Evaluation Games for Shapiro's Logic of Vagueness in Context

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1 Introduction

Stewart Shapiro (Shapiro, 2006) presents a model for reasoning with vague propositions with a special focus on Sorites situations (Hyde, 2008). He maintains that the extensions and anti-extensions of vague predicates such as *bald* and *red* strongly depend on the conversational context. At the beginning of a conversation this context is empty; the extensions and anti-extensions of vague predicates are undefined for many objects, the so-called *borderline cases.* During a conversation these notions are sharpened, such that borderline cases, which have been undecided so far, get assigned to the (anti-)extension of the vague predicates in question.(It is the counterpart to the notion of *supertruth* in supervaluationist theories) Shapiro introduces logical connectives operating on formulas containing such vague predicates. Additionally to the classical connectives, he introduces new ones operating globally on trees of possible contexts.

This contribution introduces a Hintikka-style game for evaluating formulas according to Shapiro's model of vagueness. This is motivated by the following two observations:

- Shapiro's main setting, a so-called forced march version of the Sorites paradox (Hyde, 2008), already includes dialogue situations and conversational records. A dialogue game to evaluate composite propositions is just a natural consolidation of this concept.

- The game provides an explicit mechanism for the evaluation of formulas. In particular Shapiro's falsehood and indefiniteness conditions for global connectives and quantifiers are rather indirect. The dialogue rules provide a much more direct and mechanical characterization of truth in a model. As we will see, the defined connectives can be expressed in terms of a finite two-player zero-sum game with perfect information.

*This work is supported by Eurocores-ESF/FWF grant 1143-G15 (LogICCC-LoMoReVI)

2 Shapiro's approach to vagueness

Below I sketch the main points of Shapiro's account of vagueness as presented in (Shapiro, 2006) and (Shapiro, 2008). This sketch is in no way complete; topics irrelevant for the dialogue game are left out. For example, Shapiro's treatments of higher order vagueness and vague objects within his framework are not subject of the presented game, and thus are omitted here.

Central notions of Shapiro's work are *judgment dependence*, *open texture*, and the *principle of tolerance*. *Judgment dependence* means that the extensions and anti-extensions for the borderline cases of vague predicates are solely determined by the decisions of competent speakers. Vagueness in Shapiro's framework is characterized by judgment dependence. More generally, Shapiro holds that the extension and anti-extension of a vague predicate uttered in a conversation depend on the conversational context. Decisions made by the conversationalists are put on the *conversational record* together with (explicit or implicit) assumptions. This includes, for example, assumptions, statements made by them so far, and (logical) consequences thereof. Moreover it is possible that statements can, explicitly or implicitly, be withdrawn from the conversational record, which plays a crucial role when in Sorites situations.

Open texture means that for a vague predicate P there exists an object a such that a competent speaker can decide that $P(a)$ holds or that $P(a)$ does not hold without her competency being compromised. Note that the notion of 'competent speaker (of the English language)' is also vague; this is where the model can be extended to higher order vagueness.

The *principle of tolerance* is closely related to open texture. Its precise formulation as used in (Shapiro, 2008) is:

> Suppose that two objects a, b in the field of P differ only marginally in the relevant respect (on which P is tolerant). Then if one competently judges a to have P, then she cannot competently judge b in any other manner.

The main settings described by Shapiro are so-called forced march Sorites situations: Imagine 2000 men lined up where man #1 has full hair and man #2000 has no hair at all. The men are ordered by their amount of hair. A group of conversationalists is repeatedly asked if they judge man #i as bald, starting with man #1, continuing until man #2000 is reached. At each step we require them to return a communal verdict. At the beginning, they will unequivocally vote for 'not bald', but at some point they will begin to discuss and finally switch to 'bald'. Shapiro holds, that at this point not only information is added to the conversational record, but also the last few judgments are implicitly retracted.

2.1 Shapiro's model theory

In order to reflect these notions in the model theory, Shapiro uses a Kripke-like tree structure, called *frame*. Each frame, denoted $\langle W, M\rangle$, consists of a set of worlds W with one designated world $M \in W$ called the *base* of the frame. A world is a partial valuation of atoms assigning either *true*, *false*, or *indefinite* to all predicates in question[1]; all worlds in a frame are over the same domain. The world N' is called a sharpening of the world N, denoted as $N' \succeq N$, if and only if each atom which is *true* or *false* at N is also *true* or *false*, respectively, at N'. At the base M propositions are fixed which are determined outside the current conversation. This includes (non-linguistic) facts, external contextual factors, relevant thoughts and practices, etc. Thus it is required that for all $N \in W$, the world N is a sharpening of M. As $\succeq$ is a partial order, a frame can be considered as a tree of precisifications with root M. Note that, in contrast to supervaluationist approaches (Fine, 1975), the completability requirement is not enforced. This means that we do not require that at the leaves of the tree structure a vague predicate P is decided for all objects, i.e., we may leave P undecided for some objects. Shapiro argues that in Sorites situations complete sharpenings (where P is decided for all instances) violate the principle of tolerance (or the externally determined facts that man #1 is not bald and man #2000 is bald), thus they are artifacts of the model theory.

In a Sorites situation initially only the externally determined facts are available on the conversational record. Making (competent) judgments corresponds to moving alongside a branch, away from the root M and thus precifying the asserted statements. In the beginning of a forced situation the conversationalists will repeatedly vote for 'not bald' until, at some point they will switch to 'bald'. With the principle of tolerance in force they have to withdraw some statements from the conversational record; this amounts to jumping to another branch in the frame. It is possible to formalize from which worlds to which worlds such jumps are allowed and where not.

Shapiro argues that *determinate truth* in a frame is best characterized by the notion of *forcing*. A formula ϕ is forced at a sharpening N, if for each sharpening N' of N there is a further sharpening N'' of N' such that ϕ holds at N''. Intuitively, ϕ being forced at N means that ϕ will eventually get *true*: a formula ϕ is *determinately true* at if ϕ is forced at the base of F. (*Determinate truth* is the counterpart to the notion of *supertruth* in supervaluationist theories.) Moreover, the notions of validity and, more generally, consequence are defined in terms of forcing: $\Gamma \models \phi$ if and only if ϕ is forced at every sharpening in every frame in which all formulas of $\Gamma = \{\psi_1, \psi_2, \ldots\}$ are forced.

Of course not all possible frames are adequate for a given (Sorites) situation. For example, we can exclude frames which contain partial interpretations where man $\#i$ is declared to be bald, but another man $\#j$ with $j > i$, who has more hair, is judged not to be bald. Such constraints on adequate frames are called *penumbral connections*. They do not have to check each sharpening separately; it is also possible to require that some condition holds not locally at a sharpening but globally for the frame, e.g. by requiring that some proposition is forced at

[1]Shapiro defends the notion that there are conceptually only two truth values, *true* and *false*; *indefinite* is to be interpreted as the absence of a classical truth value.

the base. Note that tolerance can also be formulated as a penumbral connection.

2.2 Defining connectives and quantifiers

2.2.1 Local operators

Shapiro first defines local logical connectives for negation '$\neg$', conjunction '$\wedge$', disjunction '$\vee$', and implication '$\rightarrow$'. These all adhere to the standard Kleene truth tables as given by Figure 1, where 0 denotes *false*, 1 denotes *true*, and u denotes *indefinite*. The quantifiers '$\exists$' and '$\forall$' are defined as expected.

$\wedge$	0	u	1
0	0	0	0
u	0	u	u
1	0	u	1

$\vee$	0	u	1
0	0	u	1
u	u	u	1
1	1	1	1

$\rightarrow$	0	u	1
0	1	u	0
u	1	u	u
1	1	1	1

$\neg$	
0	1
u	u
1	0

Figure 1: Kleene truth tables for local connectives

Note that all these connectives obey to the monotonicity principle on sharpenings which we have encountered above for atomic propositions. That means, if a (compound) formula ϕ is *true* (or *false*) at a sharpening N then ϕ is *true* (or *false*, respectively) also at all sharpenings N' of N. Because of this, forcing is not present in the object language, but only at the meta-level: it is easy to construct a frame where it is *false* that a formula ϕ is forced at a sharpening N, but where it is *true* that ϕ is forced at a sharpening N' of N.

2.2.2 Global operators

Additionally to the standard logical connectives and quantifiers, Shapiro introduces new non-local ones operating on whole subtrees instead of a single sharpening. One of them is the new non-local implication '$\Rightarrow$' with the following semantics:

> $\phi \Rightarrow \psi$ is *true* at a sharpening N if at each sharpening N' of N if ϕ is *true*, then also ψ is *true*.

This connective is used extensively by Shapiro to define penumbral connections as seen by the following example: assume a Sorites situation as explained above. Then we can stipulate as a penumbral connection, that for all i and j the formulas $(B(m_j) \wedge S(m_i, m_j)) \Rightarrow B(m_i)$ and $(\neg B(m_i) \wedge S(m_i, m_j)) \Rightarrow \neg B(m_j)$, with $S(x, y)$ iff x has more hair than y, hold at the base (and thus at all sharpenings). This ensures that at a sharpening where man m_i is judged 'bald', all men with less hair are judged 'bald' as well. Vice versa, at a sharpening where man m_j is judged 'not bald', all men with more hair as m_j are judged 'not bald' as well.

In order to preserve monotonicity we also give a falsehood condition for each new connective. Just stating that $\phi \Rightarrow \psi$ is *false* if it is not *true* would violate monotonicity; this can be seen in the following example frame:

$$\begin{tikzcd} & M \arrow[dl, no head] \arrow[dr, no head] & \\ N : P(a), \neg P(b) & & N' : P(a), P(b) \end{tikzcd}$$

At the base M the formula $P(a) \Rightarrow P(b)$ is not *true* because the condition is violated at N, but it is *true* at N'. Thus, the only way not to violate the monotonicity principle is to leave $P(a) \Rightarrow P(b)$ undecided at M. Therefore, Shapiro makes use of the so-called *stable failure*:

> The formula $P(a) \Rightarrow P(b)$ is *false* at the sharpening N if and only if there is no sharpening N' of N such that $P(a) \Rightarrow P(b)$ is *true* at N.

This ensures that also falsehood is preserved in the tree structure, thus if a formula $\phi \Rightarrow \Psi$ is *false* at a sharpening N, it is also *false* at each sharpening of N.

Another new connective is the intuitionistic-style negation '$-$'. The proposition $-P(a)$ is *true* at the sharpening N if there is no sharpening N' of N where $P(a)$ is *true*.[2]

Shapiro observes that, as in supervaluationist theories, a formula $\exists x.\phi(x)$ can be forced at a sharpening N without $\phi(a)$ being forced at N for any particular witness a. In order to make the existence of such witnesses expressible in the object language, he introduces a new global existential quantifier E with the following semantics:

> The formula $Ex.\phi(x)$ is *true* at N if and only if there exists a such that $\phi(a)$ is forced at N.

Similarly it is possible to define the new global universal quantifier A:

> The formula $Ax.\phi(x)$ is *true* at N if and only if for all x it holds that $\phi(x)$ is forced at N.

As seen above, it is also necessary to give falsehood conditions for all the new connectives in order to preserve monotonicity. Therefore the falsehood conditions for '$-$', 'E', and 'A' are obtained by their stable failure analogously to '$\Rightarrow$'.

We obtain the following lemma:

Lemma 1. *Let ϕ be a formula of the form $-\psi$ or $Ax.\psi(x)$: ϕ is* indefinite *at a sharpening N in F if and only if there exist sharpenings N' and N'' of N such that ϕ is* true *at N' and* false *at N''.*

Proof. If ϕ is *true* at N' and *false* at N'' then, due to monotonicity, it can neither be *false* nor *true* at N. Therefore it must be *indefinite* at N. On the other hand, consider, e.g., $\phi = Ax.\psi(x)$, and assume that ϕ is *indefinite* at N. According to

[2]Notice that a formula ϕ is forced at a sharpening N if and only if $\neg-\phi$ is *true* at N. However, the property of being forced cannot be introduced as an unary connective at the object level. This still violates monotonicity as is not the case that $\neg - \phi$ is *false* if and only if ϕ is *not* forced at N.

the definition of stable failure there exists at least one sharpening of N where ϕ is *true* (otherwise ϕ would be *false* at N). Assume that there exists no sharpening where ϕ is *false*. Then ϕ is either *true* or *indefinite* at any given sharpening N' of N. But, as just argued, if ϕ is *indefinite* at N' there exists a further sharpening N'' where ϕ is *true*. This means that ϕ is forced at N. Shapiro shows that a formula $Ax.\psi(x)$ is forced at a sharpening exactly if it is *true* at that sharpening, and consequently we conclude that ϕ is *true* at N leading to the desired contradiction. Thus there exists a sharpening N'' of N such that ϕ is *false* at N''. For the global negation '$-$' we can reason analogously. □

Notice that Lemma 1 does not hold for formulas of the form $\phi \Rightarrow \psi$ or $Ex.\phi(x)$. Such formulas can be *indefinite* at a sharpening N and *true* at all further sharpenings of N.

3 A Hintikka-style evaluation game

3.1 Motivation and Overview

As we have seen above, Shapiro's logic directly refers to conversational situations, namely a forced march version of the Sorites paradox, but involves only atomic predicates. A dialogue game to decide the semantic status of a compound formula without leaving this dialogue setting therefore just seems a natural consolidation of this concept. Moreover, the game provides an explicit mechanism for the evaluation of formulas. All moves consist of either choosing between different alternatives how the game should proceed, selecting a representative of the domain, or selecting a sharpening of the current one. As we will see, especially falsehood and indefiniteness conditions for global operators are specified much more directly this way.

There are two players, the proponent **P** of a formula and the opponent **O**. Initially **P** asserts that a formula ϕ is either *true*, *false*, or *indefinite* at an initial sharpening N in a given frame F. This is denoted as **P** asserting $\vdash^+_N \phi$, $\vdash^-_N \phi$, or $\vdash^\sim_N \phi$, respectively. During the game ϕ is decomposed step by step into less complex formulas according to the game rules until, in the end, **P** asserts the semantic status of only an atomic formula $P(a)$. Assume that the game ends at the sharpening N'. Then, if at this point **P** asserts $\vdash^+_{N'} P(a)$ and if a is in the extension of P at N' then **P** is declared the winner of the game, otherwise **P** loses and **O** wins. Analogously, **P** wins if he asserts $\vdash^-_{N'} P(a)$ and a is in the anti-extension of P at N', or if he asserts $\vdash^\sim_{N'} P(a)$ and a is neither in the extension nor in the anti-extension of P at N'.

Both players are assumed to agree on the frame and the initial sharpening in the frame in which to evaluate the formula. The game is a finite two-player zero-sum game with perfect information. Thus, by Zermelo's Theorem (Zermelo, 1912) we conclude that the game is determined.

3.2 Dialogue Rules

As described above, at each point in the game exactly one formula is asserted by the proponent **P** to be *true*, *false* or *indefinite* at a certain sharpening. The dialogue rules then specify how this formula is to be further reduced and which player has to make which choices based on the outmost connective or quantifier. For instance, the dialogue rule for conjunction is given in Figure 2. It can be read as follows: if **P** asserts $\vdash^{+}_{N} \phi \wedge \psi$ than **O** can choose whether **P** has to further assert $\vdash^{+}_{N} \phi$ or $\vdash^{+}_{N} \psi$ at the same sharpening. For $\vdash^{-}_{N} \phi \wedge \psi$, on the other hand, **P** himself may choose. If **P** asserts $\vdash^{\sim}_{N} \phi \wedge \psi$ he first chooses whether to assert that both ϕ and ψ are *indefinite* at N, or that only ϕ is *indefinite* and ψ is *true*, or vice versa. In response **O** chooses one of the two corresponding assertions.

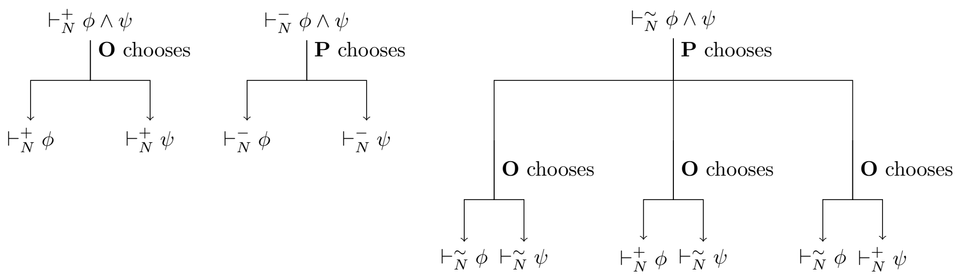

Figure 2: Dialogue rule for conjunction

As one can see, the rules can be obtained directly from the Kleene truth tables in Figure 1. Informally, $\phi \wedge \psi$ is *indefinite* at a sharpening N, if either ϕ is *true* and ψ is *indefinite*, or vice versa, or both are *indefinite* at N. Rules for the other local connectives can be constructed analogously.

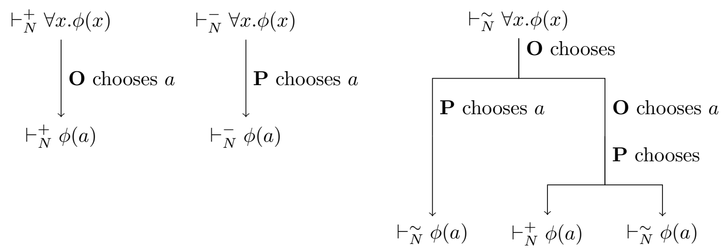

Figure 3: Dialogue rule for the local universal quantifier

For local quantifiers we proceed in the same way: Figure 3 shows the dialogue rules for the universal quantifier. For $\vdash^{+}_{N} \forall x.\phi(x)$ **O** has to choose one domain element a and the game proceeds, whereas for $\vdash^{-}_{N} \forall x.\phi(x)$ the choice is **P**'s. In the third case, $\vdash^{-}_{N} \forall x.\phi(x)$, first **O** chooses whether he wants **P** to select one element a and assert $\vdash^{\sim}_{N} a$ or if he wants to select a by himself, but let **P** choose whether to assert $\vdash_N^+ \phi(a)$ or $\vdash_N^\sim \phi(a)$. This rule can be informally motivated by observing that $\forall x.\phi(x)$ is *indefinite* if and only if for all instances a of x it holds that $\phi(a)$ is either *true* or *indefinite* and, moreover, there is at least one instance a' such that $\phi(a')$ is *indefinite*. Again, a dialogue rule for the existential quantifier can be obtained analogously.

The dialogue rules above are all *local* in the sense that the current sharpening is not changed. The rules for the other, global, connectives involve choosing sharpenings of the current one by **P** or **O**.

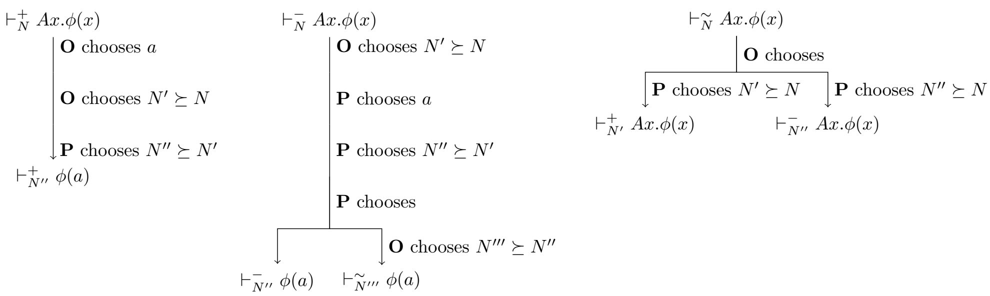

Figure 4: Dialogue rule for the global universal quantifier

The rule for the global universal quantifier 'A' is given in Figure 4. In the rule for $\vdash_N^+ Ax.\phi(x)$ first **O** selects a domain element a and then chooses a sharpening N' of N. Then **P** chooses yet a further sharpening N'' of N' and asserts $\vdash_{N''}^+ \phi(a)$. According to Shapiro's definition, in order for $Ax.\phi(x)$ to be *true* at N, after **O** has chosen a, the formula $Ax.\phi(x)$ must be forced at N. By letting players alternatively select further sharpenings we obtain a literal translation of Shapiro's forcing condition to dialogue rules. The rule $\vdash_N^- Ax.\phi(x)$ involves Shapiro's definition of the stable failure of 'A'. According to this definition **P** has to show that there is no sharpening of N where $Ax.\phi(x)$ is *true*. Thus, after **O** chooses $N' \succeq N$, player **P** selects a domain element a and then shows that $\phi(a)$ is not forced at N'. This is the case, if he can find a sharpening $N'' \succeq N'$ where either $\phi(a)$ is *false*, or $\phi(a)$ is *indefinite* and remains so in all further sharpenings. The rule $\vdash_N^\sim Ax.\phi(x)$ is directly obtained from Lemma 1 stating that $Ax.\phi(x)$ is *indefinite* at N if and only if there exists sharpenings N' and N'' such that $Ax.\phi(x)$ is *true* at N' and *false* at N''.

The dialogue rules in Figure 5 for the global negation '$-$' follow the same scheme: Rules for $\vdash_N^+ -\phi$ and $\vdash_N^- -\phi$ are obtained directly from Shapiro's truth and falsehood conditions; the rule for $\vdash_N^\sim -\phi$ is again obtained from Lemma 1. Since, as noted above, forcing can be expressed in terms of this operator, we can read the dialogue rule for $\vdash_N^- -\phi$ directly as a rule for forcing: if the proponent **P** wants to state that a formula ϕ is forced at a sharpening N, he does so by asserting $\vdash_N^- -\phi$.

Figure 6 shows the dialogue rules for the global existential quantifier 'E'. The difference between the rules for $\vdash_N^+ Ex.\phi(x)$ and $\vdash_N^- Ex.\phi(x)$ and their counter-

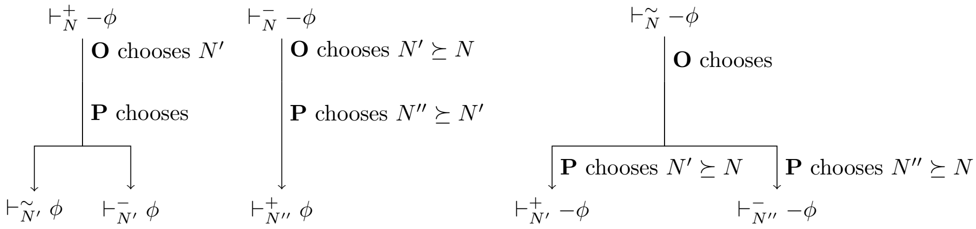

Figure 5: Dialogue rule for the global negation

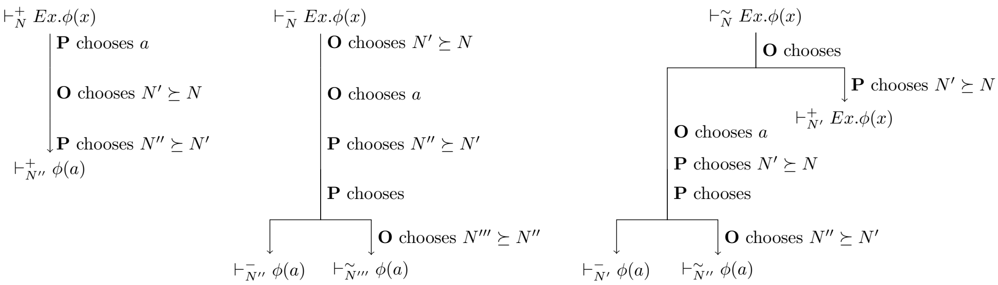

Figure 6: Dialogue rule for the global existential quantifier

parts for the global universal quantifier is only that here the proponent **P** has to pick one element of the domain instead of the opponent **O**. However, as Lemma 1 does not hold for the global existential quantifier 'E', we have to give another rule for $\vdash^{\sim}_{N} Ex.\phi(x)$: if **P** asserts that $Ex.\phi(x)$ is *indefinite* at N, he has to be able to show that it is not *true* and to show that it is not *false* at N. The former case amounts to the left branch: for any given element a, player **P** asserts that $\phi(a)$ is not forced by providing a sharpening N' of N where $\phi(a)$ is either *false*, or *indefinite* and remains *indefinite* at each further sharpening. In the latter case **P** shows that the $Ex.\phi(x)$ is not *false* at N by providing a sharpening of N where the formula is *true*.

The connective for the global implication '$\Rightarrow$' is used by Shapiro solely to formulate penumbral connections, that are constraints on possible frames. As the game is an evaluation game which takes place in a given frame, such contraints are not directly subject to the game. However, one can still specify dialogue rules for the '$\Rightarrow$' connective in the same way as for the others. Due to space restrictions the exact formulation of these rules is omitted here.

3.3 Adequacy of the game

We claim that the dialogue rules are adequate for Shapiro's logic in the following sense:

Theorem 1. *Given a frame F and a sharpening N in F, a formula ϕ is* true *at N in F if and only if the player* **P** *has a winning strategy for the game where* *he initially asserts* $\vdash^{+}_{N} \phi$*.* ϕ *is* false *at* N *if and only if* **P** *has a winning strategy for the game where he initially asserts* $\vdash^{-}_{N} \phi$ *and* indefinite *if and only if* **P** *has a winning strategy for the game where he initially asserts* $\vdash^{\sim}_{N} \phi$*.*

Proof. We proof by induction on the complexity of ϕ that the game rules are adequate for Shapiro's logic. If ϕ is atomic, this is obvious. Otherwise, applying one of the dialogue rules reduces ϕ to a less complex formula except for the rules for $\vdash^{\sim}_{N} Ax.\phi(x)$, $\vdash^{\sim}_{N} Ex.\phi(x)$, and $\vdash^{\sim}_{N} -\phi$. However, the latter cases reduce to the respective rules for $\vdash^{+}_{N} Ax.\phi(x)$, $\vdash^{-}_{N} Ax.\phi(x)$, $\vdash^{+}_{N} Ex.\phi(x)$, $\vdash^{+}_{N} -\phi$, and $\vdash^{-}_{N} -\phi$ and therefore are covered by the induction as well. Due to space restrictions this checking of rules is only carried out here for some exemplary ones.

Assume, for example, that **P** asserts $\vdash^{+}_{N} \forall x.\psi(x)$. If $\forall x.\psi(x)$ is *true* at N, then no matter which domain element a player **O** chooses, $\psi(a)$ is *true* at N. By the induction hypothesis player **P** asserting $\vdash^{+}_{N} \psi(a)$ wins the game. On the other hand, if $\vdash^{+}_{N} \phi$ is not *true* at N, then there exists an element b such that $\psi(b)$ is not *true* at N. If **O** selects b, player **P** has to assert $\psi(b)$, and, again by applying the induction hypothesis we see that **P** loses the game.

As a slightly more complex example assume that **P** asserts $\vdash^{\sim}_{N} Ex.\psi(x)$. Player **P** wins if and only he can show that $Ex.\psi(x)$ is neither *true* nor *false* at N. Assume, **O** chooses the left branch. Then **P** wins if for each domain element a he can find a sharpening N' such that either $\psi(a)$ is *false* at N' or there for all sharpenings of N' it holds that $\psi(a)$ is *indefinite* at N'. But this exactly means that there is a sharpening of N where there is no further sharpening such that $\psi(a)$ is *true*; in short, $\psi(a)$ is not forced at N. Since a was chosen by **O**, player **P** wins if there is no a which is forced at N. In other words, $Ex.\psi(x)$ is not *true* at N. On the other hand, if **O** chooses the right branch, **P** wins if there exists a sharpening of N where $Ex.\psi(x)$ is *true*, thus, according to the definition of stable failure, **P** wins when $Ex.\psi(x)$ is not *false*. Since **O** chooses between the left and the right branch, it is the case that **P** wins exactly if $Ex.\psi(x)$ is neither *true* nor *false* at N. □

As noted above, forcing can be expressed in terms of the global negation '$-$'. The following corollary follows immediately from Theorem 1.

Corollary 1. *Given a frame* F *and a sharpening* N *in* F*, a formula* ϕ *is forced at* N *in* F *if and only if the player* **P** *has a winning strategy for the game starting in* $\vdash^{-}_{N} -\phi$*.*

4 Conclusion and future work

In this contribution we have presented a dialogue game for the evaluation of formulas in Shapiro's logic in a given frame. At each point in the game the initial proponent of the formula ϕ in question asserts that a subformula of ϕ is *true*, *false*, or *indefinite* at a given sharpening. Compound formulas are being subsequently reduced to less complex formulas until, in the end, an atomic formula can easily be evaluated by checking the (anti-)extensions of the vague predicates in question at the final sharpening reached. The dialogue rules consist of simple operations like choosing a domain element, choosing a sharpening, or choosing between different succeeding assertions, which yields a rather mechanic characterisation for the connectives of Shapiro's logic.

In future work we plan to investigate other types of games adequate for this logic. In particular evaluation games in the spirit of dialogue games as defined by Paul Lorenzen (Lorenzen, 1960) and, more specifically, by Robin Giles (Giles, 1974) for Łukasiewicz logic seem promising. Such games strictly separate the stepwise decomposition of compound formulas into their atomic parts from the evaluation of atomic game states. In contrast to the game presented here, both players may assert a multiset of formulas at each point in the game. The characterisation of indefiniteness is an interesting property of this game: we can observe that *truth* of a formula ϕ coincides with the existence of a winning strategy for the player asserting ϕ in the beginning, while *falsehood* coincides with the existence of a winning strategy for the other player. For indefinite formulas neither player has a winning strategy, which fits Shapiro's point of view that indefiniteness in his logic is not just a third truth value, but merely signifies the lack of a classical one.

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