

Dialogue Games for Fuzzy Logics

Masterstudium
Computational Intelligence

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Giles's Game

Overview & Motivation

- dialogue game introduced by Robin Giles in the 1970s
- models reasoning in physical theories
- asserting a proposition means committing oneself to pay a certain amount of money if the associated experiment(s) fail(s)
- separates evaluation of atomic formulas from decomposing compound formulas

Betting for Positive Results:

- each atomic proposition a is associated with a binary (yes/no) experiment E_a
- experiments may be probabilistic, i.e. show dispersion
- for each assertion of an atomic proposition an experiment is made
- each player places bets on positive outcomes of experiments corresponding to his claims

Decomposing Compound Formulas:

- arguments about complex formulas are systematically reduced to arguments about less complex formulas
- dialogue rules have already been introduced by Lorenzen for Intuitionistic Logic
- these rules characterize the *meaning* of logical connectives, independently of the underlying betting scheme

Rules

Atomic Evaluation: Let a be an atomic proposition. He who asserts a agrees to pay his opponent $\in 1$ if a trial of the experiment associated with a yields the outcome "no".

Implication: He who asserts $A \rightarrow B$ agrees to assert B if his opponent will assert A .

Negation: He who asserts $\neg A$ agrees to assert \perp if his opponent will assert A where \perp is associated with an experiment that always evaluates to "no".

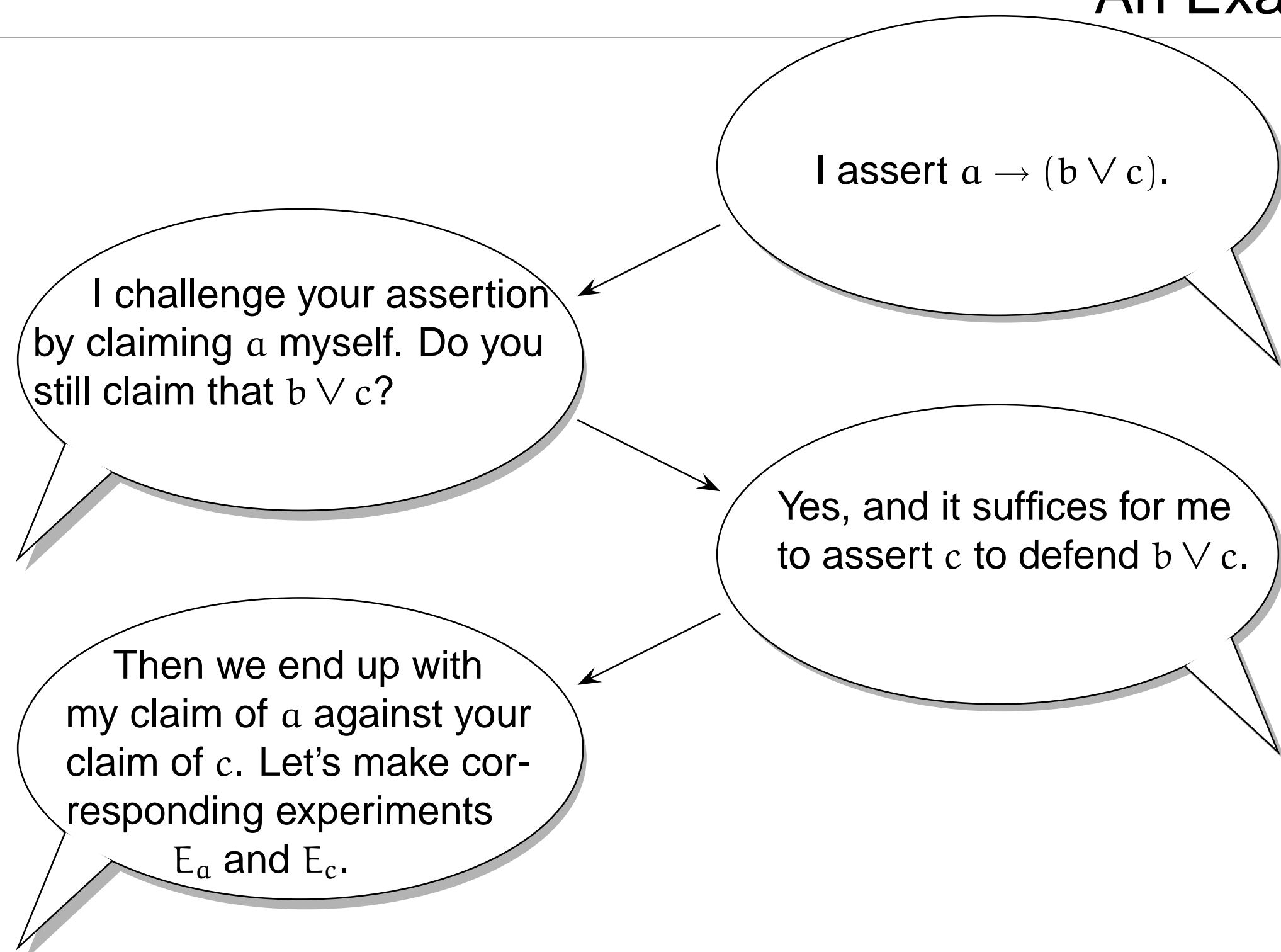
Disjunction: He who asserts $A \vee B$ commits himself to assert either A or B at his own choice.

Conjunction: He who asserts $A \wedge B$ commits himself to assert either A or B at his opponent's choice.

Strong conjunction: He who asserts $A \& B$ commits himself either to assert both A and B or to admit falsity by asserting \perp .

After being attacked, a formula is being deleted from the game.

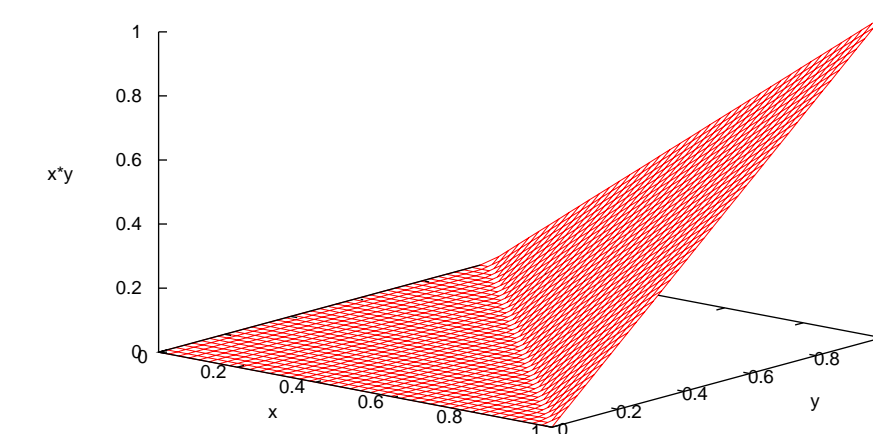
An Example Dialogue



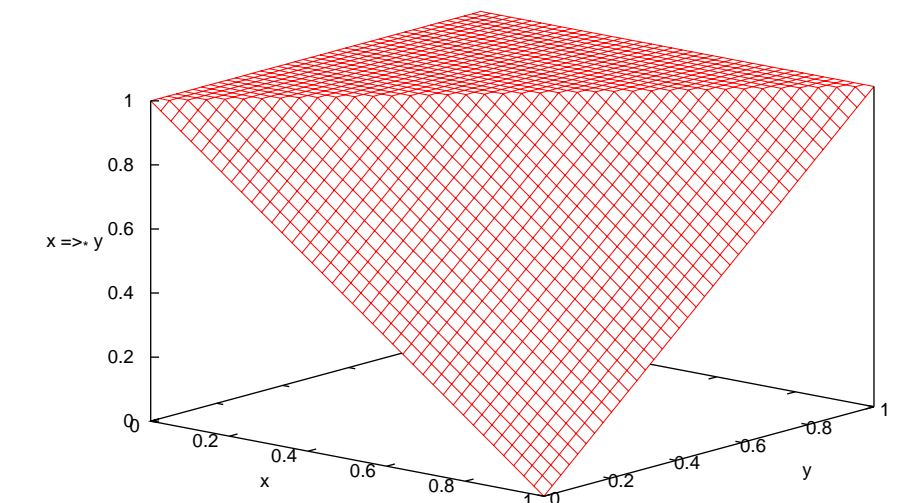
Łukasiewicz Logic

T-Norm Based Fuzzy Logics

- many valued logics: 0 stands for absolute falsity, 1 for truth, but infinitely many intermediate degrees of truth between 0 and 1
- truth function for (strong) conjunction $\&$ is a continuous t-norm
- a t-norm is a commutative, associative function $*$: $[0, 1]^2 \rightarrow [0, 1]$ with unit 1 which is order preserving
- truth function for implication \rightarrow is the residuum of a t-norm
- the residuum \Rightarrow_* of a t-norm $*$ is determined by $x \Rightarrow_* y := \sup\{z \mid x * z \leq y\}$
- other connectives \wedge , \vee , and \neg are derived from $\&$, \rightarrow , and \perp



Łukasiewicz t-Norm $*_L$



Residuum \Rightarrow_L

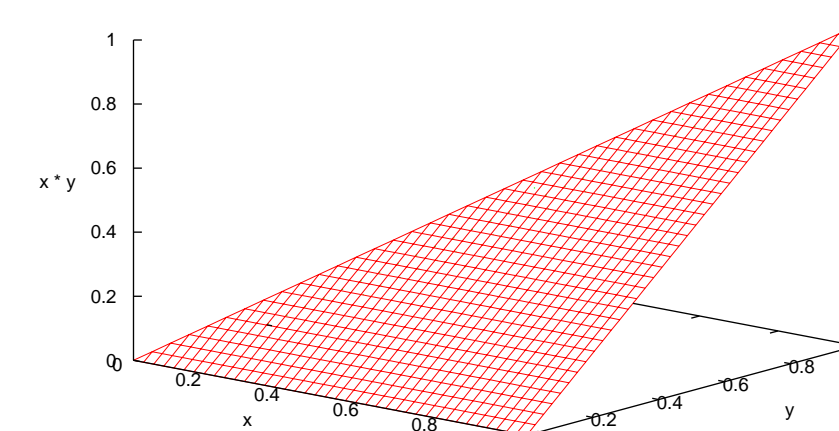
Łukasiewicz Logic

- one of three *fundamental* t-norm based fuzzy logics
- originally J. Łukasiewicz defined a three-valued logic for modelling future contingents, which has later been extended to infinitely many truth values
- Łukasiewicz t-norm: $x *_L y = \max(0, x + y - 1)$
- associated residuum: $x \Rightarrow_L y = \min(1, 1 - x + y)$
- the *unique* fuzzy logic where all truth functions are continuous
- all connectives can be derived from \rightarrow and \perp

Other Fuzzy Logics

Gödel Logic

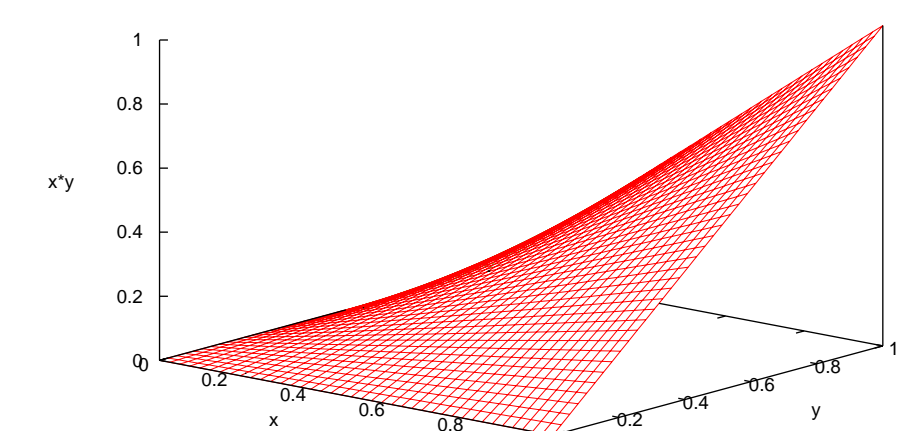
- also known as *Intuitionistic Fuzzy Logic*
- based on the Gödel t-norm $x *_G y = \min(x, y)$
- associated residuum: $x \Rightarrow_G y = y$ if $x > y$, and is 1 otherwise
- only the order of truth values is relevant for evaluating formulas



Gödel t-Norm $*_G$

Product Logic

- introduced in 1996 by Hajek, Godo, and Esteva
- based on the Product t-norm $x *_\Pi y = x \cdot y$
- associated residuum: $x \Rightarrow_\Pi y = y/x$ if $x > y$, and is 1 otherwise



Product t-Norm $*_\Pi$

Adequateness of Giles's Game

For Łukasiewicz Logic

- Already proved by Giles in the 1970s:
- A formula F is valid in Łukasiewicz Logic iff I have a strategy to avoid risk (expected loss) in a game starting with me asserting F for any assignment of probability values to experiments.
- Moreover: given a fixed interpretation, my expected loss of money from asserting a formula in the game directly corresponds to a valuation in Łukasiewicz Logic.

For Gödel & Product Logic

- Variants of Giles's Game presented by Fermüller recently,
- alternative betting schemes: *selecting representatives* (Gödel Logic) and *joint bets* (Product Logic)
- dialogue rule for implication has to be extended as well,
- dialogue rules correspond to the logical rules of an analytic proof system based on relational hypersequents.

Alternative Dialogue Rules

- Presented in this thesis,
- another way to adapt the dialogue rule for implication for Gödel Logic and Product Logic,
- game gets simpler compared to the other approach,
- connection to the hypersequent calculus is lost.

Accompanying Implementation

Webgame

- Web-based application which allows playing Giles's Game interactively,
- simulates evaluation by dispersive experiments.
- see <http://logic.at/people/roschger/thesis/webgame>

Giles

- Small Haskell-program to display game trees of Giles's game,
- given a formula, computes a game tree of the corresponding game and outputs the tree as a dot-Graph specification.

Hypseq

- Utility to find derivations of hypersequents in the relational hypersequent calculus rH ,
- computes all possible derivations and outputs the one with the smallest height.

TCGame

- Utility to find a winning strategy for the proponent P in a Truth Comparison Game,
- for Gödel Logic,
- winning strategy for P can be seen as a proof of the starting formula.