathbf{0} + (nat_{22} * \mathbf{p}_0)$

## 4 Clauses with Reflexivity

- (C136)  $\vdash \mathbf{p}_3 = \mathbf{p}_3$
- (C141)  $\vdash \mathbf{p}_2 = \mathbf{p}_2$
- (C146)  $\vdash \mathbf{p}_1 = \mathbf{p}_1$
- (C151)  $\vdash \mathbf{p}_0 = \mathbf{p}_0$
- (C186)  $\vdash m_0 = \mathbf{1}, m_0 = m_0$
- (C206)  $\vdash \mathbf{p}_3 = \mathbf{p}_3$
- (C212)  $\vdash \mathbf{p}_2 = \mathbf{p}_2$
- (C217)  $\vdash \mathbf{p}_1 = \mathbf{p}_1$
- (C221)  $\vdash \mathbf{p}_0 = \mathbf{p}_0$
- (C222)  $\vdash \mathbf{1} = \mathbf{1}$

## 5 Explicit Axioms

- (A1)  $n + \mathbf{1} < m \vdash n < m$  (subsumes 4 clauses)
- (A2)  $\vdash k + l = l + k$
- (A3)  $\vdash k + (l + m) = (k + l) + m$
- (A4)  $\vdash (k + l) + m = k + (l + m)$  (subsumes 4 clauses)
- (A5)  $\vdash k + \mathbf{0} = k$
- (A6)  $k + l = k + m \vdash l = m$
- (A7)  $\vdash \mathbf{0} + k = k$  (subsumes 9 clauses)
- (A8)  $k + l = m + l \vdash k = m$

- (A9)  $k = l + k \vdash l = \mathbf{0}$
- (A10)  $k = k + l \vdash l = \mathbf{0}$
- (A11)  $k + l = k \vdash l = \mathbf{0}$
- (A12)  $k + l = l \vdash k = \mathbf{0}$
- (A13)  $k = l \vdash m + k = m + l$
- (A14)  $\mathbf{1} + (k + \mathbf{1}) = \mathbf{1} \vdash$  (subsumes 1 clause)
- (A15)  $\vdash k * l = l * k$  (subsumes 1 clause)
- (A16)  $k + \mathbf{1} = \mathbf{0} \vdash$
- (A17)  $\vdash k * (l * m) = (k * l) * m$
- (A18)  $\vdash (k * l) * m = k * (l * m)$
- (A19)  $\vdash k * \mathbf{1} = k$
- (A20)  $\vdash \mathbf{1} * k = k$
- (A21)  $\vdash k * (l + m) = (k * l) + (k * m)$
- (A22)  $\vdash (k * l) + (k * m) = k * (m + l)$
- (A23)  $\vdash (k + l) * m = (k * m) + (l * m)$
- (A24)  $\vdash (k * l) + (m * l) = (k + m) * l$  (subsumes 4 clauses)
- (A25)  $\vdash (k * l) + k = k * (l + \mathbf{1})$
- (A26)  $\vdash (k + l) * m = (l * m) + (k * m)$
- (A27)  $\mathbf{1} = k * l \vdash k = \mathbf{1}$
- (A28)  $\mathbf{1} = l * k \vdash k = \mathbf{1}$
- (A29)  $k < l, k < m, l < m, k + (i * m) = l + (j * m) \vdash$  (subsumes 4 clauses)
- (A30)  $k < l, k < m, l < m, l + (i * m) = k + (j * m) \vdash$  (subsumes 4 clauses)
- (A31)  $\mathbf{1} < k, k = \mathbf{1} \vdash$
- (A32)  $\vdash k = l, k < l, l < k$  (subsumes 4 clauses)
- (A33)  $\vdash \mathbf{0} < k + \mathbf{1}$
- (A34)  $\mathbf{1} < k, \mathbf{1} = l * k \vdash$