

## 1 Active Clauses

- (C9)  $\vdash m_0 + (((k * (l_0 + (\mathbf{1} + \mathbf{1}))) + (l_0 * (m_0 + \mathbf{1}))) + \mathbf{1}) = k + ((k + (m_0 + \mathbf{1})) * (l_0 + \mathbf{1}))$
- (C17)  $m_0 = k_0 + (r_0 * ((t_0 + \mathbf{1}) * (t_1 + \mathbf{1}))) \vdash m_0 = k_0 + ((r_0 * (t_0 + \mathbf{1})) * (t_1 + \mathbf{1}))$
- (C19)  $m_0 = k_0 + (r_0 * ((t_0 + \mathbf{1}) * (t_1 + \mathbf{1}))) \vdash m_0 = k_0 + ((r_0 * (t_1 + \mathbf{1})) * (t_0 + \mathbf{1}))$
- (C20)  $\vdash (((t_0 + \mathbf{1}) * t_1) + t_0) + \mathbf{1} = (t_0 + \mathbf{1}) * (t_1 + \mathbf{1})$
- (C36)  $\mathbf{0} < \mathbf{p}_0 \vdash \mathbf{p}_0 = s_7(\mathbf{p}_0) + \mathbf{1}$
- (C37)  $\mathbf{0} < \mathbf{p}_0 \vdash t_0 = s_5(\mathbf{p}_0, t_0) + (s_6(\mathbf{p}_0, t_0) * \mathbf{p}_0)$
- (C38)  $\mathbf{0} < \mathbf{p}_0, s_5(\mathbf{p}_0, t_0) = \mathbf{0} \vdash t_0 = \mathbf{0} + (s_6(\mathbf{p}_0, t_0) * \mathbf{p}_0)$
- (C39)  $\mathbf{0} < \mathbf{p}_0 \vdash s_5(\mathbf{p}_0, t_0) < \mathbf{p}_0$
- (C53)  $\mathbf{0} < \mathbf{p}_1 \vdash \mathbf{p}_1 = s_7(\mathbf{p}_1) + \mathbf{1}$
- (C54)  $\mathbf{0} < \mathbf{p}_1 \vdash t_0 = s_5(\mathbf{p}_1, t_0) + (s_6(\mathbf{p}_1, t_0) * \mathbf{p}_1)$
- (C55)  $\mathbf{0} < \mathbf{p}_1, s_5(\mathbf{p}_1, t_0) = \mathbf{0} \vdash t_0 = \mathbf{0} + (s_6(\mathbf{p}_1, t_0) * \mathbf{p}_1)$
- (C56)  $\mathbf{0} < \mathbf{p}_1 \vdash s_5(\mathbf{p}_1, t_0) < \mathbf{p}_1$
- (C79)  $t_0 = \mathbf{p}_1, \text{nat}_{20} * \text{nat}_{21} = t_0 \vdash \text{nat}_{20} = \mathbf{1}, \text{nat}_{20} = t_0$
- (C80)  $t_0 = \mathbf{p}_1 \vdash \mathbf{1} < t_0$
- (C81)  $t_0 = \mathbf{p}_0, \text{nat}_{20} * \text{nat}_{21} = t_0 \vdash \text{nat}_{20} = \mathbf{1}, \text{nat}_{20} = t_0$
- (C82)  $t_0 = \mathbf{p}_0 \vdash \mathbf{1} < t_0$
- (C88)  $\vdash m_0 = \mathbf{1}, s_1(m_0) * s_4(m_0) = m_0$
- (C91)  $\vdash m_0 = \mathbf{1}, s_1(m_0) = \mathbf{p}_0, s_1(m_0) = \mathbf{p}_1$
- (C94)  $t_0 = \mathbf{p}_1, \mathbf{1} = n_0 * t_0 \vdash$
- (C97)  $t_0 = \mathbf{p}_0, \mathbf{1} = n_0 * t_0 \vdash$

## 2 Subsumed

- (C1)  $\mathbf{1} + (l_0 + \mathbf{1}) = \mathbf{1} \vdash$  by (A13)
- (C31)  $\mathbf{0} < \mathbf{p}_0 \vdash (q_0 * \mathbf{p}_0) + (r_0 * \mathbf{p}_0) = (q_0 + r_0) * \mathbf{p}_0$  by (A23)
- (C32)  $\mathbf{0} < \mathbf{p}_0 \vdash (i_0 + (q_0 * \mathbf{p}_0)) + (r_0 * \mathbf{p}_0) = i_0 + ((q_0 * \mathbf{p}_0) + (r_0 * \mathbf{p}_0))$  by (A3)
- (C40)  $\mathbf{0} < i_0, \mathbf{0} < \mathbf{p}_0, i_0 < \mathbf{p}_0, i_0 + (q_0 * \mathbf{p}_0) = \mathbf{0} + (r_0 * \mathbf{p}_0) \vdash$  by (A29)
- (C41)  $i_0 < \mathbf{0}, \mathbf{0} < \mathbf{p}_0, i_0 < \mathbf{p}_0, i_0 + (q_0 * \mathbf{p}_0) = \mathbf{0} + (r_0 * \mathbf{p}_0) \vdash$  by (A28)
- (C42)  $\vdash i_0 = \mathbf{0}, i_0 < \mathbf{0}, \mathbf{0} < i_0$  by (A30)
- (C44)  $\mathbf{0} + \mathbf{1} < \mathbf{p}_0 \vdash \mathbf{0} < \mathbf{p}_0$  by (A34)
- (C45)  $\vdash \mathbf{0} + \mathbf{1} = \mathbf{1}$  by (A6)

- (C48)  $\mathbf{0} < \mathbf{p}_1 \vdash (q_0 * \mathbf{p}_1) + (r_0 * \mathbf{p}_1) = (q_0 + r_0) * \mathbf{p}_1$  by (A23)
- (C49)  $\mathbf{0} < \mathbf{p}_1 \vdash (i_0 + (q_0 * \mathbf{p}_1)) + (r_0 * \mathbf{p}_1) = i_0 + ((q_0 * \mathbf{p}_1) + (r_0 * \mathbf{p}_1))$  by (A3)
- (C57)  $\mathbf{0} < i_0, \mathbf{0} < \mathbf{p}_1, i_0 < \mathbf{p}_1, i_0 + (q_0 * \mathbf{p}_1) = \mathbf{0} + (r_0 * \mathbf{p}_1) \vdash$  by (A29)
- (C58)  $i_0 < \mathbf{0}, \mathbf{0} < \mathbf{p}_1, i_0 < \mathbf{p}_1, i_0 + (q_0 * \mathbf{p}_1) = \mathbf{0} + (r_0 * \mathbf{p}_1) \vdash$  by (A28)
- (C59)  $\vdash i_0 = \mathbf{0}, i_0 < \mathbf{0}, \mathbf{0} < i_0$  by (A30)
- (C61)  $\mathbf{0} + \mathbf{1} < \mathbf{p}_1 \vdash \mathbf{0} < \mathbf{p}_1$  by (A34)
- (C62)  $\vdash \mathbf{0} + \mathbf{1} = \mathbf{1}$  by (A6)
- (C89)  $\vdash m_0 = \mathbf{1}, s_4(m_0) * s_1(m_0) = s_1(m_0) * s_4(m_0)$  by (A14)
- (C90)  $\vdash m_0 = \mathbf{1}, \mathbf{0} + (s_4(m_0) * s_1(m_0)) = s_4(m_0) * s_1(m_0)$  by (A6)
- (C96)  $t_0 = \mathbf{p}_1 \vdash \mathbf{0} + (n_0 * t_0) = n_0 * t_0$  by (A6)
- (C99)  $t_0 = \mathbf{p}_0 \vdash \mathbf{0} + (n_0 * t_0) = n_0 * t_0$  by (A6)

### 3 Clauses with Tautologies

- (C2)  $nat_2 + (nat_3 + \mathbf{1}) = \mathbf{1} \vdash nat_2 + (nat_3 + \mathbf{1}) = \mathbf{1}$
- (C3)  $nat_4 = \mathbf{1} \vdash nat_4 = \mathbf{1}$
- (C4)  $nat_7 = \mathbf{1} \vdash nat_7 = \mathbf{1}$
- (C5)  $nat_7 = nat_5 + (nat_8 * (nat_6 + \mathbf{1})) \vdash nat_7 = nat_5 + (nat_8 * (nat_6 + \mathbf{1}))$
- (C6)  $nat_5 = \mathbf{1} \vdash nat_5 = \mathbf{1}$
- (C7)  $m_0 + (n_0 + \mathbf{1}) \in x \vdash m_0 + (n_0 + \mathbf{1}) \in x$
- (C8)  $m_0 + (n_0 + \mathbf{1}) = k + (nat_1 * (l_0 + \mathbf{1})) \vdash m_0 + (n_0 + \mathbf{1}) = k + (nat_1 * (l_0 + \mathbf{1}))$
- (C10)  $k \in x \vdash k \in x$
- (C11)  $nat_{11} = \mathbf{1} \vdash nat_{11} = \mathbf{1}$
- (C12)  $nat_{11} = nat_9 + (nat_{12} * (nat_{10} + \mathbf{1})) \vdash nat_{11} = nat_9 + (nat_{12} * (nat_{10} + \mathbf{1}))$
- (C13)  $nat_9 = \mathbf{1} \vdash nat_9 = \mathbf{1}$
- (C14)  $n_0 = \mathbf{1} \vdash n_0 = \mathbf{1}$
- (C15)  $n_0 = \mathbf{1} \vdash n_0 = \mathbf{1}$
- (C16)  $m_0 = \mathbf{0} + (nat_{45} * \mathbf{p}_1) \vdash m_0 = \mathbf{0} + (nat_{45} * \mathbf{p}_1)$
- (C18)  $m_0 = \mathbf{0} + (nat_{44} * \mathbf{p}_0) \vdash m_0 = \mathbf{0} + (nat_{44} * \mathbf{p}_0)$
- (C21)  $k_0 = \mathbf{0} + (nat_{43} * \mathbf{p}_1) \vdash k_0 = \mathbf{0} + (nat_{43} * \mathbf{p}_1)$
- (C22)  $k_0 = \mathbf{0} + (nat_{42} * \mathbf{p}_0) \vdash k_0 = \mathbf{0} + (nat_{42} * \mathbf{p}_0)$
- (C23)  $n_0 = \mathbf{0} + (nat_{40} * \mathbf{p}_1) \vdash n_0 = \mathbf{0} + (nat_{40} * \mathbf{p}_1)$

- (C24)  $n_0 = \mathbf{0} + (\text{nat}_{41} * \mathbf{p}_0) \vdash n_0 = \mathbf{0} + (\text{nat}_{41} * \mathbf{p}_0)$
- (C25)  $n_0 = \mathbf{0} + (\text{nat}_{38} * \mathbf{p}_1) \vdash n_0 = \mathbf{0} + (\text{nat}_{38} * \mathbf{p}_1)$
- (C26)  $n_0 = \mathbf{0} + (\text{nat}_{39} * \mathbf{p}_0) \vdash n_0 = \mathbf{0} + (\text{nat}_{39} * \mathbf{p}_0)$
- (C27)  $\text{nat}_{35} = \mathbf{0} + (\text{nat}_{37} * \mathbf{p}_0) \vdash \text{nat}_{35} = \mathbf{0} + (\text{nat}_{37} * \mathbf{p}_0)$
- (C28)  $\text{nat}_{35} = \text{nat}_{32} + (\text{nat}_{36} * (\text{nat}_{34} + \mathbf{1})) \vdash \text{nat}_{35} = \text{nat}_{32} + (\text{nat}_{36} * (\text{nat}_{34} + \mathbf{1}))$
- (C29)  $\text{nat}_{32} = \mathbf{0} + (\text{nat}_{33} * \mathbf{p}_0) \vdash \text{nat}_{32} = \mathbf{0} + (\text{nat}_{33} * \mathbf{p}_0)$
- (C30)  $\mathbf{0} < \mathbf{p}_0, t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_0) \vdash t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_0)$
- (C33)  $\mathbf{0} < \mathbf{p}_0, m_0 = i_0 + (q_0 * \mathbf{p}_0) \vdash m_0 = i_0 + (q_0 * \mathbf{p}_0)$
- (C34)  $\mathbf{0} < \mathbf{p}_0, i_0 = \mathbf{0} \vdash i_0 = \mathbf{0}$
- (C35)  $\mathbf{0} < \mathbf{p}_0, i_0 < \mathbf{p}_0 \vdash i_0 < \mathbf{p}_0$
- (C43)  $t_0 = i_0 + (q_0 * \mathbf{p}_0) \vdash t_0 = i_0 + (q_0 * \mathbf{p}_0)$
- (C46)  $\mathbf{1} < \mathbf{p}_0 \vdash \mathbf{1} < \mathbf{p}_0$
- (C47)  $\mathbf{0} < \mathbf{p}_1, t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_1) \vdash t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_1)$
- (C50)  $\mathbf{0} < \mathbf{p}_1, m_0 = i_0 + (q_0 * \mathbf{p}_1) \vdash m_0 = i_0 + (q_0 * \mathbf{p}_1)$
- (C51)  $\mathbf{0} < \mathbf{p}_1, i_0 = \mathbf{0} \vdash i_0 = \mathbf{0}$
- (C52)  $\mathbf{0} < \mathbf{p}_1, i_0 < \mathbf{p}_1 \vdash i_0 < \mathbf{p}_1$
- (C60)  $t_0 = i_0 + (q_0 * \mathbf{p}_1) \vdash t_0 = i_0 + (q_0 * \mathbf{p}_1)$
- (C63)  $\mathbf{1} < \mathbf{p}_1 \vdash \mathbf{1} < \mathbf{p}_1$
- (C64)  $\text{nat}_{26} = \mathbf{p}_1 \vdash \text{nat}_{26} = \mathbf{p}_1$
- (C65)  $\text{nat}_{26} = \mathbf{1} \vdash \text{nat}_{26} = \mathbf{1}$
- (C66)  $\text{nat}_{26} * \text{nat}_{27} = \mathbf{p}_1 \vdash \text{nat}_{26} * \text{nat}_{27} = \mathbf{p}_1$
- (C67)  $\mathbf{1} < \mathbf{p}_1 \vdash \mathbf{1} < \mathbf{p}_1$
- (C69)  $\text{nat}_{24} = \mathbf{p}_0 \vdash \text{nat}_{24} = \mathbf{p}_0$
- (C70)  $\text{nat}_{24} = \mathbf{1} \vdash \text{nat}_{24} = \mathbf{1}$
- (C71)  $\text{nat}_{24} * \text{nat}_{25} = \mathbf{p}_0 \vdash \text{nat}_{24} * \text{nat}_{25} = \mathbf{p}_0$
- (C72)  $\mathbf{1} < \mathbf{p}_0 \vdash \mathbf{1} < \mathbf{p}_0$
- (C74)  $\text{nat}_{22} = t_0 \vdash \text{nat}_{22} = t_0$
- (C75)  $\text{nat}_{22} = \mathbf{1} \vdash \text{nat}_{22} = \mathbf{1}$
- (C76)  $\text{nat}_{22} * \text{nat}_{23} = t_0 \vdash \text{nat}_{22} * \text{nat}_{23} = t_0$
- (C77)  $\mathbf{1} < t_0 \vdash \mathbf{1} < t_0$

- (C78)  $t_0 = \mathbf{p}_0 \vdash t_0 = \mathbf{p}_0$
- (C83)  $m_0 = \mathbf{0} + (\text{nat}_{14} * t_0) \vdash m_0 = \mathbf{0} + (\text{nat}_{14} * t_0)$
- (C84)  $t_0 = \mathbf{p}_1 \vdash t_0 = \mathbf{p}_1$
- (C85)  $m_0 = \mathbf{0} + (\text{nat}_{13} * t_0) \vdash m_0 = \mathbf{0} + (\text{nat}_{13} * t_0)$
- (C86)  $t_0 = \mathbf{p}_0 \vdash t_0 = \mathbf{p}_0$
- (C92)  $m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C93)  $m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C95)  $t_0 = \mathbf{p}_1, m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C98)  $t_0 = \mathbf{p}_0, m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C100)  $m_0 = \mathbf{0} + (\text{nat}_{17} * \mathbf{p}_1) \vdash m_0 = \mathbf{0} + (\text{nat}_{17} * \mathbf{p}_1)$
- (C102)  $m_0 = \mathbf{0} + (\text{nat}_{19} * t_0) \vdash m_0 = \mathbf{0} + (\text{nat}_{19} * t_0)$
- (C103)  $t_0 = \mathbf{p}_0 \vdash t_0 = \mathbf{p}_0$
- (C104)  $m_0 = \mathbf{0} + (\text{nat}_{18} * \mathbf{p}_0) \vdash m_0 = \mathbf{0} + (\text{nat}_{18} * \mathbf{p}_0)$

## 4 Clauses with Reflexivity

- (C68)  $\vdash \mathbf{p}_1 = \mathbf{p}_1$
- (C73)  $\vdash \mathbf{p}_0 = \mathbf{p}_0$
- (C87)  $\vdash m_0 = \mathbf{1}, m_0 = m_0$
- (C101)  $\vdash \mathbf{p}_1 = \mathbf{p}_1$
- (C105)  $\vdash \mathbf{p}_0 = \mathbf{p}_0$
- (C106)  $\vdash \mathbf{1} = \mathbf{1}$

## 5 Explicit Axioms

- (A1)  $\vdash k + l = l + k$
- (A2)  $\vdash k + (l + m) = (k + l) + m$
- (A3)  $\vdash (k + l) + m = k + (l + m)$  (subsumes 2 clauses)
- (A4)  $k + l = k + m \vdash l = m$
- (A5)  $\vdash k + \mathbf{0} = k$
- (A6)  $\vdash \mathbf{0} + k = k$  (subsumes 5 clauses)
- (A7)  $k + l = m + l \vdash k = m$
- (A8)  $k = l + k \vdash l = \mathbf{0}$

- (A9)  $k = k + l \vdash l = \mathbf{0}$
- (A10)  $k + l = k \vdash l = \mathbf{0}$
- (A11)  $k + l = l \vdash k = \mathbf{0}$
- (A12)  $k = l \vdash m + k = m + l$
- (A13)  $\mathbf{1} + (k + \mathbf{1}) = \mathbf{1} \vdash$  (subsumes 1 clause)
- (A14)  $\vdash k * l = l * k$  (subsumes 1 clause)
- (A15)  $k + \mathbf{1} = \mathbf{0} \vdash$
- (A16)  $\vdash k * (l * m) = (k * l) * m$
- (A17)  $\vdash (k * l) * m = k * (l * m)$
- (A18)  $\vdash k * \mathbf{1} = k$
- (A19)  $\vdash \mathbf{1} * k = k$
- (A20)  $\vdash k * (l + m) = (k * l) + (k * m)$
- (A21)  $\vdash (k + l) * m = (k * m) + (l * m)$
- (A22)  $\vdash (k * l) + (k * m) = k * (m + l)$
- (A23)  $\vdash (k * l) + (m * l) = (k + m) * l$  (subsumes 2 clauses)
- (A24)  $\vdash (k * l) + k = k * (l + \mathbf{1})$
- (A25)  $\vdash (k + l) * m = (l * m) + (k * m)$
- (A26)  $\mathbf{1} = k * l \vdash k = \mathbf{1}$
- (A27)  $\mathbf{1} = l * k \vdash k = \mathbf{1}$
- (A28)  $k < l, k < m, l < m, k + (i * m) = l + (j * m) \vdash$  (subsumes 2 clauses)
- (A29)  $k < l, k < m, l < m, l + (i * m) = k + (j * m) \vdash$  (subsumes 2 clauses)
- (A30)  $\vdash k = l, k < l, l < k$  (subsumes 2 clauses)
- (A31)  $\vdash \mathbf{0} < k + \mathbf{1}$
- (A32)  $\mathbf{1} < k, k = \mathbf{1} \vdash$
- (A33)  $\mathbf{1} < k, \mathbf{1} = l * k \vdash$
- (A34)  $n + \mathbf{1} < m \vdash n < m$  (subsumes 2 clauses)