Bridges Between Contextual Linguistic Models of Vagueness and T-norm Based Fuzzy Logic

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September 2009

Abstract

Linguistic models of vagueness usually record contexts of possible precisifications. A link between such models and fuzzy logic is established by extracting fuzzy sets from context based word meanings and analyzing standard logical connectives in this setting. In a further step Lawry's voting semantics for fuzzy logics is used to re-interpret standard t-norm based truth functions from the point of view of context update semantics.

1 Introduction

Vagueness is a significant and ubiquitous phenomenon of human communication. Adequate models of reasoning with vague information are not only of perennial interest to philosophers and logicians (see, e.g., [15, 14, 31, 5, 28] and references there), but are also a topic of current linguistic research. Of particular interest from a logical point of view are approaches to formal semantics of natural language that can be traced back to Richard Montague's ground breaking work, firmly connecting formal logic and linguistics (see, e.g., [24, 12]).

At a first glimpse it seems that most linguistic models of vagueness are *incompatible* with the degree based approach offered by fuzzy logic. In particular, there are indeed good reasons why we should not simply replace Montague's type $\mathbf{t} = \{0, 1\}$ for sentences, i.e. the classical truth values *false* and *true*, by the unit interval $[0, 1]$ if we aim at a realistic and adequate model of meaning in natural language. What is rather needed, as is made clear e.g. in [25, 3, 1, 16, 18], are models that systematically take into account contexts of utterance that record relevant possible precisifications of vague word meanings. Our aim is to bridge the seemingly wide gap between such linguistic models and fuzzy logic by demonstrating how *fuzzy sets* can be systematically *extracted* from the meaning of predicates in a given context. To make this concrete we will refer to a specific linguistic framework—dynamic context semantics—as used by Chris Barker [1] for the analysis of vagueness. Building on this connection between contexts and fuzzy sets we will also investigate how the truth functional approach of fuzzy logic can be justified under certain conditions. Again, we will refer to a specific example, namely Lawry's [19] voting semantics, to illustrate how a corresponding re-interpretation of logical operators could look like.

2 Linguistic approaches to vagueness

Linguists, like logicians, often focus on *predicates* and *predicate modifiers* in modeling the semantics of vague language. It is impossible to provide a survey on the relevant literature that does justice to all linguistic approaches to vagueness in short space.[1] For our purpose it suffices to note that there seems to be wide agreement that adequate truth conditions for vague sentences have to refer not only to fixed lexical entries, but also to *contexts of utterance* that may be identified with sets of contextually relevant possible *precisifications*. Indeed, many authors take it for granted that a realistic and complete formal semantics of natural languages has to take into account the context dependence of truth conditions, anyway, e.g., to be able to resolve ambiguities and to handle anaphora. However, some care has to be taken, since 'context' can mean different things here that may operate on different levels. For example, it is obviously relevant to know, whether in applying the adjective *tall* the reference is to trees in a forest, to basket ball players, to women in central Europe, to school kids, or to *a tall story*. But even if, say, it is clear that the *general context* of asserting *Jana is tall* is a discussion about my students and not about basket players, arguably something like Lewis's *conversational score* [20] (cf. also [28]) is needed in addition to understand whether *Jana is tall* is meant to communicate information about Jana's height to someone who doesn't know her or whether speaker and hearer both have precise common information about Jana's height and the speaker intends to establish a standard of tallness by making this utterance. Reference to such 'conversational contexts of possible precisifications' is convincingly argued to be an essential ingredient of adequate models of communication with vague notions and propositions (see, e.g., [25, 3, 1, 16, 28]).

Instead of surveying the mentioned arguments, we will illustrate the versatile use of contexts in formal semantics by outlining just one particular, rather recent approach, due to Chris Barker [1]. This will serve as motivation and bridgehead—to stick with the metaphor in the title of this contribution—for exploring connections to fuzzy logic in the following sections. Barker casts his analysis of various linguistic features of vagueness in terms of so-called *dynamic semantics* (see [10]), that has been successfully employed to handle, e.g., anaphora. In this approach the meaning $[\![\phi]\!]$ of a declarative sentence (propositional expression) ϕ is given by an *update function* operating on the set of

[1] For this we refer to the handbook article [26], but also to the classic monograph [25], the more recent papers [1, 16, 18] and the references there.

contexts. As already indicated above, semantic theories differ in their intended meaning and formal manifestation of the notion of contexts. Barker [1], following Stalnaker [29], identifies a context with a set of 'worlds', where in each world the extension of all relevant predicates with respect to the actual universe of discourse is *completely precisified*; i.e., each (relevant) atomic proposition is either true or false in a given world. For gradable adjectives these precisifications are specified by a *delineation* δ that, for each world, maps every gradable adjective—or more precisely: every reference to the meaning of a gradable adjective—into a particular value or degree of a corresponding scale. These values represent local standards of acceptance. For instance, if $\delta(c)$ is the delineation function associated with world c, then $d = \delta(c)({}^{\uparrow}[\![tall]\!])$ yields the standard of tallness in c expressed, say, in cm; i.e. every individual that is at least d cm tall in c will be accepted as *tall* in c.

In fact, only a simple form of update functions is needed; namely *filters*, where $[\![\phi]\!](C) \subseteq C$ holds for all contexts C—the result $[\![\phi]\!](C)$ being the set of worlds in C that survive the update of C with the assertion that ϕ. This observation entails that dynamic semantics is just a notational variant of a more traditional specification of 'truth at a world': ϕ is true (accepted) at c if $[\![\phi]\!](\{c\}) = \{c\}$ and ϕ is false (rejected) at c if $[\![\phi]\!](\{c\}) = \{\}$. Moreover, we assume that every world c of a given context C refers to the *same domain* (relevant universe of discourse) D_C.

Gradable predicates, like *tall*, express a relation involving degrees and individuals. The denotation of *tall* is modeled by a function tall such that $\mathsf{tall}(d, \mathsf{a})$ returns the set of worlds in which the individual a is at least d cm tall. Accordingly Barker presents the (dynamic) *meaning* of *tall* by[2]

$$[\![tall]\!] \quad =_{df} \quad \lambda x \lambda C.\{c \in C : c \in \mathsf{tall}(\delta(c)({}^{\uparrow}[\![tall]\!]), x)\}$$

Among other features, this semantic setup allows Barker to capture the intuitive difference in the meaning of the modifiers *very*, *definitely*, and *clearly*. To define $[\![very]\!]$ an underlying relation very over degrees is used, such that $\mathsf{very}(s, d, d')$ holds iff the difference between d and d' is larger than the (vague, i.e., world dependent) standard s:

$$\begin{aligned}[\![very]\!] \quad =_{df} \quad \lambda\alpha\lambda x\lambda C.\{c \in \alpha(x)(C) : \exists d(c[d/\alpha] \in \alpha(x)(C) \wedge \\ \mathsf{very}(\delta(c)({}^{\uparrow}[\![very]\!]), \delta(c)({}^{\uparrow}\alpha), d)\}\end{aligned}$$

where $c[d/\alpha]$ denotes a world that is like c, except for setting $\delta(c)({}^{\uparrow}\alpha) = d$. E.g., in $c[185cm/[\![tall]\!]]$ the standard of tallness is $185cm$. Thus $[\![Ann\ is\ very\ tall]\!] = ([\![very]\!]([\![tall]\!]))(\mathsf{Ann})$ is a filter (update) that is survived by exactly those worlds of a given context where *Ann* exceeds the standard of tallness by at least some amount s. This amount s not only depends on the meaning of *tall* and *very*, but also on the world itself. Thus the vagueness of *very* is modeled by a twofold context dependence: the meaning of *very* may obviously vary from context to

[2] In fact Barker does not distinguish between $[\![tall]\!]$ and the purely indexical reference ${}^{\uparrow}[\![tall]\!]$ to it. Our notation is meant to indicate that the circularity is of a harmless type.

context, but even within a fixed context different worlds may have different standards of accepting that an individual is *very tall*, granted that it is *tall*.

Note that, on the level of an individual world c, the update function for *very* refers only to information pertaining to c. In contrast, Barker suggests to model *definitely* as a type of modal operator:

$$[\![\mathit{definitely}]\!] =_{\mathit{df}} \lambda\alpha\lambda x\lambda C.\{c \in \alpha(x)(C) : \forall d(c[d/\alpha] \in C \rightarrow c[d/\alpha] \in \alpha(x)(C))\}$$

This means that a world $c \in C$ survives the update with $[\![\mathit{Ann\ is\ definitely\ tall}]\!]$ iff all worlds in C in which Ann has the same height as in c judge Ann as tall according to their local standard.[3]

Finally, essential elements of $[\![\mathit{very}]\!]$ and $[\![\mathit{definitely}]\!]$ are combined in the following suggestion for the meaning of *clearly*:[4]

$$[\![\mathit{clearly}]\!] \quad =_{\mathit{df}} \quad \lambda\alpha\lambda x\lambda C.\{c \in \alpha(x)(C) : \mathsf{very}(\delta(c)(^{\uparrow}[\![\mathit{clearly}]\!]), \max\nolimits_\alpha, \max\nolimits_C)\}$$

where $\max_\alpha = \{d : c[d/\alpha] \in \alpha(x)\}$ and $\max_C = \{d : c[d/\alpha] \in C\}$. The reference to $[\![\mathit{clearly}]\!]$ in the first argument of the relation very entails that, while the same comparison relation is used, the (world dependent) amount that the difference between the second and the third value has to exceed, may be different for *clearly* and *very*, respectively. However the essential difference between $[\![\mathit{very}]\!]$ and $[\![\mathit{clearly}]\!]$ is another one: while for *very tall* one compares the local standard of tallness with the local value for an individuals' height in each world, *clearly tall* involves a comparison of the highest standard of tallness in the whole context with the maximal height that the individual may have according to any world of the context.

3 Extracting fuzzy sets from contexts

Our main pillar in building a bridge between linguistics and fuzzy logics consists in connecting the meaning of predicates like *tall* with fuzzy sets. We define logical operators *and*, *or*, and *not* directly on predicates[5] in a natural way and explore how they relate to the corresponding operations on fuzzy sets. Note that linguists may seek to preserve the difference between statements like *Jana is tall and clever* and *Jana is tall and Jana is clever*, respectively. However, it will be straightforward to lift our analysis of predicate operators to the propositional level.

We introduce the notion of an *element filter*. These are filters parametrized by a domain element. Element filters that we have already encountered are e.g. $[\![\mathit{tall}]\!]$ but also $[\![\mathit{very}]\!]([\![\mathit{tall}]\!])$, where for a domain element x both $[\![\mathit{tall}]\!](x)$ and $([\![\mathit{very}]\!]([\![\mathit{tall}]\!]))(x)$ are filters.

[3] Note that there might be uncertainty about Ann's height. I.e., Ann may have different heights in different worlds. Therefore *definitely tall* is not just equivalent to '*tall* in all worlds of the context'.

[4] Our version of $[\![\mathit{clearly}]\!]$ differs in inessential details from Barker's in [1].

[5] For brevity we focus on monadic predicates, but the concepts can easily be extended to relations of higher arity.

Given a context C we can extract a fuzzy set from the meaning $\alpha = [\![A]\!]$ of a predicate A by applying for each domain element x the filter $\alpha(x)$ to C and measuring the amount of surviving worlds of C. For simplicity we stipulate contexts to be finite sets of worlds and identify fuzzy sets with their membership functions to obtain the following:

Definition 1. *Let C be a context with domain D_C and α an element filter. Then the fuzzy set $[\alpha]_C$ is given by*

$$[\alpha]_C : D_C \to [0,1] : \quad x \mapsto \frac{|\alpha(x)(C)|}{|C|}$$

Note that the collection of fuzzy sets $[\alpha]_C$ for all relevant element filters α carries less information that C itself. This will get apparent when we compare logical operators defined on predicates with corresponding operations on fuzzy sets.

Extending the framework of Barker, we model compound predicates (like *tall and clever*), built up from logically simpler predicates (*tall*, *clever*), in a straightforward manner:

Definition 2.

- $[\![\mathit{and}]\!] =_{df} \lambda\alpha\lambda\beta\lambda x\lambda C.\alpha(x)(C) \cap \beta(x)(C)$
- $[\![\mathit{or}]\!] =_{df} \lambda\alpha\lambda\beta\lambda x\lambda C.\alpha(x)(C) \cup \beta(x)(C)$[6]
- $[\![\mathit{not}]\!] =_{df} \lambda\alpha\lambda\beta\lambda x\lambda C.C\backslash(\alpha(x)(C))$

Note that in the above definition $\alpha = [\![A]\!]$ and $\beta = [\![B]\!]$ are element filters representing the meaning of the predicates A and B, respectively. Using the usual infix notation, $[\![A\, \mathit{and}\, B]\!]$ is an element filter as well. In general, applying $[\![A\, \mathit{and}\, B]\!]$ is not equivalent to applying the element filters $[\![A]\!]$ and $[\![B]\!]$ consecutively. We may additionally define

- $[\![\mathit{and}^*]\!]$[7] $=_{df} \lambda\alpha\lambda\beta\lambda x\lambda C.[\![B]\!](x)([\![A]\!](x)(C))$

Then $[\![A\ \mathit{and}^*\ B]\!]$ is, in general, different from $[\![A\, \mathit{and}\, B]\!]$ (and from $[\![B\ \mathit{and}^*\ A]\!]$).

The membership degree of x in the fuzzy set $[A\, \mathit{and}\, B]_C$[8] is determined by applying the filter $[\![A\, \mathit{and}\, B]\!](x)$ to the context C and calculating the fraction of worlds in C that survive this update. Proceeding a step further on our bridge from linguistics to fuzzy logics, the question arises if we can determine $[A\, \mathit{and}\, B]_C(x)$ from the membership degrees $[A]_C(x)$ and $[B]_C(x)$ alone. This, of course, would give us a fully truth-functional semantics for *and*, *or*, and *not*. However, fuzzy sets abstract away from the internal structure of contexts that

[6] In natural language one can also find *exclusive* disjunction, e.g. *Jana is either tall or clever (but not both)*. He we focus on *inclusive* disjunction as this directly corresponds to disjunction as it is normally used in logics.

[7] Arguably, *and** corresponds to certain uses of *and even* and of *but*, respectively.

[8] For the sake of readability we write $[X]_C$ instead of $[[\![X]\!]]_C$.

may show various possible dependencies of worlds. We illustrate this by the following example.

Let C be a context consisting of the five possible worlds c_1 to c_5 as in Table 1. Furthermore, let $[\![jana]\!] = \mathsf{j}$ be a domain element and let tall, clever, and heavy be the denotations of the unary predicates *tall*, *clever*, and *heavy*, respectively, just as already demonstrated for tall and *tall* in Section 2.

c	$\delta(c)(^{\uparrow}[\![tall]\!])$	$\text{maxd}^{\mathsf{j}}_{\uparrow[\![tall]\!]}$	$\delta(c)(^{\uparrow}[\![clever]\!])$	$\text{maxd}^{\mathsf{j}}_{\uparrow[\![clever]\!]}$	$\delta(c)(^{\uparrow}[\![heavy]\!])$	$\text{maxd}^{\mathsf{j}}_{\uparrow[\![heavy]\!]}$
c_1	170	175	100	105	80	75
c_2	160	170	120	125	75	70
c_3	170	180	100	95	90	100
c_4	180	175	105	100	85	75
c_5	170	165	110	115	70	65

with maxd^x_p denoting the maximum degree to which to individual x fulfills the predicate referenced by p.

Table 1: Example Context C

Then $[\![heavy]\!]$ is an element filter with $[\![heavy]\!](\mathsf{j})(C) = \{c_3\}$. Accordingly, $[heavy]_C(\mathsf{j}) = 1/5$. Likewise we have $[clever]_C(\mathsf{j}) = [tall]_C(\mathsf{j}) = 3/5$. Since these latter are equal, also the membership degrees of j in the fuzzy sets $[tall\ and\ heavy]_C$ and $[clever\ and\ heavy]_C$. respectively, had to be equal if the (context update) meaning of *and* were truth functional. But $[\![tall\ and\ heavy]\!](\mathsf{j})(C) = \{c_3\}$, thus $[tall\ and\ heavy]_C(\mathsf{j}) = 1/5$, while $[clever\ and\ heavy]_C(\mathsf{j}) = 0$. As we see, by extracting the three fuzzy sets from the corresponding element filters we lose the information about the specific overlap of the corresponding updates in the given context.

The following bounds encode our best knowledge about membership degrees for fuzzy sets extracted from to composite predicates with respect to membership degrees referring to the corresponding components.

Theorem 1. *Let C be a context, $d \in D_C$, and let $\alpha = [\![A]\!]$ and $\beta = [\![B]\!]$ be two element filters. Then the following bounds are tight:*

- $\max\{0, [\alpha]_C(d) + [\beta]_C(d) - 1\} \leq [A\ and\ B]_C(d) \leq \min\{[\alpha]_C(d), [\beta]_C(d)\}$
- $\max\{[\alpha]_C(d), [\beta]_C(d)\} \leq [A\ or\ B]_C(d) \leq \min\{1, [\alpha]_C(d) + [\beta]_C(d)\}$
- $[not\ A]_C(d) = 1 - [\alpha]_C(d)$

Proof. The value $1 - [\alpha]_C(d)$ for negation follows directly from the relevant definitions.

For conjunction and disjunction we focus on the extremal cases: the sets $\alpha(d)(C)$ and $\beta(d)(C)$ may either be 'as disjoint as possible' or one set may contain the other one. In the latter case we have $\min\{[\alpha]_C(d), [\beta]_C(d)\}$ as a tight upper bound for conjunction, but also as a tight lower bound for disjunction.

Now assume that both sets are as disjoint as possible. We distinguish:

Case 1. $[\alpha]_C(d) + [\beta]_C(d) \leq 1$: Then $\alpha(d)(C) \cap \beta(d)(C) = \{\}$, thus $[A\ and\ B]_C(d) = 0$ and $[A\ or\ B]_C(d) = [\alpha]_C(d) + [\beta]_C(d)$.

Case 2. $[\alpha]_C(d)+[\beta]_C(d) > 1$: Then $\alpha(d)(C)\cap\beta(d)(C) \neq \{\}$. As we assume the sets to be as disjoint as possible, their intersection is as small as possible; therefore $|\alpha(d)(C) \cap \beta(d)(C)| = [\alpha]_C(d)+[\beta]_C(d)-1$, and $\alpha(d)(C)\cup\beta(d)(C) = 1$

Combining the cases yields the specified bounds. □

Remark. Note that $*_\mathrm{G} = \min$ and $\bar{*}_\mathrm{G} = \max$ are the Gödel t-norm and co-t-norm, respectively. Moreover, $*_\mathrm{L} = \lambda x, y.\max\{0, x + y - 1\}$ and $\bar{*}_\mathrm{L} = \lambda x, y.\min\{1, x + y\}$ are the Łukasiewicz t-norm and co-t-norm, respectively. In other words, Theorem 1 shows that the truth functions of (strong) conjunction and (strong) disjunction in Gödel and Łukasiewicz logic (see [11]) correspond to opposite extremal cases of context based evaluations of conjunction and disjunction.

The above analysis on logical predicate operators can easily be lifted to the propositional level. For a sentence like *Jana is tall* its meaning $[\![\textit{Jana is tall}]\!]$ is a filter (rather than an element filter). Usual logical connectives on propositions can be defined in analogy to Definition 2:

Definition 3.

- $[\![\phi \wedge \psi]\!] =_{df} \lambda C.[\![\phi]\!](C) \cap [\![\psi]\!](C)$
- $[\![\phi \vee \psi]\!] =_{df} \lambda C.[\![\phi]\!](C) \cup [\![\psi]\!](C)$
- $[\![\neg\phi]\!] =_{df} \lambda C.C\backslash[\![\phi]\!](C)$

In the following the set of all propositions formed in this way is called Prop.

Similarly to the predicate level, we can associate a 'degree of truth' $\|\phi\|_C$ for every $\phi \in$ Prop by applying the filter $[\![\phi]\!]$ to context C:

$$\|\phi\|_C =_{df} \frac{|[\![\phi]\!](C)|}{|C|}.$$

In other words we identify the degree of truth of ϕ in a context C with the fraction of worlds in C that survive the update with the filter $[\![\phi]\!]$. E.g., returning to the context C specified in the example following Definition 2, *Jana is tall* is true to degree 3/5 in C since three out of five worlds in C classify *Jana*'s height as above the relevant local standard of tallness.

Once more we note that contexts allow to model specific constraints on the worlds (i.e. contextually relevant possible precisifications) of which they consist. Therefore, in general, there are no truth functions that determine $\|\phi \wedge \psi\|_C$ and $\|\phi \vee \psi\|_C$ in terms of $\|\phi\|_C$ and $\|\psi\|_C$ alone. However the optimal bounds of Theorem 1 also apply at the level of sentences:

- $*_\mathrm{L}(\|\phi\|_C, \|\psi\|_C) \leq \|\phi \wedge \psi\|_C \leq *_\mathrm{G}(\|\phi\|_C, \|\psi\|_C)$, and
- $\bar{*}_\mathrm{G}(\|\phi\|_C, \|\psi\|_C) \leq \|\phi \vee \psi\|_C \leq \bar{*}_\mathrm{L}(\|\phi\|_C, \|\psi\|_C)$,

where $*_\mathrm{G}(\bar{*}_\mathrm{G})$ and $*_\mathrm{L}(\bar{*}_\mathrm{L})$ are the Gödel and Łukasiewicz t-norms (co-t-norms), respectively.

4 Translating voting semantics to contexts

As we have seen in Section 3, the context based semantics of logical connectives is more fine grained than any specification by some particular truth function over degrees. The fraction of worlds surviving an update with $[\![\phi \wedge \psi]\!]$ is not determined by the fractions of worlds surviving the filters $[\![\phi]\!]$ and $[\![\psi]\!]$, respectively: t-norm based truth functions provide optimal bounds, but in general the internal structure of contexts determines the corresponding fractions of worlds surviving updates with logically complex propositions. The following question arises: can one constrain and/or modify the structure of contexts in a manner that leads to standard fuzzy truth functions at the level of such contexts. For a positive answer we rely on an analogy between Lawry's *voting semantics* [19] and our (or rather Barker's) version of contextual semantics.

To explain the assignment of truth values $\in [0, 1]$ to a statement ϕ Lawry [19], but also many other reseachers (e.g., [7, 13]) suggest to consider the following scenario. Ask each of N agents whether she accepts the statement ϕ. It is assumed that the agents are all competent speakers of the respective language and are fully informed about the relevant facts. Therefore they will all agree on whether ϕ is to be accepted or to be rejected if ϕ is a precise statement. However, if ϕ is vague[9] then they may diverge on their judgements in spite of their linguistic competence and factual knowledge. In this setting one assigns the 'truth value' $v = n/N$ to ϕ, where n is the number of agents that accept ϕ.

Let us write $a_s(\phi) = 1$ if agent s accepts ϕ and $a_s(\phi) = 0$ otherwise. If the agents have to satisfy the following consistency conditions

$$\begin{aligned} a_s(\phi \wedge \psi) = 1 &\iff a_s(\phi) = 1 \text{ and } a_s(\psi) = 1 \\ a_s(\phi \vee \psi) = 1 &\iff a_s(\phi) = 1 \text{ or } a_s(\psi) = 1 \\ a_s(\neg\phi) = 1 &\iff a_s(\phi) = 0 \end{aligned}$$

then the resulting global 'fuzzy truth value assignment' turns out to be simply a probability function (see, e.g., [21]) and therefore does not justify a truth functional semantics of fuzzy logic if the agents' votes are independent. However, if we require that the agent's voting behaviour is determined by an associated 'degree of scepticism' in a particular way, than usual fuzzy truth functions emerge.

Definition 4. *A family of functions $a_\sigma : \mathrm{Prop} \mapsto \{0, 1\}$, where $\sigma \in [0, 1]$ is called a* scepticism degree based voting behaviour *if the following conditions hold:*

$$\begin{aligned} &\textit{if } \sigma \leq \sigma' \textit{ and } a_\sigma(\phi) = 0 \textit{ then } a_{\sigma'}(\phi) = 0 \\ a_\sigma(\phi \wedge \psi) = 1 &\iff a_\sigma(\phi) = 1 \textit{ and } a_\sigma(\psi) = 1 \\ a_\sigma(\phi \vee \psi) = 1 &\iff a_\sigma(\phi) = 1 \textit{ or } a_\sigma(\psi) = 1 \\ a_\sigma(\neg\phi) = 1 &\iff a_{1-\sigma}(\phi) = 0 \end{aligned}$$

The intended interpretation of the *scepticism degree* σ is the level of willingness to assert a positive statement. The first condition means that an agent

[9] We deliberately focus on vagueness and ignore other forms of indeterminateness and uncertainty here.

rejects at least all those propositions that are rejected by less skeptic agents. The condition for negated statements implies that an agent with a high degree of scepticism is willing to accept $\neg\phi$ whenever an agent with inverted (low) degree of scepticism is willing to reject ϕ. This implies that, in general, agents do not evaluate classically: we may have $a_\sigma(\phi \vee \neg\phi) = 0$ but also $a_\sigma(\phi \wedge \neg\phi) = 1$; only $a_{0.5}$ is always a classic valuation. To obtain a (global) fuzzy valuation from such families of (local) para-consistent $\{0,1\}$-valuations, we have to measure 'amounts of acceptance'.

Definition 5. *Let $\Lambda = \{a_\sigma : \sigma \in [0,1]\}$ be a scepticism degree based voting behaviour and let μ be a measure on the Borel subsets of $[0,1]$. Then the corresponding fuzzy truth value assignment is defined by*

$$v^\mu_A(\phi) = \mu\{\sigma \in [0,1] : a_\sigma(\phi) = 1\}$$

Proposition 1. ([19]) *For all scepticism degree based voting behaviours Λ and measures μ, as above, we have:*

$$\begin{aligned} v^\mu_A(\phi \wedge \psi) &= \min(v^\mu_A(\phi), v^\mu_A(\psi)) \\ v^\mu_A(\phi \vee \psi) &= \max(v^\mu_A(\phi), v^\mu_A(\psi)) \end{aligned}$$

Moreover, if μ is symmetric, i.e. if $\mu[a,b] = \mu[1-b, 1-a]$ for $0 \leq a \leq b \leq 1$, then

$$v^\mu_A(\neg\phi) = 1 - v^\mu_A(\phi)$$

How does this relate to contextual dynamic semantics? The most obvious transfer of voting semantics to contexts is to associate with each world a value that directly corresponds to the scepticism degree of an agent and to evaluate logically complex statements as specified above. But remember that this entails that local evaluations violate either the law of excluded middle ($\phi \vee \neg\phi$) or the law of contradiction ($\neg(\phi \wedge \neg\phi)$) in general. Of course, a world c of a context C is something different than a voter among many voting agents. But c can be viewed as a *local semantic test*: it specifies for each sentence ϕ whether ϕ holds according to certain precisified standards or not. It does not seem to be unnatural to compare these semantic tests with respect to their *strictness* in analogy to the comparison of agents with respect to degrees of scepticism. Moreover, considering the intended application of contextual semantics, we may assume that only one or at most a few directly related predicates are relevant in a given context. Also the domain of any particular context can realistically be assumed to be small. This makes it plausible that worlds of a context may often be characterized solely by their *degree of strictness*. Let us illustrate this by an example from natural language. A realistic context for evaluating

(1) *Jana is tall*

might be represented by worlds (i.e. precisifications) that agree on Jana's actual height (say 178cm) but differ in their standards of accepting 178cm as being above the local standard of tallness. Obviously we can then define linearly ordered degrees of strictness induced by increasing standards of tallness for the worlds of such a context. A similar observation holds for

(2) *The weather is cold today*

Again, we have no troubles to extract degrees of strictness corresponding to decreasing threshold values (temperatures) for accepting (2). Using our re-interpretation of voting semantics we can extract truth values $\in [0,1]$ for (1) and for (2) in the respective contexts indicated above. In contrast, one might argue that there simply is *no natural context* in which the *conjunction* of (1) and (2) has to evaluated, which nicely fits our model.

While the above remarks may be sufficient to justify the focus on contexts with associated linearly ordered degrees of strictness, the fact that the translation of voting semantics to contexts calls for 'non-classical worlds' seems to be more problematic. However we claim that this is compatible with Barker's context based model [1], as introduced in Section 2. Note that Barker does not provide a semantics for logical connectives. Only non-compound vague predicates and vagueness-related predicate modifiers are investigated. While, following voting semantics, one can straightforwardly generalize to include *conjunctions* and *disjunctions* at the local level of individual worlds, *negation* is viewed in this model as an inherently global operator, which only receives meaning at the level of whole contexts.

5 Summary and outlook

We started by noting the fact that linguists usually analyze the semantics of vague words by reference to contexts of utterance that register relevant possible precisifications. This seems to be at variance with the degree based approach to vagueness suggested by fuzzy logic. However, taking Barker's [1] version of dynamic (update) semantics as a concrete point of reference, we have demonstrated that fuzzy sets can be associated in a systematic manner with contexts and corresponding filters as used in Barker's model. While the structure of context filters used to specify the different meanings of modifiers like *very*, *definitely*, and *clearly* allows to take into account information that is abstracted away in corresponding fuzzy sets, standard t-norm based operators faithfully register the extremal cases that may result from applying logical connectives to vague predicates and sentences.

While it is rather straightforward to identify intermediate truth values with the fraction of worlds in a given context that survive certain updates codifying the meaning of vague expressions, it is not clear how one might derive specific truth functions in such a setting (beyond providing the indicated bounds). This problem, of course, is just a particular instance of a well known challenge for deductive fuzzy logic: how to justify particular truth functions with respect to more fundamental semantic notions, like votes or arguments for and against accepting a vague assertion. In [23] Jeff Paris provides an overview over semantic frameworks for fuzzy logics that support truth functionality. Here we picked a particular approach, namely so-called voting semantics as suggested by Lawry [19] to illustrate how one might connect context based update semantics with frameworks that model the meaning of logical connectives by particular t-norm based truth functions.

We emphasize that both, Barker's specific update functions over contexts and Lawry's voting semantics, should be understood as just two particular spots on either side of the river dividing linguistics from fuzzy logic, that may be chosen as end points of a bridge crossing that troubled water. On the linguistic side context and precisification based approaches suggested, e.g., by Kennedy [16], Kyburg and Moreau [18], and already earlier by Pinkal [25] and Bosch [3] are certainly worth investigating from this perspective. On the fuzzy logic side we just mention similarity semantics [27, 17, 30], Robin Giles's dialogue and betting game based characterisation of Łukasiewicz logic [9, 8] (extended to other logics in [4, 6]), acceptability semantics [22], rerandomising semantics [13, 11], and approximation semantics [2, 23] as alternative candidates for corresponding bridge heads. We plan to explore at least some of these options in future work. In any case, we hope to have shown already here that constructing such a bridge is neither a futile nor a completely trivial matter.

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