# Disambiguating permissions: A contribution from Mīmāṃsā

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**Abstract**

The notion of permission has received less attention than obligation from the deontic logic community, that has often taken for granted the interdefinability of deontic operators (obligations, prohibitions and permissions). Yet, permission has proven to be a complex topic with various nuances that require careful treatment, and can lead to unwanted consequences if the interdefinability is kept. In contrast, the Sanskrit philosophical school of Mīmāṃsā refuted this interdefinability and instead established independent definitions for deontic concepts. This article focuses on the exploration of permission within Mīmāṃsā and its formalization through Hilbert axioms and semantics. We also compare the Mīmāṃsā approach to contemporary deontic logic discussions, and show that the central paradoxes of permission do not arise in the Mīmāṃsā paradigm.

*Keywords:* Permission; Interdefinability of deontic operators; Mīmāṃsā; Sanskrit philosophy; Deontic paradoxes; Free choice paradox; Neighbourhood semantics.

## 1 Introduction

Permission is of crucial importance in several settings, from law to ethics to artificial intelligence. Despite its significance, it has been the subject of fewer investigations in the deontic logic literature compared to obligation.

The concept of permission is inherently ambiguous and can be expressed in various manners such as "you are allowed to", "it is open for you to", and "you have the right to". Since the introduction of deontic logic by von Wright, permission has been often viewed simply as the dual of obligation [40], similar to how possibility serves as the dual of necessity in modal logic. Due to the unintuitive consequences (aka deontic paradoxes) mainly arising from this assumption, different varieties of permissions have been considered in the deontic logic literature; these include weak and strong permissions (e.g. [41,1,5,43,6]), bilateral and unilateral permissions (e.g. [12,26,27,23]), positive and negative permissions (e.g. [32,34]), and explicit, tacit or implicit permissions (e.g. [26]).

This paper contributes to the debate, by revealing and formalizing the concept of permission in Mīmāṃsā, which is one of the main Sanskrit philosophical schools and is a largely unexplored source for deontic investigations. Mainly active between the last centuries BCE and the 20th c. CE, Mīmāṃsā centred around the analysis of the prescriptive portions of the Vedas – the sacred texts of (what is now called) Hinduism. Mīmāṃsā authors interpreted the Vedas independently of the will of any speaker, as a consistent and self-sufficient corpus of laws. Thus, Mīmāṃsā authors have thoroughly discussed and analyzed normative statements in order to explain "what has to be done" in the presence of seemingly conflicting obligations. Since the Vedas are assumed to be not contradictory, Mīmāṃsā authors invested all their efforts in creating a consistent deontic [1] system. Key to their enterprise was the formulation of reasoning principles called *nyāya*s (see e.g. [19]), that lend themselves to a formalization through logic. Some *nyāya*s can be transformed into properties (Hilbert axioms) for the corresponding deontic operator in Mīmāṃsā, others (e.g. the specificity principle discussed in Kumārila's *Tantravārttika*) are instead metarules to resolve seeming contradictions in the Vedas.

The deontic theory of Mīmāṃsā has been progressively formalized through a series of works [14,29,8], each introducing new deontic operators and properties found in the original texts. The initial paper [14] presented the base logic, which considered only obligation, whose properties were extracted by analyzing around 40 *nyāya*s. Subsequently, prohibition was added in [29], and [8] included a weaker form of obligation, corresponding to elective duties, which are sacrifices to be performed only if one desires their specific outcome.

Our work has involved an interdisciplinary team effort that began with the discovery of the relevant *nyāya*s in Sanskrit texts, followed by their translation into English, their interpretation, and their formalization as Hilbert axioms. It is important to remark that our logics are solely based on principles extracted by Mīmāṃsā texts. Our aim is indeed to faithfully formalize the deontic theories of the Mīmāṃsā authors and use them to provide a better understanding of their debates, as well as new insights for contemporary deontic logic.

A distinctive feature of Mīmāṃsā deontics is the non-interdefinability of obligation and prohibition. As we have recently discovered, the independence of the deontic concepts extends also to permission, which is the focus of the present paper. Here we extend the logic discussed in [8] with the axioms for permission, and with newly formalised *nyāya*s, one of which corresponds to a version of the 'ought implies can' principle, see e.g. [9]. We propose a neighbourhood semantics for the resulting logic, which we call $LM_P$ (Mīmāṃsā Logic with permission). To analyze the behaviour of $LM_P$ we confront it with the best known deontic paradoxes concerning permission: free choice inference [42],

[1] Different Mīmāṃsā authors interpret commands differently (see [8]), but most of them looked at the Veda as a text having only deontic, i.e., normative authority.

Ross' paradox [37] and the paradox of the privacy act [22]. These paradoxes do not arise in $LM_P$; its well-behaved nature can be attributed to the millennia-old philosophical and juridical foundation upon which it is built.

The paper is organized as follows: Section 2 summarizes our previous findings on Mīmāṃsā deontics. Section 3 introduces the notion of permission in Mīmāṃsā and compares it to the literature of contemporary deontic logic. Mīmāṃsā permission is formalized in Section 4 by extending the logic in [8] with suitable Hilbert axioms and their semantics. In Section 5, the resulting logic is examined in light of the main deontic paradoxes related to permission, and it is demonstrated that it effectively addresses them.

*Sanskrit sources:* Throughout this paper, we refer to Jaimini's *Mīmāṃsā Sūtra* (or *Pūrva Mīmāṃsā Sūtra*, henceforth PMS, approximately 250 BCE) and Śabara's *Bhāṣya* 'commentary' thereon (henceforth ŚBh, approx. 5th c. CE), whose authority has been recognized by all Mīmāṃsā authors. We refer also to the following Mīmāṃsā texts: Kumārila's *Tantravārttika* (7th c., a key subcommentary on the PMS and ŚBh), and Rāmānujācārya's *Tantrarahasya* (14th c.?), as well as to a key text of Sanskrit jurisprudence (called Dharmaśāstra), Vijñāneśvara's *Mītākṣarā* (early 12th c.), a commentary on Yajñavalkya's code of norms.

## 2 Preliminaries on Mīmāṃsā Deontics

The Mīmāṃsā school focused on the rational interpretation and systematization of the prescriptive portions of the Vedas. These include *commands* of various kinds, such as prescriptions concerning the performance of sacrifices, and prohibitions applying either to the context of a sacrifice or to the entire life of a person (e.g. "One should not harm any living being"). Sometimes the commands seem to be contradictory, like in the case of the Śyena sacrifice, that should be performed if one wants to kill their enemy. [2] Mīmāṃsā thinkers introduced and applied metarules (called *nyāya*s) in order to rigorously analyze the Vedic commands and solve seeming contradictions among them.

The *nyāya*s are not listed explicitly by Mīmāṃsā authors, and have to be carefully distilled from their concrete applications within the texts. An example of a *nyāya* is "if a certain action is obligatory but it implies other activities, then these other activities are also obligatory" (Rāmānujācārya's *Tantrarahasya*).

Mīmāṃsā authors distinguish between obligations (*vidhi*) and prohibitions (*niṣedha*). [3] The former are determined by the fact of leading one to a desired result, if fulfilled, whereas the latter by the risk of sanction, if not fulfilled. This implies that negative obligations are different from prohibitions, and these two concepts are not mutually definable. "It is forbidden to lie" means that one will be liable to a sanction if one lies. "It is obligatory not to lie" means that one will receive a reward if one avoids any lie. Commands are always uttered with regard

[2] See [8] for the solutions to the Śyena controversy provided by the main Mīmāṃsā authors.

[3] Obligations and prohibitions in Mīmāṃsā have been discussed in [20], and formalized as suitable logics in [29,8].

to a specific person, called 'eligible' or 'responsible' (*adhikārin*), or to a specific situation in which an *adhikārin* might be in. In terms of deontic logic, this means that commands are always dyadic. For instance, the obligation to recite the Vedas is incumbent only on male members of the highest three classes who have undergone initiation, which can be rendered as $O(reciteVedas/initiated)$.

The use of logic to formalize Mīmāṃsā reasoning is justified by the rigorous theory of inference implemented by the school, that implicitly refers to logical principles and methods [14,19].

A further salient characteristic of Mīmāṃsā deontics is that commands have always one goal, hence they do not have conjunctions or disjunctions within them. A seemingly unitary command like "You should offer clarified butter and pour milk" would be interpreted as two separate commands, namely "You should offer clarified butter" and "You should pour milk".

Last, a metarule prescribes that commands should always convey something new (*apūrva*). A command that seems to prescribe an action one is already inclined to do should therefore be interpreted otherwise. For instance, "One should eat the five five-nailed animals" cannot be interpreted as enjoining the eating of certain animals, because one is naturally inclined to eat the meat of each animal. The command is instead interpreted as a prohibition of eating the meat of any other animal. A connected *nyāya* prescribes that the Vedas are always purposeful and do not enjoin anything without purpose. Altough the scope of these two *nyāya* may overlap, they are different as it is possible to imagine a sentence being purposeful but not novel. As a consequence of these *nyāya*, for instance, prohibitions need to refer to actions one would be naturally inclined to undertake (*rāgaprāpta*) or that have already been enjoined (*śāstraprāpta*). Prohibiting something one would never undertake, e.g. "building an altar in the sky" would be purposeless and hence is not a viable interpretation of a command.

## 3 Permissions and new discoveries in Mīmāṃsā Deontics

One of the most striking features of Mīmāṃsā deontics is the non-interdefinability of deontic concepts, that also applies to the concept of permission. Its main characteristic is that a permission is always an exception to a prohibition or negative obligation. In Mīmāṃsā, saying "it is permitted to do $X$ given $Y$", always entails that $X$ is negatively obligatory or forbidden given a condition $Z$ that is more general than $Y$. This can be illustrated by the following applications of an underlying *nyāya* (i.e. "A permission is always an exception to a pre-existing prohibition or negative obligation"):

(a) The permission to take a second wife can only occur as an exception to a general prohibition or negative obligation not to remarry (ŚBh on PMS 6.8.17–18).
(b) The permission to take up the occupation of a lower class in times of distress depends on the underlying prohibition to take up any occupation other than the ones admitted for one's own class (*Mitākṣarā* on 3.35).

(c) The permission to eat after buying Soma implies the prohibition to eat (or the obligation not to eat) before it (*Tantravārttika* on 1.3.4).

(d) The permission to sell while being a Brāhmaṇa in distress implies the prohibition to sell while being a Brāhmaṇa in normal circumstances (*Mitākṣarā* on 3.35).

Thus, these permissions are interpreted as presupposing an underlying prohibition or negative obligation, and not as stand-alone permissions.

The permission to sell while being a Brāhmaṇa in distress, for instance, implies that a Brāhmaṇa not in distress should not be selling anything. Similarly, the permission to take up the occupation of a lower class in times of distress depends on the underlying prohibition to take up any occupation other than the ones admitted for one's own class (see Vijñāneśvara's *Mitākṣarā* commentary on Yājñavalkya 3.35) and the permission to eat after a certain moment of the sacrifice implies the prohibition to eat before it (*Tantravārttika* on 1.3.4).

Hence permissions only make sense for Mīmāṃsā authors with regard to acts which were previously prohibited or the abstention from which was obligatory. To define the realm of "whatever is not prohibited is permitted", Mīmāṃsā authors introduce the concept of "normatively indifferent actions". These are actions that are possible, but neither prohibited *nor* enjoined (nor permitted in the Mīmāṃsā sense) and that constitute most of our everyday life. Normatively indifferent actions are the ones on which normative texts make an intervention. For instance, offering a ritual substance is not permitted in a Mīmāṃsā sense, because it is enjoined. In the following, we will call whatever is neither prohibited nor permitted nor enjoined "extra-normative". In sum, for Mīmāṃsā there are either normed actions (enjoined, prohibited or permitted) or extra-normative ones.

A last feature of Mīmāṃsā permission is the following: if $X$ is permitted given $Y$, doing $X$ is not on the same level as not doing it, or as doing $X$ while $X$ is an extra-normative action. Rather, permissions allow an option that is less desirable than its counterpart. One of the main consequences of this approach is that performing a permitted $X$ exposes one to the risk of restrictions, insofar as the permitted action is actually an action one should have "better-not" performed. Thus, eating after having bought Soma is permissible, but not eating is the preferred option (for more details, see [18], Section 4).

Related to permission, we have newly (identified and) formalized a characteristic of Mīmāṃsā deontics, that is a version of the 'ought implies can' principle, usually attributed to Immanuel Kant, see [38], and that in Mīmāṃsā's case can be formulated as "each command must be actionable", thus including the claim that also forbidden entails can. This metarule is extracted from the *nyāya*s "Prescriptions can only prescribe actions that can be performed" and "Prohibitions can only prohibit actions that can be performed", whose application is found below:

(e) Commands prescribing complicated sacrifices in order to get *svarga* (that is, heaven, to be understood as happiness) are addressed to men who are able to perform them (see *Tantravārttika* on 1.3.4).

(f) The seeming prohibition "The fire is not to be kindled on the earth, nor in the sky, nor in heaven" cannot be taken as a prohibition, because fire cannot be kindled in the sky nor in heaven (see ŚBh on 1.2.5 and 1.2.18).

The metarule regarding novelty (*apūrva*, see Section 2) also implies that it is impossible to have more than one deontic operator applied to the same action under the same circumstances. Rather, each deontic operator needs to make a novel intervention and is therefore applied to an extra-normative situation, or, in the case of permissions, to a pre-existing negative obligation or prohibition. With regard to permissions, this also means that the same action cannot be at the same time obligatory and permitted given the same circumstances (pace SDL [40]), since the operator for permission would not add anything novel if applied to a situation already normed by the deontic operator for obligation. For instance, if one already knows that male married Brahmins ought to perform a certain ritual at dawn, receiving the information that it is permitted to perform the same ritual at the same time and given the same circumstances would be redundant and purposeless, and no command in the Veda can be purposeless.

### 3.1 Mīmāṃsā Permission vs Permission in Deontic Logic

The interdefinability between obligation and permission is an old problem in Deontic Logic, dating back to the observation by Von Wright in [40] of the similarity with the relation between necessity and possibility. The deontic axiom D included in Standard Deontic Logic SDL therein introduced says that obligation implies permission.

As emphasized in [1], a main problem with this interdefinability is that the resulting system does not allow for gaps. If everything that is not permitted is prohibited and everything that is not prohibited is permitted, then any normative system would regulate all possible states of affairs. This is counterintuitive since not all situations are subject to regulation, as also acknowledged by the Mīmāṃsā school and its recognition of extra-normative actions.

Mīmāṃsā's concept of extra-normativity aligns with the idea of indifference as defined by McNamara in relation to supererogation [33]. In McNamara's definition, an indifferent action is neither obligatory nor forbidden. Moreover, the author links an operator for indifference with one for "moral significance" and thus differentiates between indifference and supererogation. Both indifferent and supererogatory actions are neither obligatory nor forbidden, but supererogatory actions hold moral significance.

In [41], von Wright treats the notion of permission more carefully than in his previous writings and introduces a distinction between weak permission and strong permission. Weak permission is permission as the absence of prohibition, whereas strong permission is a modality by itself. The latter is defined as follows: (i) an act "will be said to be permitted in the strong sense if it is not forbidden but subject to norm", and (ii) "an act is permitted in the strong sense if the authority has considered its normative status and decided to permit it". A formalization of strong permission is contained, e.g., in [36]. Like in the case of Mīmāṃsā, it functions as a dyadic operator, but, unlike in Mīmāṃsā, it can be granted under general conditions (and not just as an exception to a prohibition or negative obligation), and all tautologies are trivially permitted, which is not the case in Mīmāṃsā.

Many authors have attempted a formalization of von Wright's definition of strong permission, mainly with the purpose of obtaining a consistent formalization of the so-called 'free choice inference', introduced in [42]. This inference is of the form 'If it is permitted to do $A$ or $B$, it is permitted to do $A$ and it is permitted to do $B$'. Generally, a disjunction of permissions implies that any of the disjuncts is a possible option, and this is clearly an inference scheme that is desirable for a permission to follow. However, accepting the free choice inference might lead to deriving counterintuitive conclusions, e.g., an obligation to pay your taxes implies a permission to murder someone. Among the works that have endeavored to establish a formalization of free choice permissions that are immune to undesired consequences are [3,5,6,16]. The introduced systems are quite complex, and use, e.g., substructural logics as underlying logics or semantical elements added to the language.

Hansson's paper [26] explores a third form of permission: implicit permission, which is implied by an obligation. For instance, the obligation to testify in court implies the permission to enter the courtroom. In contrast, for Mīmāṃsā an act cannot be both obligatory and permitted under the same circumstance and the obligation to perform $X$ extends to the obligation to perform whatever is necessarily entailed by $X$. Thus, entering the courtroom is not the content of an implicit permission but of an obligation.

Alchourrón famously recounts a story (originally from [17]) about a hunting tribe and its new chief, who emits a norm permitting hunting on certain days, but without prohibiting it on the others. The tribe is utterly dissatisfied, because one expected from the chief an intervention in the status quo ("The moral of this story is valuable. It shows that purely permissory norms are of little if any practical interest" [2]). Alchourrón's conclusion, is different from the Mīmāṃsā one, as he highlights the importance of permissions in the case of more than one source of norms, see [2]. However the tribe reasoned according to Mīmāṃsā principles, based on which each command needs to change something which was previously the case (see the novelty requirement discussed above and the examples mentioned in Section 3).

A Mīmāṃsā permission is always an exception to a more general prohibition or negative obligation. This approach reflects a common practice in normative texts, such as legal codes in European jurisprudence, where permissions are typically stated only when there is an expectation of the opposite due to a general prohibition. Norms granting permissions usually derogate from what is stated in other norms, as Bouvier notes in his definition of permission in his legal dictionary in [11]. He distinguishes between express permissions that "derogate from something which before was forbidden," and implied permissions, "which arise from the fact that the law has not forbidden the act to be done". The latter are therefore different from Hansson's "implicit permission" and rather correspond to what Hansson calls "tacit permissions" in [26], and to what von Wright calls "weak permissions" in [41]. Similarly, the idea that permissions grant one a different degree of freedom if compared to the non-normed space of indifferent actions is neatly reflected by the comparison of cases like "You are permitted to run 2km per day" (said by a physician to her patient, who is recovering from a heart attack), as opposed to the same person's freedom to run prior to the heart attack. The permission rules the realm of running by introducing a space of possibility that is, however, not as absolute as the space of extra-normative actions. Accordingly, permitted actions are actions one would be naturally inclined to do, prior to the intervention of a normative text prohibiting them (or obliging one to refrain from them). In Mīmāṃsā deontics, it would not make sense to have a permission that regards impossible actions like flying or undesirable actions like harming oneself (assuming that harming oneself is not desirable for anyone). The Mīmāṃsā position is neatly distinguished from the one of, e.g., [26], who thinks that introducing permissions even in the absence of general prohibitions are useful to define rights.

A last trait of Mīmāṃsā permissions is that they always lead to less desirable options. This offers a solution to seeming problems like the "Interrupted promise", discussed by Zylberman [44]. There, one promises to participate in a conference, but then one's daughter has an accident and the previous duty is overruled by the duty to stand by one's daughter during surgery. Zylberman notes that despite having permission to withdraw, there is still an obligation to apologize or make reparations to the conference organizers. This sentiment contradicts the standard account of permissions, which does not mandate such actions. For instance, if it is permitted to drive at 18, no 18-years-old is expected to apologise because they are in fact driving. By contrast, the "interrupted promise" problem is instantly solved if we realise that the permission Zylberman is referring to is a Mīmāṃsā permission ("better-not") and therefore requires some expiation (such as offering an apology).

The concept of preference in deontic logic is well known, see, e.g., [15,24,39,30,4,25]. However, its application to a "less preferred" permission has not been explored in depth. We defer to future research the examination of the preference aspect of permission. Instead, we focus below on the formalization of the remaining properties.

## 4 Formalizing Mīmāṃsā Permission

Following a bottom-up approach of extracting deontic principles from the Mīmāṃsā texts, we transform the properties of the permission operator into suitable Hilbert axioms, which are added to the logic $LKu^+$ of [8]. We call the resulting logic $LM_P$ (Mīmāṃsā Logic with permission). In this section, we present and justify its Hilbert axiomatization, we introduce a neighbourhood semantics, and demonstrate soundness, completeness and consistency of $LM_P$.

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|---|
| Ax1. $(\Box_u(\phi \to \psi) \wedge \mathcal{O}(\phi/\theta) \wedge \neg\Box_u\psi) \to \mathcal{O}(\psi/\theta)$ |
| Ax2. $(\Box_u(\phi \to \psi) \wedge \mathcal{F}(\psi/\theta) \wedge \Diamond_u\phi) \to \mathcal{F}(\phi/\theta)$ |
| Ax3. $\neg(X(\phi/\theta) \wedge X(\neg\phi/\theta))$ for $X \in \{\mathcal{O}, \mathcal{F}\}$ |
| Ax4. $\neg(\mathcal{O}(\phi/\theta) \wedge \mathcal{F}(\phi/\theta))$ |
| Ax5. $(\Box_u(\psi \leftrightarrow \theta) \wedge X(\phi/\psi)) \to X(\phi/\theta)$ for $X \in \{\mathcal{O}, \mathcal{F}\}$ |
| Ax6. $(\Diamond_u(\phi \wedge \theta) \wedge \mathcal{O}(\phi/\top) \wedge \mathcal{O}(\theta/\top)) \to \mathcal{O}(\phi \wedge \theta/\top)$ |

Table 1
Axioms regarding obligation and prohibition from [8]

### 4.1 Syntax

The logic $LM_P$ is an extension of the logic $LKu^+$ of [8]. Recall that the language of $LKu^+$ consists of the modalities[4] $\mathcal{O}(\phi/\psi)$ and $\mathcal{F}(\phi/\psi)$ for obligation and prohibition (read as "$\phi$ is obligatory/prohibited given $\psi$"). Here we add the permission operator $\mathcal{P}(\phi/\psi)$, to be read as "$\phi$ is permitted, given $\psi$". This operator is treated as a primitive modality, that is, $\mathcal{P}(\phi/\psi)$ is not equivalent to $\neg\mathcal{F}(\phi/\psi)$ or $\neg\mathcal{O}(\neg\phi/\psi)$. As explained in Section 2, all the deontic operators in Mīmāṃsā are dyadic. The language $\mathcal{L}_{LM_P}$ is defined as follows:

$$\phi ::= p \in \mathit{Atom} \mid \neg\phi \mid \phi \vee \phi \mid \Box_u\phi \mid \mathcal{O}(\phi/\phi) \mid \mathcal{F}(\phi/\phi) \mid \mathcal{P}(\phi/\phi)$$

(where $\mathit{Atom}$ is the set of atomic propositions). We take the classical logic[5]. connective $\neg$ and $\vee$ as primitive, and define $\wedge$, $\to$, $\leftrightarrow$ in the usual way. The constants $\top$ and $\bot$ are abbreviations for $\neg\phi \vee \phi$ and $\neg\top$, respectively. $\Box_u$ is the universal S5 modality, read as 'in all scenarios, $\phi$ is true' and its dual $\Diamond_u\phi = \neg\Box_u\neg\phi$ as 'there is at least one scenario where $\phi$ is true'.

**Definition 4.1** The logic $LM_P$ extends $LKu^+$ – whose axiomatization consists of the axiomatization for the modal logic S5 for $\Box_u$ and the axioms of Table 1 – with the following axioms:

**P1.** $\mathcal{P}(\phi/\psi) \to (\mathcal{F}(\phi/\top) \vee \mathcal{O}(\neg\phi/\top))$
**P2.** a) $\neg(\mathcal{P}(\phi/\psi) \wedge \mathcal{F}(\phi/\psi))$
b) $\neg(\mathcal{P}(\phi/\psi) \wedge \mathcal{O}(\phi/\psi))$
c) $\neg(\mathcal{P}(\phi/\psi) \wedge \mathcal{O}(\neg\phi/\psi))$
**P3.** $(\mathcal{O}(\phi/\psi) \vee \mathcal{F}(\phi/\psi)) \to \Diamond_u(\phi \wedge \psi) \wedge \neg\Box_u\phi$
**P4.** a) $(\Box_u(\psi \leftrightarrow \theta) \wedge \mathcal{P}(\phi/\psi)) \to \mathcal{P}(\phi/\theta)$
b) $(\Box_u(\phi \leftrightarrow \psi) \wedge \mathcal{P}(\phi/\theta)) \to \mathcal{P}(\psi/\theta)$
**P5.** $(\mathcal{P}(\phi/\psi) \wedge (\mathcal{F}(\phi/\theta) \vee \mathcal{O}(\neg\phi/\theta))) \to \Box_u(\psi \to \theta)$

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[4] The logic $LKu^+$ formalizes the deontic theories of two main Mīmāṃsā authors: Kumārila and Prabhākara (both 7 CE?). Their theories differ from the way elective duties are interpreted: as an obligation for Prabhākara, and as a recipe that guarantees to obtain a desired result, for Kumārila. The latter has been formalized in [8] with a modality $\mathcal{E}(\phi/\psi)$ having no deontic force. As this modality does not interact with the deontic operators, to simplify the matter we omit it from the language of $LM_P$.

[5] The classical logic base is justified by the presence, e.g., of the reduction ad absurdum law in Mīmāṃsā, see [14]

Introduced in [8] the axioms in Table 1 are based on the following principles extracted from suitable *nyāya*s:

1. If the accomplishment of an action presupposes the accomplishment of another action, the obligation to perform the first prescribes also the second. Conversely, if an action necessarily implies a prohibited action, this will also be prohibited. This corresponds to the *nyāya* given as an example in Section 2, and formalized by Ax1 and Ax2.
2. Two actions that exclude each other can neither be prescribed nor prohibited simultaneously to the same group of eligible people under the same conditions. This principle is the base for Ax3 and Ax4.
3. If two sets of conditions always identify the same group of eligible agents, then a command valid under the conditions in one of the sets is also enforceable under the conditions in the other set. This is formalized by Ax5.
4. If two fixed duties are prescribed and compatible, their conjunction is obligatory as well. This corresponds to Ax6.

**Remark 4.2** In this paper we use a slightly different formulation of the axioms Ax1 and Ax2, w.r.t. [8], as their original version leads to contradictions in the presence of our new axiom P3. Ax1 was indeed presented as $(\boxdot(\phi \to \psi) \wedge \mathcal{O}(\phi/\theta)) \to \mathcal{O}(\psi/\theta)$. Since $\boxdot(\phi \to \top)$ is true for any formula $\phi$, we would derive $\mathcal{O}(\top/\theta)$ whenever we have $\mathcal{O}(\phi/\theta)$ for any $\phi$ and $\theta$, contradicting axiom P3. Ax2 was presented in [8] as $(\boxdot(\phi \to \psi) \wedge \mathcal{F}(\psi/\theta)) \to \mathcal{F}(\phi/\theta)$. The formula $\boxdot(\bot \to \psi)$ is true for any formula $\psi$, and therefore we derive $\mathcal{F}(\bot/\theta)$ from $\mathcal{F}(\psi/\theta)$ for any $\psi$ and $\theta$, contradicting P3, as well.

We discuss the properties of permission that led to the definition of axioms P1-P5 in Def. 4.1. We start by presenting the abstract principles behind them.

(i) Permissions are always exceptions to more general prohibitions or negative obligations.

This principle is extracted from the *nyāya* applied in (a)-(d) from Section 3, and is the base for axioms P1 and P5. P1 represents the fact that a permission is always an exception to a general prohibition or negative obligation (cf. (a)-(c)). From the application (d), we conclude that if something is allowed in one context and prohibited (or negatively obliged) in another, the context of the prohibition or negative obligation is more general, as formalized by axiom P5.

(ii) No more than one deontic operator can be applied to the same action under the same circumstances.

In the domain of Mīmāṃsā deontics, this principle represents a foundational metarule (cf. the *apūrva*-metarule discussed in Section 2 and 3) and justifies P2a-P2c. These axioms are similar to Ax4, but extended to permission. Especially interesting is axiom P2b, which states that an action cannot be permitted as well as obligatory under the same circumstances, contradicting the often-accepted inference in deontic logic that obligation implies permission.

(iii) Commands entail possibility.

The formalization of this principle is accomplished through Axiom P3, which does not pertain to permission. The principle has been extracted from various contexts, summarized by the *nyāya*-applications (e), corresponding to 'ought implies can', and (f), corresponding to 'forbidden implies can' (cf. Section 3). As commands must be meaningful, this axiom also excludes the possibility of obligatory or prohibited tautologies. Although we have not found an explicit statement that principle (iii) applies also to permissions, the fact that permitted actions are exceptions to prohibited or negatively obliged (possible) actions, is enough to conclude that this axiom should be present; as shown by Lemma 4.4 it is indeed derivable in $LM_P$.

Axiom P4a and P4b do not follow from any explicit discussion by Mīmāṃsā authors. P4a is implicitly used in Dharmaśāstra discussions of permissions under extreme circumstances. For instance, Vijñāneśvara states that it is permitted to sell certain vegetables if one has assumed the occupation of the *vaiśya* class, and then refers to the permission to sell the same vegetables if one is working as a merchant, given that assuming the occupation of a *vaiśya* implies being a merchant (*Mitākṣarā* on Yājñavalkya 3.35). Axiom P4b is also implicitly used in the same context when $\mathcal{P}$(act as a *vaiśya*/(being a Brāhmaṇa $\wedge$ being in distress)) leads to the $\mathcal{P}$(selling/(being a Brāhmaṇa $\wedge$ being in distress)) because acting as a *vaiśya* is synonymous of selling.

**Remark 4.3** In contrast with obligation and prohibition (as well as contrary to the notion of permission in [36]), $LM_P$ does not contain a monotonicity axiom for permission, i.e., $(\mathcal{P}(\phi/\theta) \wedge \Box_U(\phi \to \psi)) \to \mathcal{P}(\psi/\theta)$. The main reason is that we have not found it in Mīmāṃsā texts. It is also unlikely to find it as this axiom would lead to unwanted consequences. For instance, from "eating meat implies being alive" and $\mathcal{P}$(eating meat/during extreme circumstances), would follow $\mathcal{P}$(being alive/during extreme circumstances) which is not meaningful as we have no control over being alive. Additionally, as shown by the following derivation, monotonicity of permissions would imply an unconditional prohibition or negative obligation for any other feasible action:

(i) $\mathcal{P}(\phi/\theta) \to \mathcal{P}(\phi \vee \psi/\theta)$ (monotonicity for permissions)

(ii) $\mathcal{P}(\phi \vee \psi/\theta) \to (\mathcal{F}(\phi \vee \psi/\top) \vee \mathcal{O}(\neg(\phi \vee \psi)/\top))$ (P1)

(iii) $\Box_U(\psi \to (\phi \vee \psi)) \wedge \mathcal{F}(\phi \vee \psi/\top) \wedge \Diamond_U \psi \to \mathcal{F}(\psi/\top)$ (Ax2)

(iv) $\Box_U((\neg\phi \wedge \neg\psi) \to \neg\psi) \wedge \mathcal{O}(\neg(\phi \vee \psi)/\top) \wedge \Diamond_U \psi \to \mathcal{O}(\neg\psi/\top)$ (Ax1)

(v) $\mathcal{P}(\phi/\theta) \wedge \Diamond_U \psi \to (\mathcal{F}(\psi/\top) \vee \mathcal{O}(\neg\psi/\top))$ (from (i)-(iv))

**Lemma 4.4** *The following formulas are derivable in $LM_P$:*

1. $\Box_U(\phi \to \psi) \to \neg(\mathcal{O}(\phi/\theta) \wedge \mathcal{P}(\psi/\theta))$
2. $\Box_U(\phi \to \psi) \to \neg(\mathcal{F}(\psi/\theta) \wedge \mathcal{P}(\phi/\theta))$
3. $\mathcal{P}(\phi/\psi) \to \Diamond_U \phi \wedge \neg \Box_U \phi$
4. $\neg(\mathcal{P}(\phi/\theta) \wedge \mathcal{P}(\neg\phi/\theta))$

**Proof.** 1. follows by Ax1 and P2b. 2. follows by Ax2 and P2a. 3. follows from by axiom P1 and P3. 4. follows from P1, Ax3 and Ax4. □

The first two formulas from Lem. 4.4 can be viewed as generalizations of the D-axiom for permission. Formula 3, that will be utilized in our formalization of the free choice inference in the next section, constitutes a variation of the 'commands entail possibility' principle for permissions. Although formula 4 is not a property of permission in natural language, in the context of Mīmāṃsā, permissions are treated as exceptions to general prohibitions or negative obligations and there cannot be a prohibition or negative obligation regarding both a particular action and its negation.

### 4.2 Semantics

In line with [8], we use neighbourhood semantics to model $LM_P$.

Neighbourhood semantics generalizes Kripke semantics. It consists of a set of worlds $W$ and a valuation function $V$, and contains neighbourhood functions $N_x$ that map a world to a set of ordered pairs of sets of worlds. Each of the three modalities, obligation, permission and prohibition, has its own neighbourhood function. For example, let $w \in W$, if $(X, Y)$ is in $w$'s obligation-neighbourhood, this means that $X$ represents the worlds of compliance 'from the point of view' of $Y$. Then, if $X$ is exactly the set of worlds where $\phi$ is true, and $Y$ is exactly the set of worlds where $\psi$ is true, then $\mathcal{O}(\phi/\psi)$ is true in $w$.

**Definition 4.5** An $LM_P$-frame $F = \langle W, N_{\mathcal{O}}, N_{\mathcal{P}}, N_{\mathcal{F}} \rangle$ is a tuple where $W \neq \emptyset$ is a set of worlds $w, v, u, \ldots$ and $N_{\chi} : W \mapsto P(P(W) \times P(W))$ is a neighbourhood function for $\chi \in \{\mathcal{O}, \mathcal{P}, \mathcal{F}\}$. Let $X, Y, Z \subseteq W$, $F$ satisfies the following:

(i) If $(X, Z) \in N_{\mathcal{P}}(w)$ then $(X, W) \in N_{\mathcal{F}}(w)$ or $(\overline{X}, W) \in N_{\mathcal{O}}(w)$.
(ii) If $(X, Z) \in N_{\mathcal{P}}(w)$ then $(X, Z) \notin N_{\mathcal{F}}(w)$ and $(X, Z) \notin N_{\mathcal{O}}(w)$.
(iii) If $(X, Z) \in N_{\chi}(w)$ then $X \bigcap Z \neq \emptyset$ and $X \neq W$ for $(\chi \in \{\mathcal{O}, \mathcal{F}\})$.
(iv) If $(X, Y) \in N_{\mathcal{P}}(w)$ and $(X, Z) \in N_{\mathcal{F}}(w)$ or $(\overline{X}, Z) \in N_{\mathcal{O}}(w)$ then $Y \subset Z$.
(v) If $(X, Z) \in N_{\mathcal{P}}(w)$ then $(\overline{X}, Z) \notin N_{\mathcal{O}}(w)$.
(vi) If $(X, Z) \in \mathcal{N}_{\mathcal{O}}(w)$ and $X \subseteq Y$ and $Y \neq W$, then $(Y, Z) \in \mathcal{N}_{\mathcal{O}}(w)$.
(vii) If $(X, Z) \in \mathcal{N}_{\mathcal{F}}(w)$ and $Y \subseteq X$ and $Y \neq \emptyset$, then $(Y, Z) \in \mathcal{N}_{\mathcal{F}}(w)$.
(viii) If $(X, Y) \in \mathcal{N}_{\mathcal{X}}(w)$, then $(\overline{X}, Y) \notin \mathcal{N}_{\mathcal{X}}(w)$ for $\mathcal{X} \in \{\mathcal{O}, \mathcal{F}\}$.
(ix) If $(X, Z) \in \mathcal{N}_{\mathcal{O}}(w)$ then $(X, Z) \notin \mathcal{N}_{\mathcal{F}}(w)$.
(x) If $X \bigcap Y \neq \emptyset$ and $(X, W), (Y, W) \in \mathcal{N}_{\mathcal{O}}(w)$, then $(X \bigcap Y, W) \in \mathcal{N}_{\mathcal{O}}(w)$.

An $LM_P$-model is a tuple $M = \langle F, V \rangle$ where $F$ is an $LM_P$-frame and $V$ is a valuation function mapping atomic propositions from *Atom* to sets of worlds.

Note that (i) corresponds to axiom P1, (ii) and (vi) to axioms P2a-c, (iii) to axiom P3 and (iv) to P5. Moreover, (vi) and (vii) correspond to axioms Ax1 and Ax2, expressing the property of monotonicity in the first argument of the deontic operators; these conditions are based on the ones in [8], adjusted to comply with our new version of the monotonicity axioms (see Remark 4.2). (viii) corresponds to Ax3, avoiding the accumulation of deontic operators, (ix) corresponds to Ax4, and (x) to Ax6. Axioms P4a, P4b and Ax5 hold in any neighbourhood model [13] and do not require explicit conditions.

**Definition 4.6** Let $M$ be a $LM_P$-model and $\|\phi\| = \{w \in W \mid M, w \vDash \phi\}$. We define the satisfaction of a formula $\phi \in \mathcal{L}_{LM_P}$ at any $w \in W$ as follows:

$$
\begin{array}{lll}
M, w \vDash p & \text{iff} & w \in V(p), \text{ for } p \in Atom \\
M, w \vDash \neg\phi & \text{iff} & M, w \nvDash \phi \\
M, w \vDash \phi \vee \psi & \text{iff} & M, w \vDash \phi \text{ or } M, w \vDash \psi \\
M, w \vDash \boxed{\mathsf{U}}\,\phi & \text{iff} & \text{for all } w_i \in W \ M, w_i \vDash \phi \\
M, w \vDash \langle\mathsf{U}\rangle\,\phi & \text{iff} & \text{there exists a } w_i \in W \ M, w_i \vDash \phi \\
M, w \vDash \mathcal{X}(\phi/\psi) & \text{iff} & (\|\phi\|, \|\psi\|) \in \mathcal{N}_{\mathcal{X}}(w) \text{ for } \mathcal{X} \in \{\mathcal{O}, \mathcal{F}, \mathcal{P}\}
\end{array}
$$

We say a formula $\phi$ *holds* in a model $M$ iff $M, w \vDash \phi$ for each $w \in W$.

Using the strategy outlined in [10] and their corresponding definitions, we demonstrate that the axioms are sound and complete relative to the given neighbourhood semantics.

**Definition 4.7** A formula $\phi$ is *valid* in $LM_P$, if for all worlds $w$ in all $LM_P$-models $M$ it is the case that $M, w \vDash \phi$.

A formula $\phi$ is a *theorem* of $LM_P$, if it is derivable using only the axioms of $LM_P$, modus ponens and necessitation rule for $\boxed{\mathsf{U}}$.

**Theorem 4.8 (Soundness)** *If a formula $\phi$ is a theorem of $LM_P$, then $\phi$ is valid.*

**Proof.** We show that all axioms of $LM_P$ are true in all worlds of any $LM_P$-model $M$. For each axiom, we assume that the antecedent holds in a world, and use the neighbourhood restrictions and the truth conditions of Def. 4.6 to derive the intended consequent. Showing that modus ponens and the necessitation rule for $\boxed{\mathsf{U}}$ preserve validity is easy. We only detail the case of axiom P1 – the main property of Mīmāṃsā permission– as all other axioms are proven similarly. Assume that $\mathcal{P}(\phi/\psi) \to (\mathcal{F}(\phi/\top) \vee \mathcal{O}(\neg\phi/\top))$ is a theorem of $LM_P$. Consider a world $w$ in model $M$ such that $M, w \vDash \mathcal{P}(\phi/\psi)$. Def. 4.6 gives us $(\|\phi\|, \|\psi\|) \in N_{\mathcal{P}}(w)$, and (i) from Def. 4.5 gives us that $(\|\phi\|, W) \in N_{\mathcal{F}}(w)$ or $(\|\neg\phi\|, W) \in N_{\mathcal{O}}(w)$. Since $W = \|\top\|$, we have that $M, w \vDash \mathcal{F}(\phi/\top)$ or $M, w \vDash \mathcal{O}(\neg\phi/\top)$. Therefore, $M, w \vDash \mathcal{P}(\phi/\psi) \to (\mathcal{F}(\phi/\top) \vee \mathcal{O}(\neg\phi/\top))$. □

**Theorem 4.9 (Completeness)** *If a formula $\phi$ is valid, then $\phi$ is a theorem of $LM_P$.*

**Proof.** We use the method of canonical models from [13]. First, we define the canonical model $M^c$, in such a way that for each formula $\phi$ and world $w$, $M^c, w \vDash \phi$ iff $\phi \in w$. The formulas true in all worlds of $M^c$ are, then, exactly the theorems of $LM_P$. However, $M^c$ is not necessarily an $LM_P$-model. The universal modality $\boxed{\mathsf{U}}$ is axiomatized by S5, which is canonical for the equivalence relation, i.e. $R^c_{\boxed{\mathsf{U}}} \subseteq W \times W$ (see [10]). For the global modality, the required property is $R^c_{\boxed{\mathsf{U}}} = W \times W$. Thus, as is done in [8], we introduce a submodel –we call it $M^*$– of the canonical model $M^c$, and we show that is an $LM_P$-model. $M^*$ is then used to establish completeness.

First, we define a canonical model $M^c = \langle W^c, R^c_{\boxed{\mathsf{U}}}, N^c_{\mathcal{O}}, N^c_{\mathcal{P}}, N^c_{\mathcal{F}}, V^c \rangle$ for $LM_P$. Let $W^c$ be the set of all $LM_P$-maximally consistent sets of formulas. Let $(Y, Z) \in N^c_{\mathcal{O}}(w)$ iff $Y \neq W^c$ and there is a formula $\mathcal{O}(\phi/\psi) \in w$ such that $\{w_j \in W^c \mid \phi \in w_j\} \subseteq Y$ and $\{w_j \in W^c \mid \psi \in w_j\} = Z$. Then, let $(Y, Z) \in N^c_{\mathcal{P}}(w)$ iff there is a formula $\mathcal{P}(\phi/\psi) \in w$ such that $Y = \{w_j \in W^c \mid \phi \in w_j\}$

and $\{w_j \in W^c \mid \psi \in w_j\} = Z$. Furthermore, let $(Y, Z) \in N^c_{\mathcal{F}}(w)$ iff $Y \neq \emptyset$ and there is a formula $\mathcal{F}(\phi/\psi) \in w$ such that $Y \subseteq \{w_j \in W^c \mid \phi \in w_j\}$ and $\{w_j \in W^c \mid \psi \in w_j\} = Z$. Lastly, $w \in V^c(p)$ iff $p \in w$. We will use the following shorthand throughout the proof $\|\phi\|^c = \{w \in W^c \mid \phi \in w\}$.

To show that our canonical model satisfies the restrictions of Def. 4.5, we outline the case of (vi). The same strategy can be adopted for the other cases.

(vi) If $(X, Z) \in N^c_{\mathcal{P}}(w)$, then $(\overline{X}, Z) \notin N^c_{\mathcal{O}}(w)$. To see why, consider $(X, Z) \in N^c_{\mathcal{P}}$ for some $w \in W^c$ and $X, Z \subseteq W^c$. Note that there is a formula $\mathcal{P}(\phi/\psi) \in w$ where $\|\phi\|^c = X$ and $\|\psi\|^c = Z$. By axiom P2c, we have $\mathcal{O}(\neg\phi/\psi) \notin w$. It might be the case that $(\overline{X}, Z) \in N^c_{\mathcal{O}}(w)$ if there is a $\chi$ such that $\|\chi\|^c \subseteq \|\neg\phi\|^c$ and $\mathcal{O}(\chi/\psi) \in w$. However, since $\boxdot(\chi \to \neg\phi)$, by Ax1, we have $\mathcal{O}(\neg\phi/\psi) \in w$ contradicting P2c. Thus, $(\overline{X}, Z) \notin N^c_{\mathcal{O}}(w)$.

We show, by induction on $\phi$, that $M^c, w \vDash \phi$ iff $\phi \in w$. The base case is clear: $M^c, w \vDash p$ implies $p \in w$ by definition. For the inductive case, we consider only $\mathcal{P}(\phi/\psi)$, as the classical connectives are straightforward, and $\mathcal{O}(\phi/\psi)$ and $\mathcal{F}(\phi/\psi)$ are done similarly. If $M^c, w \vDash \mathcal{P}(\phi/\psi)$, then $(\|\phi\|^c, \|\psi\|^c) \in N^c_{\mathcal{P}}(w)$. By the canonical model, there is a formula $\mathcal{P}(\theta_1/\theta_2) \in w$ such that $\|\phi\|^c = \|\theta_1\|^c$ and $\|\psi\|^c = \|\theta_2\|^c$. By axioms P4ab, we have that $\mathcal{P}(\phi/\psi) \in w$.

Our model $M^c$ satisfies $R^c_{\boxdot} \subseteq W^c \times W^c$. However, since $\boxdot$ represents the global modality, it is necessary that $R^c_{\boxdot} = W^c \times W^c$. To meet this requirement, we generate a submodel $M^*$ of $M^c$, and show that its relation $R^*_{\boxdot}$ satisfies $R^*_{\boxdot} = W^* \times W^*$ for some $W^* \subseteq W^c$. We then prove that $M^*$ is an $LM_P$-model, and utilize this model to demonstrate completeness. To construct $M^* = \langle W^*, R^*_{\boxdot}, N^*_{\mathcal{O}}, N^*_{\mathcal{F}}, N^*_{\mathcal{P}}, V^*\rangle$ we begin by selecting a world $w \in W^c$. Its domain $W^*$ is defined as follows: $W^* = \{v \in W^c \mid$ for all $\boxdot\phi \in w$, $\phi \in v\}$. The relation $R^*_{\boxdot}$ is defined as: $R^*_{\boxdot} = R^c_{\boxdot} \cap (W^* \times W^*)$. As described in [10], it can be easily shown that $R^*_{\boxdot} = W^* \times W^*$, which is our required property. Then, $V^*(p) = V^c(p) \cap W^*$, and the neighborhood functions are defined as follows: $N^*_\chi(w) = \{(X, Y) \mid (X', Y') \in N^c_\chi(w), X = X' \cap W^*, Y = Y' \cap W^*\}$ for $\chi \in \{\mathcal{O}, \mathcal{P}, \mathcal{F}\}$. By a simple induction on the complexity of $\phi$, it follows that $\|\phi\|^* = \{w \in W^* \mid \phi \in w\} = \|\phi\|^c \cap W^*$. We can show that each neighbourhood restriction is satisfied by $M^*$, and that $M^*$ is thus a $LM_P$-model. We show the case for (i), the other cases being similar.

(i) If $(X, Y) \in N^*_{\mathcal{P}}(w)$ then $(X, W^*) \in N^*_{\mathcal{F}}(w)$ or $(\overline{X}, W^*) \in N^*_{\mathcal{O}}(w)$. To see why, consider $(X, Y) \in N^*_{\mathcal{P}}(w)$ for some $w \in W^*$. Then, by definition of the submodel $M^*$, it follows that $X = X' \cap W^*$ and $Y = Y' \cap W^*$ for some $(X', Y') \in N^c_{\mathcal{P}}(w)$. Since $M^c$ is an $LM_P$ model, we know that $(X', W^c) \in N^c_{\mathcal{F}}(w)$ or $(\overline{X'}, W^c) \in N^c_{\mathcal{O}}(w)$. Thus, $(X' \cap W^*, W^c \cap W^*) = (X, W^*) \in N^*_{\mathcal{F}}(w)$ or $(\overline{X'} \cap W^*, W^c \cap W^*) = (\overline{X}, W^*) \in N^*_{\mathcal{O}}(w)$.

Lastly, we have that for each $w \in W^*$, $M^*, w \vDash \phi \leftrightarrow M^c, w \vDash \phi$ by induction on the complexity of $\phi$, and therefore $M^*, w \vDash \phi \leftrightarrow \phi \in w$. □

**Lemma 4.10 (Consistency)** *The logic $LM_P$ is consistent.*

**Proof.** We exhibit a $LM_P$-model $M$ in which all $LM_P$ axioms hold but there is one formula that does not. Let $M = \langle W, N_{\mathcal{O}}, N_{\mathcal{F}}, N_{\mathcal{P}}, V\rangle$, where $W =$

$\{w_1, w_2\}$, $N_{\mathcal{O}}(w_i) = \{(\{w_1\}, \{w_2\})\}$, $N_{\mathcal{P}}(w_i) = N_{\mathcal{F}}(w_i) = \emptyset$ for $i \in \{1,2\}$, and $V(p) = \{w_1\}$, $V(q) = \{w_2\}$.

We show that axiom P2b holds. We have $M, w_i \vDash \mathcal{O}(p/q)$, since $(\|p\|, \|q\|) \in N_{\mathcal{O}}(w_i)$, and $M, w_i \nvDash \mathcal{P}(p/q)$. The model similarly satisfies axiom P2a, P2c, Ax3 and Ax4. It trivially satisfies all remaining axioms, since they are implications and the antecedent is false. Since $M, w_i \nvDash \mathcal{P}(p/q)$, there is a formula that does not hold in the model and therefore $LM_P$ is consistent. □

## 5 Deontic Paradoxes in Mīmāṃsā

To analyze the behaviour of $LM_P$ we use as benchmarks the main deontic paradoxes [6] involving permission: the free choice inference [42], Ross' paradox [37] and the paradox of the privacy act [22]. As demonstrated below, $LM_P$ behaves well with respect to them.

### 5.1 The Free Choice Inference

It is plausible to say that "you may have coffee or tea" implies that you may have a coffee and you may have a tea (though possibly not both at once). This very intuitive principle, first mentioned in [42], is known as the free choice inference (FCI) and is formalized in SDL as $\mathcal{P}(\phi \vee \psi) \to \mathcal{P}(\phi)$. The paradoxical consequences of accepting FCI have been widely discussed in deontic logic, see, e.g. [21,5,12,16]. Among them, as demonstrated in [21], SDL with (FCI) derives (i) $\mathcal{O}(\phi) \to \mathcal{O}(\phi \wedge \psi)$, (ii) $\mathcal{O}(\phi) \to \mathcal{P}(\psi)$, (iii) $\mathcal{P}(\phi) \to \mathcal{P}(\psi)$ and (iv) $\mathcal{P}(\phi) \to \mathcal{P}(\phi \wedge \psi)$. As a special instance of (iii), we get (v) $\mathcal{P}(\phi) \to \mathcal{P}(\bot)$, which is a particularly undesirable consequence in Mīmāṃsā, where permitted actions should be possible, as shown by Lemma 4.4. As a result, we modify the free choice inference in $LM_P$ to ensure that every inferred permission corresponds to a feasible action:

$$\mathcal{P}(\phi \vee \psi/\theta) \wedge \Diamond_{\mathsf{U}}\, \phi \to \mathcal{P}(\phi/\theta). \qquad (\mathrm{FCI}\Diamond_{\mathsf{U}})$$

We demonstrate that the (dyadic variant of) (i)-(v) cannot be derived in $LM_P$ in presence of FCI$\Diamond_{\mathsf{U}}$. We start by establishing a sufficient condition for FCI$\Diamond_{\mathsf{U}}$ to hold in an $LM_P$-model.

**Lemma 5.1** *Let $M = \langle W, N_{\mathcal{O}}, N_{\mathcal{P}}, N_{\mathcal{F}}, V\rangle$ be an $LM_P$-model, and consider non-empty $X, Y, Z \subseteq W$. For all $w \in W$, if $X \subseteq Y$ and $(Y, Z) \in N_{\mathcal{P}}(w)$ implies $(X, Z) \in N_{\mathcal{P}}(w)$, then $M, w \vDash$FCI$\Diamond_{\mathsf{U}}$.*

**Proof.** Assume $M, w \vDash \mathcal{P}(\phi \vee \psi/\theta) \wedge \Diamond_{\mathsf{U}}\, \phi$. Then, $(\|\phi\| \cup \|\psi\|, \|\theta\|) \in N_{\mathcal{P}}(w)$. Since $\|\phi\| \subseteq \|\phi\| \cup \|\psi\|$ and $\|\phi\| \neq \emptyset$ (by $M, w \vDash \Diamond_{\mathsf{U}}\, \phi$), we have that $(\|\phi\|, \|\theta\|) \in N_{\mathcal{P}}(w)$ and thus $M, w \vDash \mathcal{P}(\phi/\theta)$. □

The example below exhibit an $LM_P$-model that satisfies FCI$\Diamond_{\mathsf{U}}$ but not the unwanted consequences (i)-(v).

[6] Although called paradoxes, they are intended here in a broad sense as (un)derivable theorems that are counter-intuitive in a common-sense reading.

**Example 5.2** Let $M = \langle W, N_{\mathcal{O}}, N_{\mathcal{P}}, N_{\mathcal{F}}, V\rangle$ be the $LM_P$-model such that $W = \{w_1, w_2, w_3\}$, $V(p) = \{w_1\}$, $V(q) = \{w_2\}$, $V(r) = \{w_3\}$, $N_{\mathcal{P}}(w_i) = \{(V(q), V(r))\}$, $N_{\mathcal{O}}(w_i) = \{(X, Y) \mid V(p) \subseteq X, X \neq W, Y = V(r)\}$, $N_{\mathcal{F}}(w_i) = \{(V(p), W)\}$ for $i \in \{1, 2, 3\}$. FCI$\Diamond_{\cup}$ is true in all $w_i \in W$, by Lem. 5.1. We show that $M$ does not satisfy (i)-(v).

For (i), we see that $M, w_i \vDash \mathcal{O}(p/r)$ and $M, w_i \nvDash \mathcal{O}(p \wedge q/r)$. For (ii), $M, w_i \vDash \mathcal{O}(p/r)$ and $M, w_i \nvDash \mathcal{P}(r/r)$. For (iii), we have $M, w_i \vDash \mathcal{P}(q/r)$ and $M, w_i \nvDash \mathcal{P}(p/r)$. For (iv), we have that $M, w_i \vDash \mathcal{P}(q/r)$ and $M, w_i \nvDash \mathcal{P}(p \wedge q/r)$. Lastly, for (v), we have $M, w_i \vDash \mathcal{P}(q/r)$ and $M, w_i \nvDash \mathcal{P}(\bot/r)$.

**Remark 5.3** The undesirable consequences (i)-(iv) can be derived in SDL using instances of obligation implies permission (aka axiom D), interdefinability between the deontic operators, and monotonicity of permission. These principles do not hold in $LM_P$. Nonetheless $LM_P$ cannot get rid of *all* unwanted results. To elaborate: while the undesirable inferences regarding obligation (i.e., (i) and (ii)), and impossible actions (i.e., (v)) are blocked even when (an unrelated action) $\psi$ is possible, the debatable statement $\mathcal{P}(\phi/\theta) \wedge \Diamond_{\cup}(\phi \wedge \psi) \rightarrow \mathcal{P}(\phi \wedge \psi/\theta)$ holds in $LM_P$ due to axiom P4b.

### 5.2 Ross' paradox

Ross' paradox [37] is a frequently debated issue. Introduced as a paradox for obligation, it states that the obligation to mail a letter implies the obligation to mail the letter or burn it. Here we consider its version for permission ("the permission to mail the letter implies the permission to mail or burn the letter"), formalized as the following valid formula in SDL

$$\mathcal{P}(\phi) \rightarrow \mathcal{P}(\phi \vee \psi).$$

The prima facie version of this paradox does not apply to permissions in Mīmāṃsā, because all commands in Mīmāṃsā have only one action as their argument. Moreover, the consequences of the paradox can be avoided even if we consider the all-things-considered deontic situation. In fact, as discussed in Section 3, unconditional permissions do not exist in Mīmāṃsā and thus the dyadic version of the paradox is the following:

$$\mathcal{P}(\phi/\theta) \rightarrow \mathcal{P}(\phi \vee \psi/\theta).$$

This formula is not derivable in $LM_P$, as shown by the following countermodel: Let $M = \langle W, N_{\mathcal{O}}, N_{\mathcal{P}}, N_{\mathcal{F}}, V\rangle$ be a $LM_P$-model, such that $W = \{w_1, w_2, w_3\}$, $V(p) = \{w_1\}$, $V(q) = \{w_2\}$, $V(r) = \{w_3\}$, $N_{\mathcal{P}}(w_i) = \{(V(p), V(r))\}$, $N_{\mathcal{F}}(w_i) = \{(V(p), W)\}$ and $N_{\mathcal{O}}(w_i) = \emptyset$ for $i \in \{1, 2, 3\}$. Note that the neighborhood function of prohibition is not empty in order to satisfy condition (i) stated in Def. 4.5. We see that $(V(p), V(r)) \in N_{\mathcal{P}}(w_i)$, but $(V(p) \cup V(q), V(r)) \notin N_{\mathcal{P}}(w_i)$. Thus $M, w_i \vDash \mathcal{P}(p/r)$ while $M, w_i \nvDash \mathcal{P}(p \vee q/r)$.

**Remark 5.4** Ross' paradox does not appear in $LM_P$ as Mīmāṃsā permission is not monotonic in the first argument, see Remark 4.3. If we were to derive $\mathcal{P}(\text{mail} \vee \text{burn}/\theta)$ from $\mathcal{P}(\text{mail}/\theta)$ for some $\theta$, then we would need to have a pre-existing command $\mathcal{F}(\text{burn}/\top)$ or $\mathcal{O}(\neg\text{burn}/\top)$ (cf. Remark 4.3). This is impossible if such a pre-existing prohibition or negative obligation is not available.

### 5.3 The Paradox of the Privacy Act

Introduced in [22], this paradox consists of a privacy act containing the norms:

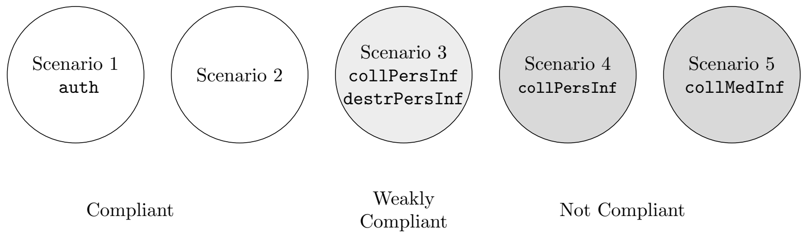
(i) The collection of personal information is forbidden unless acting on a court order authorising it.
(ii) The destruction of illegally collected personal information before accessing it is a defence against the illegal collection of personal data.
(iii) The collection of medical information is forbidden unless the entity collecting the medical information is permitted to collect personal information.

To properly assess this act, we need to consider five distinct scenarios as all other possible scenarios are variations of these. We refer to these as Scenarios 1-5. Scenario 1 involves a court order that authorizes the collection of personal data. Regardless of whether the data is ultimately collected or not, this scenario is compliant with the privacy act. Scenario 2, where a court has not authorized the collection of data and neither personal nor medical data is collected, is compliant as well. Scenario 3, where personal data is collected illegally but is compensated by its destruction, is called 'weakly compliant'. Lastly, there are two non-compliant situations: Scenario 4, involving the unauthorized collection of personal data, and Scenario 5, involving the unauthorized collection of medical data.

While SDL can formalize the norms (i)-(iii) in a consistent way, it derives a contradiction when considering the compliant Scenarios 1 and 2. For, by formalizing (i) as $\mathcal{F}(collPersInf)$ and $auth \to \mathcal{P}(collPersInf)$, when $auth$ is true (as in Scenario 1), we derive $\mathcal{P}(collPersInf)$, contradicting $\mathcal{F}(collPersInf)$.

This contradiction is prevented in $LM_P$. We formalize the norms (i)-(iii) in the following way: (i) is $\mathcal{F}(collPersInf/\top)$ and $\mathcal{P}(collPersInf/auth)$. Norm (ii) represents a contrary-to-duty obligation (see e.g. [35]) since the violation of collecting personal data must be compensated by its destruction, and is formalized as $\mathcal{O}(destrPersInf/collPersInf)$. Lastly, (iii) is formalized as $\mathcal{F}(collMedInf/\top)$ and $\mathcal{P}(collPersInf/X) \to \mathcal{P}(collMedInf/X)$ for any $X$, since the permission of collecting medical data depends on the condition $X$ of the permission for collecting personal data.

We show that $LM_P$ is suitable to model the privacy act, by giving a model where all norms (i)-(iii) holds, and each world represents one of the scenarios without contradictions: $M = \langle W, N_\mathcal{O}, N_\mathcal{P}, N_\mathcal{F}, V\rangle$, where: $W = \{w_i \mid 1 \leq i \leq 5\}$, $\|collPersInf\| = \{w_3, w_4\}$, $\|destrPersInf\| = \{w_3\}$, $\|auth\| = \{w_1\}$, $\|collMedInf\| = \{w_5\}$, $N_\mathcal{F}(w_i) = \{(X,Y) \mid X \neq \emptyset, X \subseteq \|collPersInf\|, Y = W\} \cup \{(U,Z) \mid U \neq \emptyset, U \subseteq \|collMedInf\|, Z = W\}$, $N_\mathcal{O}(w_i) = \{(X,Y) \mid \|destrPersInf\| \subseteq X, Y = \|collPersInf\|\}$, and $N_\mathcal{P}(w_i) = \{(\|collPersInf\|, \|auth\|), (\|collMedInf\|, \|auth\|)\}$. The picture below illustrates the model.




Note that the exception-based definition of permission in $LM_P$ is well-suited for the formalization of the privacy act, which considers permissions as exceptions to prohibitions.

**Remark 5.5** The paradox arising from the privacy act is resolved in $LM_P$ by the use of dyadic deontic operators. In contrast to SDL's monadic operators, $LM_P$ indeed enables the derivation of context-dependent prohibitions, permissions, and obligations, accommodating changing situations, and thus allowing, e.g., the formulas $\mathcal{F}(collPersInf/\top)$ and $\mathcal{P}(collPersInf/auth)$ to be true simultaneously.

## 6 Conclusions

Mīmāṃsā provides a treasure trove of more than 2,000 years worth of deontic investigations, including the application of deontic principles in juridical contexts and problems. In this article, we have analyzed the notion of permission in Mīmāṃsā, and formalized its properties by transforming relevant *nyāya*s (identified, translated from Sanskrit and interpreted) into suitable Hilbert axioms. The resulting deontic operator has been added to the logic of Mīmāṃsā as discussed in [8], and a sound and complete semantics has been provided. We have analyzed the behavior of the new permission operator using an established method in the deontic logic literature, which involves confronting it with deontic paradoxes, and found out that the resulting operator behaves well w.r.t. the considered paradoxes.

One might wonder whether the command we are discussing can be meaningfully described as permission at all. In fact, the term 'permission' in Euro-American philosophy or in Deontic Logic is strongly polysemic, covering, among others, acts that are not normed as well as acts that were previously prohibited and are now permitted, and even rights. Philosophers of the Mīmāṃsā school, by contrast, adopt the standard Sanskrit terms for permission (*anujñā* and *anumati*), but focus on only one aspect among the ones mentioned above, and use different terms for the others (for instance, *adhikāra* comes close to rights, see [18]). Using the term 'permission' thus highlights a single shared aspect and suggests a way out of the polysemy of 'permissions'.

Overall, this paper introduces and formalizes the concept of permission in Mīmāṃsā, contributes to the ongoing development of deontic logic, and sheds light on the importance of considering permission in normative reasoning.

There is still a missing component to capture the essence of Mīmāṃsā permission. As discussed in Section 3, while a certain condition may render a generally prohibited action permissible under specific circumstances, Mīmāṃsā still encourages avoiding such action whenever possible. To address this, we aim to incorporate in $LM_P$ the *Ceteris Paribus* preference (as e.g. in [7,31]) as future work. Specifically, we plan to compare two scenarios with identical obligations and prohibitions, but where the preference of the world depends on the fulfilled permissions.

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