Advanced Proof Viewing in PROOFTOOL
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1. Introduction

2. Requirements for a complimentary viewer

3. Sunburst View

4. Conclusion
Problem Description

Context

- Full sequent calculus formalizations of mathematical proofs are large
- Often defined schematically i.e. contains structural regularities
- Proof analysis applies several transformations:
  e.g.: skolemization, cut-elimination, Herbrand formula extraction
Problem Description

Question occurring during analysis

- Are there easily removable cuts?
- Which strong quantifier rule is related to which skolem symbol?
- How do skolem symbols propagate through the proof?
- How do we identify individual inferences from the specification after several proof transformations?
Problem Description

Approach

- Proof visualization helps to answer these questions
- Original Gentzen notation alone is insufficient
- Use a second view (Sunburst Tree) to compensate the drawbacks
- Implementation for GUI Prooftool
### Pros of the Gentzen notation
- Detailed low-level view
- Directly reflects the calculus
- Widespread & easy to understand

### Cons of the Gentzen notation
- Focus on single inferences
- Large contexts obscure the inference information
- Layout grows mainly horizontally (empirically width:height $\approx 10:1$)
- Locating / tracing inferences is hard
Small example

Commutativity

- Axioms: $0 + 1 = 1$, $1 + 0 = 1$, $x + (y + z) = (x + y) + z$
- We prove: $x + y = y + x$
- 3 instances of Induction schema in antecedent of end-sequent
  IND: $S(0) \land \forall x (S(x) \rightarrow S(x + 1)) \rightarrow \forall x S(x)$
- ca. 70 inferences, mostly equational reasoning, 1 cut
Drawbacks of the Gentzen notation (1)

Wide proofs leading into binary rules prevent display of auxiliary formulas and primary formulas on the same screen.
Drawbacks of the Gentzen notation (2)

Context causes wide horizontal gaps in binary rules

\[
\begin{align*}
(x00 + 1) & = (1 + x00) \vdash (x00 + 1) = (1 + x00) \\
(\forall x)((x + 1) = (1 + x)) & \vdash (x00 + 1) = (1 + x00) \\
(\forall x)((x + 1) = (1 + x)) & \vdash (\forall x)((x + 1) = (1 + x)) \\
\vdash ((x3 + 1) + y2) & = (x3 + (1 + y2)) \\
(x3 + (1 + y2)) & = ((1 + y2) + x3) \vdash (x3 + 1) + y2) = (x3 + (1 + y2))
\end{align*}
\]
Drawbacks of the Gentzen notation (3)

"Pseudo-linear" reasoning (trivial chains of \( \rightarrow: l \) or \( \vDash: l \)/\( \vdash: r \) ) grows to the side

\[
\begin{align*}
\vdash (0 + 1) = 1 & \quad \vdash 1 = 1 \\
\vdash (1 + 0) = (1 + 0) & \\
\vdash (x + 0 + 1) = ((x + 0) + 1) & \quad \vdash ((x + 1) + 0) = (x + (1 + 0)) \\
\vdash (x + 0) = (x + 0) & \\
\vdash (0 + x) = (x + 0) & \quad \vdash ((0 + x) + 1) = ((x + 1) + 0)
\end{align*}
\]
Criteria

- Displaying of large proofs
- Distinguishing between different kinds of rules
- Easy navigation
- Traceable formula ancestors
- Focus on different aspects of the proof
- Draw information from the structural shape of the proof
Alternatives: GraphViz
Alternatives: Hyperbolic Tree
Alternatives: Rectangular Treemap
Alternatives: Circular Treemap
Alternatives: Icicle Tree
Alternatives: Sunburst
Combination with Inference view

Inference View

- Necessary because sequents are invisible in Sunburst View
- Shows primary and auxiliary formulas
- Room for additional information (quantifier instantiations, etc.)
Sunburst within GAPT

Inference:
\[ d : l \]

Auxiliary:
\[ (((\forall y)((0 + y = (y + 0)) \land (\forall x)((\forall y)((x + y) = x + y))) \rightarrow d = l) \]

Primary:

Substitution:
Conclusion

- The Gentzen layout needs supporting viewers to handle large proofs better.
- Sunburst Trees + Single inference information can provide complementary information.
Open Questions

- Criteria definition is empirical – what other requirements are there in different contexts?
- Sunburst View works well with trees – what about DAGs? Degenerative example: refutation of 
  \[ \vdash P(0); P(f^n(y)) \vdash; P(z) \vdash P(f(z)) \]
- How do we improve navigation in face of information loss by transformations?
Thanks for your attention!