

1 Active Clauses

$$(C9) \vdash m_0 + (((k * (l_0 + (\mathbf{1} + \mathbf{1}))) + (l_0 * (m_0 + \mathbf{1}))) + \mathbf{1}) = k + ((k + (m_0 + \mathbf{1})) * (l_0 + \mathbf{1}))$$

$$(C25) \mathbf{0} < \mathbf{p}_0 \vdash \mathbf{p}_0 = s_7(\mathbf{p}_0) + \mathbf{1}$$

$$(C26) \mathbf{0} < \mathbf{p}_0 \vdash t_0 = s_5(\mathbf{p}_0, t_0) + (s_6(\mathbf{p}_0, t_0) * \mathbf{p}_0)$$

$$(C27) \mathbf{0} < \mathbf{p}_0, s_5(\mathbf{p}_0, t_0) = \mathbf{0} \vdash t_0 = \mathbf{0} + (s_6(\mathbf{p}_0, t_0) * \mathbf{p}_0)$$

$$(C28) \mathbf{0} < \mathbf{p}_0 \vdash s_5(\mathbf{p}_0, t_0) < \mathbf{p}_0$$

$$(C41) t_0 = \mathbf{p}_0, nat_{17} * nat_{18} = t_0 \vdash nat_{17} = \mathbf{1}, nat_{17} = t_0$$

$$(C42) t_0 = \mathbf{p}_0 \vdash \mathbf{1} < t_0$$

$$(C46) \vdash m_0 = \mathbf{1}, s_1(m_0) * s_4(m_0) = m_0$$

$$(C49) \vdash m_0 = \mathbf{1}, s_1(m_0) = \mathbf{p}_0$$

$$(C52) t_0 = \mathbf{p}_0, \mathbf{1} = n_0 * t_0 \vdash$$

2 Subsumed

$$(C1) \mathbf{1} + (l_0 + \mathbf{1}) = \mathbf{1} \vdash \text{by (A13)}$$

$$(C20) \mathbf{0} < \mathbf{p}_0 \vdash (q_0 * \mathbf{p}_0) + (r_0 * \mathbf{p}_0) = (q_0 + r_0) * \mathbf{p}_0 \text{ by (A23)}$$

$$(C21) \mathbf{0} < \mathbf{p}_0 \vdash (i_0 + (q_0 * \mathbf{p}_0)) + (r_0 * \mathbf{p}_0) = i_0 + ((q_0 * \mathbf{p}_0) + (r_0 * \mathbf{p}_0)) \text{ by (A1)}$$

$$(C29) \mathbf{0} < i_0, \mathbf{0} < \mathbf{p}_0, i_0 < \mathbf{p}_0, i_0 + (q_0 * \mathbf{p}_0) = \mathbf{0} + (r_0 * \mathbf{p}_0) \vdash \text{by (A29)}$$

$$(C30) i_0 < \mathbf{0}, \mathbf{0} < \mathbf{p}_0, i_0 < \mathbf{p}_0, i_0 + (q_0 * \mathbf{p}_0) = \mathbf{0} + (r_0 * \mathbf{p}_0) \vdash \text{by (A28)}$$

$$(C31) \vdash i_0 = \mathbf{0}, i_0 < \mathbf{0}, \mathbf{0} < i_0 \text{ by (A30)}$$

$$(C33) \mathbf{0} + \mathbf{1} < \mathbf{p}_0 \vdash \mathbf{0} < \mathbf{p}_0 \text{ by (A34)}$$

$$(C34) \vdash \mathbf{0} + \mathbf{1} = \mathbf{1} \text{ by (A6)}$$

$$(C47) \vdash m_0 = \mathbf{1}, s_4(m_0) * s_1(m_0) = s_1(m_0) * s_4(m_0) \text{ by (A14)}$$

$$(C48) \vdash m_0 = \mathbf{1}, \mathbf{0} + (s_4(m_0) * s_1(m_0)) = s_4(m_0) * s_1(m_0) \text{ by (A6)}$$

$$(C54) t_0 = \mathbf{p}_0 \vdash \mathbf{0} + (n_0 * t_0) = n_0 * t_0 \text{ by (A6)}$$

3 Clauses with Tautologies

$$(C2) nat_2 + (nat_3 + \mathbf{1}) = \mathbf{1} \vdash nat_2 + (nat_3 + \mathbf{1}) = \mathbf{1}$$

$$(C3) nat_4 = \mathbf{1} \vdash nat_4 = \mathbf{1}$$

$$(C4) nat_7 = \mathbf{1} \vdash nat_7 = \mathbf{1}$$

$$(C5) nat_7 = nat_5 + (nat_8 * (nat_6 + \mathbf{1})) \vdash nat_7 = nat_5 + (nat_8 * (nat_6 + \mathbf{1}))$$

$$(C6) nat_5 = \mathbf{1} \vdash nat_5 = \mathbf{1}$$

- (C7) $m_0 + (n_0 + \mathbf{1}) \in x \vdash m_0 + (n_0 + \mathbf{1}) \in x$
- (C8) $m_0 + (n_0 + \mathbf{1}) = k + (\text{nat}_1 * (l_0 + \mathbf{1})) \vdash m_0 + (n_0 + \mathbf{1}) = k + (\text{nat}_1 * (l_0 + \mathbf{1}))$
- (C10) $k \in x \vdash k \in x$
- (C11) $\text{nat}_{11} = \mathbf{1} \vdash \text{nat}_{11} = \mathbf{1}$
- (C12) $\text{nat}_{11} = \text{nat}_9 + (\text{nat}_{12} * (\text{nat}_{10} + \mathbf{1})) \vdash \text{nat}_{11} = \text{nat}_9 + (\text{nat}_{12} * (\text{nat}_{10} + \mathbf{1}))$
- (C13) $\text{nat}_9 = \mathbf{1} \vdash \text{nat}_9 = \mathbf{1}$
- (C14) $n_0 = \mathbf{1} \vdash n_0 = \mathbf{1}$
- (C15) $n_0 = \mathbf{1} \vdash n_0 = \mathbf{1}$
- (C16) $\text{nat}_{26} = \mathbf{0} + (\text{nat}_{28} * \mathbf{p}_0) \vdash \text{nat}_{26} = \mathbf{0} + (\text{nat}_{28} * \mathbf{p}_0)$
- (C17) $\text{nat}_{26} = \text{nat}_{23} + (\text{nat}_{27} * (\text{nat}_{25} + \mathbf{1})) \vdash \text{nat}_{26} = \text{nat}_{23} + (\text{nat}_{27} * (\text{nat}_{25} + \mathbf{1}))$
- (C18) $\text{nat}_{23} = \mathbf{0} + (\text{nat}_{24} * \mathbf{p}_0) \vdash \text{nat}_{23} = \mathbf{0} + (\text{nat}_{24} * \mathbf{p}_0)$
- (C19) $\mathbf{0} < \mathbf{p}_0, t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_0) \vdash t_0 = i_0 + ((q_0 + r_0) * \mathbf{p}_0)$
- (C22) $\mathbf{0} < \mathbf{p}_0, m_0 = i_0 + (q_0 * \mathbf{p}_0) \vdash m_0 = i_0 + (q_0 * \mathbf{p}_0)$
- (C23) $\mathbf{0} < \mathbf{p}_0, i_0 = \mathbf{0} \vdash i_0 = \mathbf{0}$
- (C24) $\mathbf{0} < \mathbf{p}_0, i_0 < \mathbf{p}_0 \vdash i_0 < \mathbf{p}_0$
- (C32) $t_0 = i_0 + (q_0 * \mathbf{p}_0) \vdash t_0 = i_0 + (q_0 * \mathbf{p}_0)$
- (C35) $\mathbf{1} < \mathbf{p}_0 \vdash \mathbf{1} < \mathbf{p}_0$
- (C36) $\text{nat}_{19} = \mathbf{p}_0 \vdash \text{nat}_{19} = \mathbf{p}_0$
- (C37) $\text{nat}_{19} = \mathbf{1} \vdash \text{nat}_{19} = \mathbf{1}$
- (C38) $\text{nat}_{19} * \text{nat}_{20} = \mathbf{p}_0 \vdash \text{nat}_{19} * \text{nat}_{20} = \mathbf{p}_0$
- (C39) $\mathbf{1} < \mathbf{p}_0 \vdash \mathbf{1} < \mathbf{p}_0$
- (C43) $m_0 = \mathbf{0} + (\text{nat}_{13} * t_0) \vdash m_0 = \mathbf{0} + (\text{nat}_{13} * t_0)$
- (C44) $t_0 = \mathbf{p}_0 \vdash t_0 = \mathbf{p}_0$
- (C50) $m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C51) $m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C53) $t_0 = \mathbf{p}_0, m_0 = \mathbf{1} \vdash m_0 = \mathbf{1}$
- (C55) $m_0 = \mathbf{0} + (\text{nat}_{16} * \mathbf{p}_0) \vdash m_0 = \mathbf{0} + (\text{nat}_{16} * \mathbf{p}_0)$

4 Clauses with Reflexivity

$$(C40) \vdash \mathbf{p_0} = \mathbf{p_0}$$

$$(C45) \vdash m_0 = \mathbf{1}, m_0 = m_0$$

$$(C56) \vdash \mathbf{p_0} = \mathbf{p_0}$$

$$(C57) \vdash \mathbf{1} = \mathbf{1}$$

5 Explicit Axioms

$$(A1) \vdash (k + l) + m = k + (l + m) \text{ (subsumes 1 clause)}$$

$$(A2) \vdash k + l = l + k$$

$$(A3) \vdash k + (l + m) = (k + l) + m$$

$$(A4) k + l = k + m \vdash l = m$$

$$(A5) \vdash k + \mathbf{0} = k$$

$$(A6) \vdash \mathbf{0} + k = k \text{ (subsumes 3 clauses)}$$

$$(A7) k + l = m + l \vdash k = m$$

$$(A8) k = l + k \vdash l = \mathbf{0}$$

$$(A9) k = k + l \vdash l = \mathbf{0}$$

$$(A10) k + l = k \vdash l = \mathbf{0}$$

$$(A11) k + l = l \vdash k = \mathbf{0}$$

$$(A12) k = l \vdash m + k = m + l$$

$$(A13) \mathbf{1} + (k + \mathbf{1}) = \mathbf{1} \vdash \text{(subsumes 1 clause)}$$

$$(A14) \vdash k * l = l * k \text{ (subsumes 1 clause)}$$

$$(A15) k + \mathbf{1} = \mathbf{0} \vdash$$

$$(A16) \vdash k * (l * m) = (k * l) * m$$

$$(A17) \vdash (k * l) * m = k * (l * m)$$

$$(A18) \vdash k * \mathbf{1} = k$$

$$(A19) \vdash \mathbf{1} * k = k$$

$$(A20) \vdash k * (l + m) = (k * l) + (k * m)$$

$$(A21) \vdash (k + l) * m = (k * m) + (l * m)$$

$$(A22) \vdash (k * l) + (k * m) = k * (m + l)$$

$$(A23) \vdash (k * l) + (m * l) = (k + m) * l \text{ (subsumes 1 clause)}$$

$$(A24) \vdash (k * l) + k = k * (l + \mathbf{1})$$

$$(A25) \vdash (k + l) * m = (l * m) + (k * m)$$

$$(A26) \mathbf{1} = l * k \vdash k = \mathbf{1}$$

$$(A27) \mathbf{1} = k * l \vdash k = \mathbf{1}$$

$$(A28) k < l, k < m, l < m, k + (i * m) = l + (j * m) \vdash \text{(subsumes 1 clause)}$$

$$(A29) k < l, k < m, l < m, l + (i * m) = k + (j * m) \vdash \text{(subsumes 1 clause)}$$

$$(A30) \vdash k = l, k < l, l < k \text{ (subsumes 1 clause)}$$

$$(A31) \vdash \mathbf{0} < k + \mathbf{1}$$

$$(A32) \mathbf{1} < k, \mathbf{1} = l * k \vdash$$

$$(A33) \mathbf{1} < k, k = \mathbf{1} \vdash$$

$$(A34) n + \mathbf{1} < m \vdash n < m \text{ (subsumes 1 clause)}$$