Bridges Between Contextual Linguistic Models of Vagueness and T-norm Based Fuzzy Logic

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Adequate models of reasoning with vague information are not only of perennial interest to philosophers and logicians (see, e.g., [6, 5, 12, 2, 11] and references there), but also in the focus of linguistic research (see, e.g., [10, 7, 1, 8]). Of particular interest from a logical point of view are approaches to formal semantics a natural language that can be traced back to Richard Montague's ground braking work, firmly connecting modern formal logic and linguistics (see, e.g., the handbook chapter [9] and the widely used textbook [4]). At a first glimpse, it seems that all important contemporary linguistic models of vagueness are *incompatible* with the degree based approach offered by fuzzy logic (see, e.g., [13, 14, 3]). E.g., Manfred Pinkal in his frequently cited (and translated) monograph [10] explicitly argues that many-valued, truth functional logics are inadequate for modelling central linguistic phenomena of vagueness and indeterminateness.

More specifically, contemporary linguists seem to agree that a special type of *context dependency* is the key to understand the semantics of vague predicates ('tall', 'nice', 'is a heap', 'enjoys', 'likes', ...), but also of corresponding predicate modifiers ('very', 'definitely', ...) and quantifiers ('most', 'many', 'few', ...). However, a closer look at corresponding recent papers on vagueness, in particular [1, 8, 7], reveals that contexts are primarily used to keep track of varying *standards of assertability* connected with *gradable predicates*. This observation is our starting point in exploring formal bridges concepts from t-norm based fuzzy logic and the cited linguistic models of vagueness.

We will show how *fuzzy sets* and *fuzzy relations* can be systematically extracted from a given context space endowed with a probability measure (or more generally, possibility measure) intended to model the relative salience and plausibility of different contexts (standards). Roughly speaking, the membership degree of an individual **a** (say 'Adam') in a fuzzy set modelling a predicate **T** (say 'is tall') gets identified with the probability — alternatively: degree or possibility or degree of necessity — that **a** satisfies the assertability standard associated with **T** in a ran-

domly chosen context. In this manner t-norm and co-t-norm based operators re-emerge as semantic correlates of conjunction, disjunction, and other logical connectives, if one insists on global evaluations that ignore all dependencies between context specific standards pertaining to different predicates. In contrast, local evaluations, i.e. those referring to individual contexts, lead to an *intensional semantic framework*, also for logical connectives. While an intensional evaluation, based on a specific context space, allows to model phenomena of vague language [1, 8, 7] that escape the coarser truth functional approach of fuzzy logic, the price to be paid for the more fine grained analysis is higher computational complexity. In this respect, t-norm based truth functions can be seen as *efficient extensional approximations* to potentially very complex *intensional evaluations* with respect to context dependent assertability conditions.

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