Atomic Cut-Introduction (and Proof Compression) by Resolution: Preliminary Investigations

Bruno Woltzenlogel Paleo

Theoretische Informatik und Logik
Institut für Computersprachen
Technische Universität Wien

2009, July 2nd

Outline

- Cut-Elimination by Resolution
- Atomic-Cut-Introduction by Resolution

$$
\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(u) \vdash P(u) \qquad Q(u) \vdash Q(u)}{P(u), P(u) \rightarrow Q(u) \vdash Q(u)}\rightarrow_l}{P(u) \rightarrow Q(u) \vdash P(u) \rightarrow Q(u)}\rightarrow_r}{P(u) \rightarrow Q(u) \vdash \exists y (P(u) \rightarrow Q(y))}\exists_r}{\forall x (P(x) \rightarrow Q(x)) \vdash \exists y (P(u) \rightarrow Q(y))}\forall_l}{\forall x (P(x) \rightarrow Q(x)) \vdash \forall x \exists y (P(x) \rightarrow Q(y))}\forall_r
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(a) \vdash P(a) \qquad Q(v) \vdash Q(v)}{P(a), P(a) \rightarrow Q(v) \vdash Q(v)}\rightarrow_l}{P(a) \rightarrow Q(v) \vdash P(a) \rightarrow Q(v)}\rightarrow_r}{P(a) \rightarrow Q(v) \vdash \exists y (P(a) \rightarrow Q(y))}\exists_r}{\exists y (P(a) \rightarrow Q(y)) \vdash \exists y (P(a) \rightarrow Q(y))}\exists_l}{\forall x \exists y (P(x) \rightarrow Q(y)) \vdash \exists y (P(a) \rightarrow Q(y))}\forall_l
}{\forall x (P(x) \rightarrow Q(x)) \vdash \exists y (P(a) \rightarrow Q(y))}\mathit{cut}
$$

$$\dfrac{P(u) \vdash P(u) \qquad Q(u) \vdash Q(u)}{P(u), P(u) \to Q(u) \vdash Q(u)}\ \to_l$$

$$\dfrac{P(u), P(u) \to Q(u) \vdash Q(u)}{P(u) \to Q(u) \vdash P(u) \to Q(u)}\ \to_r$$

$$\dfrac{P(u) \to Q(u) \vdash P(u) \to Q(u)}{P(u) \to Q(u) \vdash \exists y(P(u) \to Q(y))}\ \exists_r$$

$$\dfrac{P(u) \to Q(u) \vdash \exists y(P(u) \to Q(y))}{\forall x(P(x) \to Q(x)) \vdash \exists y(P(u) \to Q(y))}\ \forall_l$$

$$\dfrac{\forall x(P(x) \to Q(x)) \vdash \exists y(P(u) \to Q(y))}{\forall x(P(x) \to Q(x)) \vdash \forall x \exists y(P(x) \to Q(y))}\ \forall_r$$

$$\dfrac{P(a) \vdash P(a) \qquad Q(v) \vdash Q(v)}{P(a), P(a) \to Q(v) \vdash Q(v)}\ \to_l$$

$$\dfrac{P(a), P(a) \to Q(v) \vdash Q(v)}{P(a) \to Q(v) \vdash P(a) \to Q(v)}\ \to_r$$

$$\dfrac{P(a) \to Q(v) \vdash P(a) \to Q(v)}{P(a) \to Q(v) \vdash \exists y(P(a) \to Q(y))}\ \exists_r$$

$$\dfrac{P(a) \to Q(v) \vdash \exists y(P(a) \to Q(y))}{\exists y(P(a) \to Q(y)) \vdash \exists y(P(a) \to Q(y))}\ \exists_l$$

$$\dfrac{\exists y(P(a) \to Q(y)) \vdash \exists y(P(a) \to Q(y))}{\forall x \exists y(P(x) \to Q(y)) \vdash \exists y(P(a) \to Q(y))}\ \forall_l$$

$$\dfrac{\forall x(P(x) \to Q(x)) \vdash \forall x \exists y(P(x) \to Q(y)) \qquad \forall x \exists y(P(x) \to Q(y)) \vdash \exists y(P(a) \to Q(y))}{\forall x(P(x) \to Q(x)) \vdash \exists y(P(a) \to Q(y))}\ cut$$

$$\mathcal{S}_\varphi \equiv (\neg P(u) \otimes Q(u)) \oplus (P(a) \oplus \neg Q(v))$$

$$\mathcal{C}_\varphi^W \equiv \{P(u) \vdash Q(u) \ ; \ \vdash P(a) \ ; \ Q(v) \vdash\}$$

$$\dfrac{\dfrac{\vdash P(a) \qquad P(u) \vdash Q(u)}{\vdash Q(a)}\ R \qquad Q(v) \vdash}{\vdash}\ R$$

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{P(u) \vdash P(u) \qquad Q(u) \vdash Q(u)}{P(u), P(u) \to Q(u) \vdash Q(u)}\;{\to_l}
}{P(u) \to Q(u) \vdash P(u) \to Q(u)}\;{\to_r}
}{P(u) \to Q(u) \vdash \exists y(P(u) \to Q(y))}\;{\exists_r}
}{\forall x(P(x) \to Q(x)) \vdash \exists y(P(u) \to Q(y))}\;{\forall_l}
}{\forall x(P(x) \to Q(x)) \vdash \forall x \exists y(P(x) \to Q(y))}\;{\forall_r}
\qquad
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{P(a) \vdash P(a) \qquad Q(v) \vdash Q(v)}{P(a), P(a) \to Q(v) \vdash Q(v)}\;{\to_l}
}{P(a) \to Q(v) \vdash P(a) \to Q(v)}\;{\to_r}
}{P(a) \to Q(v) \vdash \exists y(P(a) \to Q(y))}\;{\exists_r}
}{\exists y(P(a) \to Q(y)) \vdash \exists y(P(a) \to Q(y))}\;{\exists_l}
}{\forall x \exists y(P(x) \to Q(y)) \vdash \exists y(P(a) \to Q(y))}\;{\forall_l}
}{\forall x(P(x) \to Q(x)) \vdash \exists y(P(a) \to Q(y))}\;\mathit{cut}
$$

$$
\dfrac{
\dfrac{\vdash P(a) \qquad P(u) \vdash Q(u)}{\vdash Q(a)}\;R
\qquad Q(v) \vdash
}{\vdash}\;R
$$

$\lfloor \varphi \rfloor^{O}_{\vdash P(a)}$:

$$
\dfrac{
\dfrac{
\dfrac{P(a) \vdash P(a)}{P(a) \vdash P(a), Q(v)}\;w_r
}{\vdash P(a), P(a) \to Q(v)}\;{\to_r}
}{\vdash P(a), \exists y(P(a) \to Q(y))}\;{\exists_r}
$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(u)\vdash P(u) \qquad Q(u)\vdash Q(u)}{P(u), P(u)\to Q(u)\vdash Q(u)}\to_l}{P(u)\to Q(u)\vdash P(u)\to Q(u)}\to_r}{P(u)\to Q(u)\vdash \exists y(P(u)\to Q(y))}\exists_r}{\forall x(P(x)\to Q(x))\vdash \exists y(P(u)\to Q(y))}\forall_l}{\forall x(P(x)\to Q(x))\vdash \forall x\exists y(P(x)\to Q(y))}\forall_r \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(a)\vdash P(a) \qquad Q(v)\vdash Q(v)}{P(a), P(a)\to Q(v)\vdash Q(v)}\to_l}{P(a)\to Q(v)\vdash P(a)\to Q(v)}\to_r}{P(a)\to Q(v)\vdash \exists y(P(a)\to Q(y))}\exists_r}{\exists y(P(a)\to Q(y))\vdash \exists y(P(a)\to Q(y))}\exists_l}{\forall x\exists y(P(x)\to Q(y))\vdash \exists y(P(a)\to Q(y))}\forall_l}{\forall x(P(x)\to Q(x))\vdash \exists y(P(a)\to Q(y))}\mathit{cut}$$

$$\dfrac{\dfrac{\vdash P(a) \qquad P(u)\vdash Q(u)}{\vdash Q(a)}R \qquad Q(v)\vdash}{\vdash}R$$

$\lfloor\varphi\rfloor^{O}_{Q(v)\vdash}$:

$$\dfrac{\dfrac{\dfrac{Q(v)\vdash Q(v)}{Q(v), P(a)\vdash Q(v)}w_l}{Q(v)\vdash P(a)\to Q(v)}\to_r}{Q(v)\vdash \exists y(P(a)\to Q(y))}\exists_r$$

$$
\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(u)\vdash P(u) \qquad Q(u)\vdash Q(u)}{P(u), P(u)\rightarrow Q(u)\vdash Q(u)}\rightarrow_l}{P(u)\rightarrow Q(u)\vdash P(u)\rightarrow Q(u)}\rightarrow_r}{P(u)\rightarrow Q(u)\vdash \exists y(P(u)\rightarrow Q(y))}\exists_r}{\forall x(P(x)\rightarrow Q(x))\vdash \exists y(P(u)\rightarrow Q(y))}\forall_l}{\forall x(P(x)\rightarrow Q(x))\vdash \forall x\exists y(P(x)\rightarrow Q(y))}\forall_r
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(a)\vdash P(a) \qquad Q(v)\vdash Q(v)}{P(a), P(a)\rightarrow Q(v)\vdash Q(v)}\rightarrow_l}{P(a)\rightarrow Q(v)\vdash P(a)\rightarrow Q(v)}\rightarrow_r}{P(a)\rightarrow Q(v)\vdash \exists y(P(a)\rightarrow Q(y))}\exists_r}{\exists y(P(a)\rightarrow Q(y))\vdash \exists y(P(a)\rightarrow Q(y))}\exists_l}{\forall x\exists y(P(x)\rightarrow Q(y))\vdash \exists y(P(a)\rightarrow Q(y))}\forall_l
}{\forall x(P(x)\rightarrow Q(x))\vdash \exists y(P(a)\rightarrow Q(y))}\mathit{cut}
$$

$$
\dfrac{\dfrac{\vdash P(a) \qquad P(u)\vdash Q(u)}{\vdash Q(a)}R \qquad Q(v)\vdash}{\vdash}R
$$

$\lfloor\varphi\rfloor^{O}_{P(u)\vdash Q(u)}$:

$$
\dfrac{\dfrac{P(u)\vdash P(u) \qquad Q(u)\vdash Q(u)}{P(u), P(u)\rightarrow Q(u)\vdash Q(u)}\rightarrow_l}{P(u), \forall x(P(x)\rightarrow Q(x))\vdash Q(u)}\forall_l
$$

$$
\begin{prooftree}
\AxiomC{$P(u) \vdash P(u)$}
\AxiomC{$Q(u) \vdash Q(u)$}
\RightLabel{$\rightarrow_l$}
\BinaryInfC{$P(u), P(u) \rightarrow Q(u) \vdash Q(u)$}
\RightLabel{$\rightarrow_r$}
\UnaryInfC{$P(u) \rightarrow Q(u) \vdash P(u) \rightarrow Q(u)$}
\RightLabel{$\exists_r$}
\UnaryInfC{$P(u) \rightarrow Q(u) \vdash \exists y(P(u) \rightarrow Q(y))$}
\RightLabel{$\forall_l$}
\UnaryInfC{$\forall x(P(x) \rightarrow Q(x)) \vdash \exists y(P(u) \rightarrow Q(y))$}
\RightLabel{$\forall_r$}
\UnaryInfC{$\forall x(P(x) \rightarrow Q(x)) \vdash \forall x \exists y(P(x) \rightarrow Q(y))$}
\AxiomC{$P(a) \vdash P(a)$}
\AxiomC{$Q(v) \vdash Q(v)$}
\RightLabel{$\rightarrow_l$}
\BinaryInfC{$P(a), P(a) \rightarrow Q(v) \vdash Q(v)$}
\RightLabel{$\rightarrow_r$}
\UnaryInfC{$P(a) \rightarrow Q(v) \vdash P(a) \rightarrow Q(v)$}
\RightLabel{$\exists_r$}
\UnaryInfC{$P(a) \rightarrow Q(v) \vdash \exists y(P(a) \rightarrow Q(y))$}
\RightLabel{$\exists_l$}
\UnaryInfC{$\exists y(P(a) \rightarrow Q(y)) \vdash \exists y(P(a) \rightarrow Q(y))$}
\RightLabel{$\forall_l$}
\UnaryInfC{$\forall x \exists y(P(x) \rightarrow Q(y)) \vdash \exists y(P(a) \rightarrow Q(y))$}
\RightLabel{cut}
\BinaryInfC{$\forall x(P(x) \rightarrow Q(x)) \vdash \exists y(P(a) \rightarrow Q(y))$}
\end{prooftree}
$$

$$
\begin{prooftree}
\AxiomC{$P(a) \vdash P(a)$}
\RightLabel{w_r}
\UnaryInfC{$P(a) \vdash P(a), Q(v)$}
\RightLabel{$\rightarrow_r$}
\UnaryInfC{$\vdash P(a), P(a) \rightarrow Q(v)$}
\RightLabel{$\exists_r$}
\UnaryInfC{$\vdash P(a), \exists y(P(a) \rightarrow Q(y))$}
\AxiomC{$P(u) \vdash P(u)$}
\AxiomC{$Q(u) \vdash Q(u)$}
\RightLabel{$\rightarrow_l$}
\BinaryInfC{$P(u), P(u) \rightarrow Q(u) \vdash Q(u)$}
\RightLabel{$\forall_l$}
\UnaryInfC{$P(u), \forall x(P(x) \rightarrow Q(x)) \vdash Q(u)$}
\RightLabel{R}
\BinaryInfC{$\vdash Q(a)$}
\AxiomC{$Q(v) \vdash Q(v)$}
\RightLabel{w_l}
\UnaryInfC{$Q(v), P(a) \vdash Q(v)$}
\RightLabel{$\rightarrow_r$}
\UnaryInfC{$Q(v) \vdash P(a) \rightarrow Q(v)$}
\RightLabel{$\exists_r$}
\UnaryInfC{$Q(v) \vdash \exists y(P(a) \rightarrow Q(y))$}
\RightLabel{R}
\BinaryInfC{$\vdash$}
\end{prooftree}
$$

$$
\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(u) \vdash P(u) \qquad Q(u) \vdash Q(u)}{P(u), P(u) \to Q(u) \vdash Q(u)}\,{\to_l}}{P(u) \to Q(u) \vdash P(u) \to Q(u)}\,{\to_r}}{P(u) \to Q(u) \vdash \exists y(P(u) \to Q(y))}\,{\exists_r}}{\forall x(P(x) \to Q(x)) \vdash \exists y(P(u) \to Q(y))}\,{\forall_l}}{\forall x(P(x) \to Q(x)) \vdash \forall x \exists y(P(x) \to Q(y))}\,{\forall_r}
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(a) \vdash P(a) \qquad Q(v) \vdash Q(v)}{P(a), P(a) \to Q(v) \vdash Q(v)}\,{\to_l}}{P(a) \to Q(v) \vdash P(a) \to Q(v)}\,{\to_r}}{P(a) \to Q(v) \vdash \exists y(P(a) \to Q(y))}\,{\exists_r}}{\exists y(P(a) \to Q(y)) \vdash \exists y(P(a) \to Q(y))}\,{\exists_l}}{\forall x \exists y(P(x) \to Q(y)) \vdash \exists y(P(a) \to Q(y))}\,{\forall_l}
}{\forall x(P(x) \to Q(x)) \vdash \exists y(P(a) \to Q(y))}\,cut
$$

$$
\dfrac{\dfrac{
\dfrac{
\dfrac{\dfrac{\dfrac{P(a) \vdash P(a)}{P(a) \vdash P(a), Q(v)}\,{w_r}}{\vdash P(a), P(a) \to Q(v)}\,{\to_r}}{\vdash P(a), \exists y(P(a) \to Q(y))}\,{\exists_r}
\qquad
\dfrac{\dfrac{P(u) \vdash P(u) \qquad Q(u) \vdash Q(u)}{P(u), P(u) \to Q(u) \vdash Q(u)}\,{\to_l}}{P(u), \forall x(P(x) \to Q(x)) \vdash Q(u)}\,{\forall_l}
}{\forall x(P(x) \to Q(x)) \vdash Q(a), \exists y(P(a) \to Q(y))}\,R
\qquad
\dfrac{\dfrac{\dfrac{Q(v) \vdash Q(v)}{Q(v), P(a) \vdash Q(v)}\,{w_l}}{Q(v) \vdash P(a) \to Q(v)}\,{\to_r}}{Q(v) \vdash \exists y(P(a) \to Q(y))}\,{\exists_r}
}{\forall x(P(x) \to Q(x)) \vdash \exists y(P(a) \to Q(y)), \exists y(P(a) \to Q(y))}\,R}{\forall x(P(x) \to Q(x)) \vdash \exists y(P(a) \to Q(y))}\,{c_r}
$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(u) \vdash P(u) \qquad Q(u) \vdash Q(u)}{P(u), P(u) \to Q(u) \vdash Q(u)}\ \to_l}{P(u) \to Q(u) \vdash P(u) \to Q(u)}\ \to_r}{P(u) \to Q(u) \vdash \exists y (P(u) \to Q(y))}\ \exists_r}{\forall x (P(x) \to Q(x)) \vdash \exists y (P(u) \to Q(y))}\ \forall_l}{\forall x (P(x) \to Q(x)) \vdash \forall x \exists y (P(x) \to Q(y))}\ \forall_r \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(a) \vdash P(a) \qquad Q(v) \vdash Q(v)}{P(a), P(a) \to Q(v) \vdash Q(v)}\ \to_l}{P(a) \to Q(v) \vdash P(a) \to Q(v)}\ \to_r}{P(a) \to Q(v) \vdash \exists y (P(a) \to Q(y))}\ \exists_r}{\exists y (P(a) \to Q(y)) \vdash \exists y (P(a) \to Q(y))}\ \exists_l}{\forall x \exists y (P(x) \to Q(y)) \vdash \exists y (P(a) \to Q(y))}\ \forall_l}{\forall x (P(x) \to Q(x)) \vdash \exists y (P(a) \to Q(y))}\ cut$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(a) \vdash P(a)}{P(a) \vdash P(a), Q(a)}\ w_r}{\vdash P(a), P(a) \to Q(a)}\ \to_r}{\vdash P(a), \exists y (P(a) \to Q(y))}\ \exists_r \qquad \dfrac{\dfrac{P(a) \vdash P(a) \qquad Q(a) \vdash Q(a)}{P(a), P(a) \to Q(a) \vdash Q(a)}\ \to_l}{P(a), \forall x (P(x) \to Q(x)) \vdash Q(a)}\ \forall_l}{\forall x (P(x) \to Q(x)) \vdash Q(a), \exists y (P(a) \to Q(y))}\ cut \qquad \dfrac{\dfrac{\dfrac{Q(a) \vdash Q(a)}{Q(a), P(a) \vdash Q(a)}\ w_l}{Q(a) \vdash P(a) \to Q(a)}\ \to_r}{Q(a) \vdash \exists y (P(a) \to Q(y))}\ \exists_r}{\forall x (P(x) \to Q(x)) \vdash \exists y (P(a) \to Q(y)), \exists y (P(a) \to Q(y))}\ cut}{\forall x (P(x) \to Q(x)) \vdash \exists y (P(a) \to Q(y))}\ c_r$$

- ACNFs produced by Reductive methods have atomic cuts in the top of the proof (closer to leaves).
- ACNFs produced by CERes have atomic cuts in the bottom of the proof (closer to the root).
- If CERes is applied to a proof already in ACNF, but with cuts in the top, CERes produces a new ACNF with cuts in the bottom.
- The ACNF produced by CERes might be smaller than the original ACNF. (CERes is somehow capable of exploiting redundancies when pushing down the atomic cuts).
- This leads to a simple idea for atomic-cut-introduction (and possibly proof-compression) by resolution.

- The method CIRes for introducing atomic cuts in a *cut-free* proof φ follows the two steps below:
  1. Obtain φ' by replacing every axiom-sequent $A \vdash A$ in φ by an atomic cut:
  $$\frac{A \vdash A \qquad A \vdash A}{A \vdash A}\ cut$$
  2. Apply CERes to φ' (thus obtaining a possibly compressed proof φ^* with atomic cuts in the bottom)

A short proof ψ with cut (length 5, atomic size 26, symbolic size 32):

$$\dfrac{\dfrac{\dfrac{A \vdash A \qquad A \vdash A}{A \vee A \vdash A, A}\,\vee_l}{A \vee A \vdash A}\,c_r \qquad \dfrac{\dfrac{A \vdash A \qquad A \vdash A}{A, A \vdash A \wedge A}\,\wedge_r}{A \vdash A \wedge A}\,c_l}{A \vee A \vdash A \wedge A}\,cut$$

One of the shortest proofs without cut, φ (length 6, atomic size 32, symbolic size 41):

$$\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A \qquad A \vdash A}{A, A \vdash A \wedge A}\,\wedge_r}{A \vdash A \wedge A}\,c_l \qquad \dfrac{\dfrac{A \vdash A \qquad A \vdash A}{A, A \vdash A \wedge A}\,\wedge_r}{A \vdash A \wedge A}\,c_l}{A \vee A \vdash A \wedge A, A \wedge A}\,\vee_l}{A \vee A \vdash A \wedge A}\,c_r$$

Can CIRes output ψ when given φ as input?

φ' (with atomic cuts added to the top):

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\ cut \qquad \dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\ cut}{A, A \vdash A \wedge A}\ \wedge_r}{A \vdash A \wedge A}\ c_l \qquad \dfrac{\dfrac{\dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\ cut \qquad \dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\ cut}{A, A \vdash A \wedge A}\ \wedge_r}{A \vdash A \wedge A}\ c_l}{A \vee A \vdash A \wedge A, A \wedge A}\ \vee_l}{A \vee A \vdash A \wedge A}\ c_r$$

Testing the Method (can it really compress proofs?)

A Simple Example: $A \vee A \vdash A \wedge A$

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut
        \quad
        \dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut
      }{A, A \vdash A \wedge A}\,\wedge_r
    }{A \vdash A \wedge A}\,c_l
    \quad
    \dfrac{
      \dfrac{
        \dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut
        \quad
        \dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut
      }{A, A \vdash A \wedge A}\,\wedge_r
    }{A \vdash A \wedge A}\,c_l
  }{A \vee A \vdash A \wedge A, A \wedge A}\,\vee_l
}{A \vee A \vdash A \wedge A}\,c_r
$$

$$\mathcal{S}_{\varphi'} \equiv ((A \oplus \neg A) \otimes^{**} (A \oplus \neg A)) \otimes^{****} ((A \oplus \neg A) \otimes^{**} (A \oplus \neg A))$$

$$\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut \qquad \dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut}{A, A \vdash A \wedge A}\,\wedge_r}{A \vdash A \wedge A}\,c_l \qquad \dfrac{\dfrac{\dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut \qquad \dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut}{A, A \vdash A \wedge A}\,\wedge_r}{A \vdash A \wedge A}\,c_l}{\dfrac{A \vee A \vdash A \wedge A, A \wedge A}{A \vee A \vdash A \wedge A}\,c_r}\,\vee_l$$

$$\begin{aligned}
\mathcal{S}_{\varphi'} \quad & \equiv \quad ((A \oplus \neg A) \otimes^{**} (A \oplus \neg A)) \otimes^{****} ((A \oplus \neg A) \otimes^{**} (A \oplus \neg A)) \\
& \rightsquigarrow_{\oplus\otimes W} \quad (A \oplus A \oplus (\neg A \otimes^{**} \neg A)) \otimes^{****} ((A \oplus \neg A) \otimes^{**} (A \oplus \neg A)) \\
& \rightsquigarrow_{\oplus\otimes W} \quad (A \oplus A \oplus (\neg A \otimes^{**} \neg A)) \otimes^{****} (A \oplus A \oplus (\neg A \otimes^{**} \neg A)) \\
& \rightsquigarrow_{\oplus\otimes W} \quad (A \otimes^{****} A) \oplus (A \otimes^{****} A) \oplus (A \otimes^{****} A) \oplus (A \otimes^{****} A) \oplus (\neg A \otimes^{**} \neg A) \oplus (\neg A \otimes^{**} \neg A)
\end{aligned}$$

$$\mathcal{C}^W_{\varphi'} \equiv \{\ \vdash A, A \ ;\ \vdash A, A \ ;\ \vdash A, A \ ;\ \vdash A, A \ ;\ A, A \vdash \ ;\ A, A \vdash\ \}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut \quad \dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut}{A, A \vdash A \wedge A}\,\wedge_r}{A \vdash A \wedge A}\,c_l \qquad \dfrac{\dfrac{\dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut \quad \dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut}{A, A \vdash A \wedge A}\,\wedge_r}{A \vdash A \wedge A}\,c_l}{\dfrac{A \vee A \vdash A \wedge A, A \wedge A}{A \vee A \vdash A \wedge A}\,c_r}\,\vee_l$$

$$C_{\varphi'}^{W} \equiv \{ \vdash A, A \;;\; \vdash A, A \;;\; \vdash A, A \;;\; \vdash A, A \;;\; A, A \vdash \;;\; A, A \vdash \}$$

δ:

$$\dfrac{\dfrac{\vdash A, A}{\vdash A}\,f_r \qquad \dfrac{A, A \vdash}{A \vdash}\,f_l}{\vdash}\,R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut \qquad \dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut}{A, A \vdash A \wedge A}\,\wedge_r}{A \vdash A \wedge A}\,c_l \qquad \dfrac{\dfrac{\dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut \qquad \dfrac{A \vdash A \quad A \vdash A}{A \vdash A}\,cut}{A, A \vdash A \wedge A}\,\wedge_r}{A \vdash A \wedge A}\,c_l}{A \vee A \vdash A \wedge A, A \wedge A}\,\vee_l}{A \vee A \vdash A \wedge A}\,c_r$$

$$\dfrac{\dfrac{\vdash A, A}{\vdash A}\,f_r \qquad \dfrac{A, A \vdash}{A \vdash}\,f_l}{\vdash}\,R$$

$\lfloor \varphi' \rfloor^{O}_{\vdash A, A}$:

$$\dfrac{A \vdash A \qquad A \vdash A}{A \vee A \vdash A, A}\,\vee_l$$

$\lfloor \varphi' \rfloor^{O}_{A, A \vdash}$:

$$\dfrac{A \vdash A \qquad A \vdash A}{A, A \vdash A \wedge A}\,\wedge_r$$

φ^*:

$$\dfrac{\dfrac{\dfrac{A \vdash A \qquad A \vdash A}{A \vee A \vdash A, A}\ \vee_l}{A \vee A \vdash A}\ c_r \qquad \dfrac{\dfrac{A \vdash A \qquad A \vdash A}{A, A \vdash A \wedge A}\ \wedge_r}{A \vdash A \wedge A}\ c_l}{A \vee A \vdash A \wedge A}\ cut$$

- Can CIRes output ψ when given φ as input?
  - Yes, φ^* equal to ψ!

- Consider the sequence of unsatisfiable sets of clauses:

$$T_m = \{\pm P^1 \vee \pm P^2_{\pm} \vee \pm P^3_{\pm\pm} \vee \ldots \vee \pm P^m_{\pm\ldots\pm}\}$$

- For example:

$$T_2 = \{P^1 \vee P^2_+, \neg P^1 \vee P^2_-, P^1 \vee \neg P^2_+, \neg P^1 \vee \neg P^2_-\}$$

- It is known (from [Cook-Reckhow, 1971]) that:
  - $|T_m|_s = s(m) \in O((2m)2^m)$.
  - $|\delta_m|_l = l_R(m) \in O(2^m)$, if δ_m is an optimal resolution refutation of T_m (or a sequent calculus refutation (*possibly with cuts*)).
  - $|\psi_m|_l = l_T(m) \in O(2^{2^{cm}})$, if ψ_m is an optimal tableaux refutation of T_m (or a *cut-free* sequent calculus refutation).

Testing the Method
A More Interesting Example

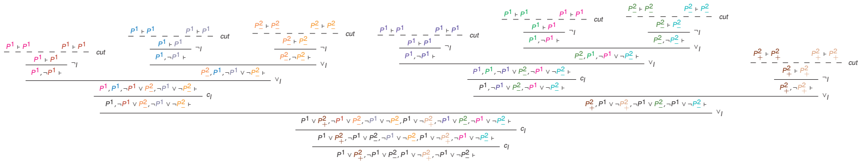
φ (length 17, atomic size 97, symbolic size 169):

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P^1 \vdash P^1}{P^1, \neg P^1 \vdash}\neg_l \quad \dfrac{\dfrac{P^1 \vdash P^1}{P^1, \neg P^1 \vdash}\neg_l \quad \dfrac{P^2_- \vdash P^2_-}{P^2_-, \neg P^2_- \vdash}\neg_l}{P^2_-, P^1, \neg P^1 \vee \neg P^2_- \vdash}\vee_l}{P^1, P^1, \neg P^1 \vee P^2_-, \neg P^1 \vee \neg P^2_- \vdash}\vee_l}{P^1, \neg P^1 \vee P^2_-, \neg P^1 \vee \neg P^2_- \vdash}c_l \quad \dfrac{\dfrac{\dfrac{\dfrac{P^1 \vdash P^1}{P^1, \neg P^1 \vdash}\neg_l \quad \dfrac{\dfrac{P^1 \vdash P^1}{P^1, \neg P^1 \vdash}\neg_l \quad \dfrac{P^2_- \vdash P^2_-}{P^2_-, \neg P^2_- \vdash}\neg_l}{P^2_-, P^1, \neg P^1 \vee \neg P^2_- \vdash}\vee_l}{P^1, P^1, \neg P^1 \vee P^2_-, \neg P^1 \vee \neg P^2_- \vdash}\vee_l}{P^1, \neg P^1 \vee P^2_-, \neg P^1 \vee \neg P^2_- \vdash}c_l \quad \dfrac{P^2_+ \vdash P^2_+}{P^2_+, \neg P^2_+ \vdash}\neg_l}{P^2_+, P^1 \vee \neg P^2_+, \neg P^1 \vee P^2_-, \neg P^1 \vee \neg P^2_- \vdash}\vee_l}{P^1 \vee P^2_+, \neg P^1 \vee P^2_-, \neg P^1 \vee \neg P^2_-, P^1 \vee \neg P^2_+, \neg P^1 \vee P^2_-, \neg P^1 \vee \neg P^2_- \vdash}\vee_l}{P^1 \vee P^2_+, \neg P^1 \vee P^2_-, \neg P^1 \vee \neg P^2_-, P^1 \vee \neg P^2_+, \neg P^1 \vee \neg P^2_- \vdash}c_l}{P^1 \vee P^2_+, \neg P^1 \vee P^2_-, P^1 \vee \neg P^2_+, \neg P^1 \vee \neg P^2_- \vdash}c_l
$$

Testing the Method

A More Interesting Example

φ':



$$\mathcal{S}_{\varphi'} \equiv ((P^1 \oplus \neg P^1) \otimes^{**} ((P^1 \oplus \neg P^1) \otimes^{**} (P^2_- \oplus \neg P^2_-))) \otimes^{***} (((P^1 \oplus \neg P^1) \otimes^{**} ((P^1 \oplus \neg P^1) \otimes^{**} (P^2_- \oplus \neg P^2_-))) \otimes^{***} (P^2_+ \oplus \neg P^2_+))$$

$$\begin{aligned}
\mathcal{S}_{\varphi'} \quad \equiv \quad & ((P^1 \oplus \neg P^1) \otimes^{**} ((P^1 \oplus \neg P^1) \otimes^{**} (P^2_- \oplus \neg P^2_-))) \otimes^{***} (((P^1 \oplus \neg P^1) \otimes^{**} ((P^1 \oplus \neg P^1) \otimes^{**} (P^2_- \oplus \neg P^2_-))) \otimes^{***} (P^2_+ \oplus \neg P^2_+)) \\
\rightsquigarrow_{\oplus\otimes W} \quad & ((P^1 \oplus \neg P^1) \otimes^{**} (P^1 \oplus P^2_- \oplus (\neg P^1 \otimes^{**} \neg P^2_-))) \otimes^{***} (((P^1 \oplus \neg P^1) \otimes^{**} ((P^1 \oplus \neg P^1) \otimes^{**} (P^2_- \oplus \neg P^2_-))) \otimes^{***} (P^2_+ \oplus \neg P^2_+)) \\
\rightsquigarrow_{\oplus\otimes W} \quad & (P^1 \oplus P^1 \oplus (\neg P^1 \otimes^{**} P^2_-) \oplus (\neg P^1 \otimes^{**} \neg P^2_-)) \otimes^{***} (((P^1 \oplus \neg P^1) \otimes^{**} ((P^1 \oplus \neg P^1) \otimes^{**} (P^2_- \oplus \neg P^2_-))) \otimes^{***} (P^2_+ \oplus \neg P^2_+)) \\
\rightsquigarrow_{\oplus\otimes W} \quad & (P^1 \oplus P^1 \oplus (\neg P^1 \otimes^{**} P^2_-) \oplus (\neg P^1 \otimes^{**} \neg P^2_-)) \otimes^{***} (((P^1 \oplus \neg P^1) \otimes^{**} (P^1 \oplus P^2_- \oplus (\neg P^1 \otimes^{**} \neg P^2_-))) \otimes^{***} (P^2_+ \oplus \neg P^2_+)) \\
\rightsquigarrow_{\oplus\otimes W} \quad & (P^1 \oplus P^1 \oplus (\neg P^1 \otimes^{**} P^2_-) \oplus (\neg P^1 \otimes^{**} \neg P^2_-)) \otimes^{***} ((P^1 \oplus P^1 \oplus (\neg P^1 \otimes^{**} P^2_-) \oplus (\neg P^1 \otimes^{**} \neg P^2_-)) \otimes^{***} (P^2_+ \oplus \neg P^2_+)) \\
\rightsquigarrow_{\oplus\otimes W} \quad & (P^1 \oplus P^1 \oplus (\neg P^1 \otimes^{**} P^2_-) \oplus (\neg P^1 \otimes^{**} \neg P^2_-)) \otimes^{***} ((P^1 \otimes^{***} \neg P^2_+) \oplus (P^1 \otimes^{***} \neg P^2_+) \oplus (\neg P^1 \otimes^{**} P^2_-) \oplus (\neg P^1 \otimes^{**} \neg P^2_-) \oplus P^2_+) \\
\rightsquigarrow_{\oplus\otimes W} \quad & ((P^1 \otimes^{***} P^2_+) \oplus (P^1 \otimes^{***} P^2_+) \oplus (\neg P^1 \otimes^{**} P^2_-) \oplus (\neg P^1 \otimes^{**} \neg P^2_-)) \oplus ((P^1 \otimes^{***} \neg P^2_+) \oplus (P^1 \otimes^{***} \neg P^2_+) \oplus (\neg P^1 \otimes^{**} P^2_-) \oplus (\neg P^1 \otimes^{**} \neg P^2 \\
\equiv \quad & (P^1 \otimes^{***} P^2_+) \oplus (P^1 \otimes^{***} P^2_+) \oplus (\neg P^1 \otimes^{**} P^2_-) \oplus (\neg P^1 \otimes^{**} \neg P^2_-) \oplus (P^1 \otimes^{***} \neg P^2_+) \oplus (P^1 \otimes^{***} \neg P^2_+) \oplus (\neg P^1 \otimes^{**} P^2_-) \oplus (\neg P^1 \otimes^{**} \neg P^2_-)
\end{aligned}$$

$$\mathcal{C}^W_{\varphi'} \equiv \{ \vdash P^1, P^2_+ \;;\; \vdash P^1, P^2_+ \;;\; P^1 \vdash P^2_- \;;\; P^1, P^2_- \vdash \;;\; P^2_+ \vdash P^1 \;;\; P^2_+ \vdash P^1 \;;\; P^1 \vdash P^2_- \;;\; P^1, P^2_- \vdash \}$$

δ:

$$\dfrac{\dfrac{\dfrac{\vdash P^1, P^2_+ \qquad P^2_+ \vdash P^1}{\vdash P^1, P^1}\,R}{\vdash P^1}\,f_r \qquad \dfrac{\dfrac{P^1 \vdash P^2_- \qquad P^1, P^2_- \vdash}{P^1, P^1 \vdash}\,R}{P^1 \vdash}\,f_r}{\vdash}\,R$$

φ':

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P^{1} \vdash P^{1} \quad P^{1} \vdash P^{1}}{P^{1} \vdash P^{1}}\,cut}{P^{1}, \neg P^{1} \vdash}\,\neg_l \quad \dfrac{\dfrac{\dfrac{P^{1} \vdash P^{1} \quad P^{1} \vdash P^{1}}{P^{1} \vdash P^{1}}\,cut}{P^{1}, \neg P^{1} \vdash}\,\neg_l \quad \dfrac{\dfrac{P^{2}_{-} \vdash P^{2}_{-} \quad P^{2}_{-} \vdash P^{2}_{-}}{P^{2}_{-} \vdash P^{2}_{-}}\,cut}{P^{2}_{-}, \neg P^{2}_{-} \vdash}\,\neg_l}{P^{2}_{-}, P^{1}, \neg P^{1} \vee \neg P^{2}_{-} \vdash}\,\vee_l}{P^{1}, P^{1}, \neg P^{1} \vee P^{2}_{-}, \neg P^{1} \vee \neg P^{2}_{-} \vdash}\,\vee_l}{P^{1}, \neg P^{1} \vee P^{2}_{-}, \neg P^{1} \vee \neg P^{2}_{-} \vdash}\,c_l \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P^{1} \vdash P^{1} \quad P^{1} \vdash P^{1}}{P^{1} \vdash P^{1}}\,cut}{P^{1}, \neg P^{1} \vdash}\,\neg_l \quad \dfrac{\dfrac{\dfrac{P^{1} \vdash P^{1} \quad P^{1} \vdash P^{1}}{P^{1} \vdash P^{1}}\,cut}{P^{1}, \neg P^{1} \vdash}\,\neg_l \quad \dfrac{\dfrac{P^{2}_{-} \vdash P^{2}_{-} \quad P^{2}_{-} \vdash P^{2}_{-}}{P^{2}_{-} \vdash P^{2}_{-}}\,cut}{P^{2}_{-}, \neg P^{2}_{-} \vdash}\,\neg_l}{P^{2}_{-}, P^{1}, \neg P^{1} \vee \neg P^{2}_{-} \vdash}\,\vee_l}{P^{1}, P^{1}, \neg P^{1} \vee P^{2}_{-}, \neg P^{1} \vee \neg P^{2}_{-} \vdash}\,\vee_l}{P^{1}, \neg P^{1} \vee P^{2}_{-}, \neg P^{1} \vee \neg P^{2}_{-} \vdash}\,c_l \quad \dfrac{\dfrac{P^{2}_{+} \vdash P^{2}_{+} \quad P^{2}_{+} \vdash P^{2}_{+}}{P^{2}_{+} \vdash P^{2}_{+}}\,cut}{P^{2}_{+}, \neg P^{2}_{+} \vdash}\,\neg_l}{P^{2}_{+}, P^{1} \vee \neg P^{2}_{+}, \neg P^{1} \vee P^{2}_{-}, \neg P^{1} \vee \neg P^{2}_{-} \vdash}\,\vee_l}{P^{1} \vee P^{2}_{+}, \neg P^{1} \vee P^{2}_{-}, \neg P^{1} \vee \neg P^{2}_{-}, P^{1} \vee \neg P^{2}_{+}, \neg P^{1} \vee P^{2}_{-}, \neg P^{1} \vee \neg P^{2}_{-} \vdash}\,\vee_l}{P^{1} \vee P^{2}_{+}, \neg P^{1} \vee P^{2}_{-}, \neg P^{1} \vee \neg P^{2}_{-}, P^{1} \vee \neg P^{2}_{+}, \neg P^{1} \vee \neg P^{2}_{-} \vdash}\,c_l}{P^{1} \vee P^{2}_{+}, \neg P^{1} \vee P^{2}_{-}, P^{1} \vee \neg P^{2}_{+}, \neg P^{1} \vee \neg P^{2}_{-} \vdash}\,c_l$$

δ:

$$\dfrac{\dfrac{\dfrac{\vdash P^{1}, P^{2}_{+} \quad P^{2}_{+} \vdash P^{1}}{\vdash P^{1}, P^{1}}\,R}{\vdash P^{1}}\,f_r \quad \dfrac{\dfrac{P^{1} \vdash P^{2}_{-} \quad P^{1}, P^{2}_{-} \vdash}{P^{1}, P^{1} \vdash}\,R}{P^{1} \vdash}\,f_r}{\vdash}\,R$$

$\lfloor \varphi' \rfloor^{O}_{\vdash P^{1}, P^{2}_{+}}$:

$$\dfrac{P^{1} \vdash P^{1} \quad P^{2}_{+} \vdash P^{2}_{+}}{P^{1} \vee P^{2}_{+} \vdash P^{1}, P^{2}_{+}}\,\vee_l$$

$\lfloor \varphi' \rfloor^{O}_{P^{2}_{+} \vdash P^{1}}$:

$$\dfrac{P^{1} \vdash P^{1} \quad \dfrac{P^{2}_{+} \vdash P^{2}_{+}}{P^{2}_{+}, \neg P^{2}_{+} \vdash}\,\neg_l}{P^{2}_{+}, P^{1} \vee \neg P^{2}_{+} \vdash P^{1}}\,\vee_l$$

$\lfloor \varphi' \rfloor^{O}_{P^{1} \vdash P^{2}_{-}}$:

$$\dfrac{\dfrac{P^{1} \vdash P^{1}}{P^{1}, \neg P^{1} \vdash}\,\neg_l \quad P^{2}_{-} \vdash P^{2}_{-}}{P^{1}, \neg P^{1} \vee P^{2}_{-} \vdash P^{2}_{-}}\,\vee_l$$

$\lfloor \varphi' \rfloor^{O}_{P^{1}, P^{2}_{-} \vdash}$:

$$\dfrac{\dfrac{P^{1} \vdash P^{1}}{P^{1}, \neg P^{1} \vdash}\,\neg_l \quad \dfrac{P^{2}_{-} \vdash P^{2}_{-}}{P^{2}_{-}, \neg P^{2}_{-} \vdash}\,\neg_l}{P^{1}, P^{2}_{-}, \neg P^{1} \vee \neg P^{2}_{-} \vdash}\,\vee_l$$

φ^* (length 13, atomic size 70, symbolic size 105):

$$\dfrac{\dfrac{\dfrac{\dfrac{P^1 \vdash P^1 \quad P^2_+ \vdash P^2_+}{P^1 \lor P^2_+ \vdash P^1, P^2_+}\lor_l \qquad \dfrac{P^1 \vdash P^1 \quad \dfrac{P^2_+ \vdash P^2_+}{P^2_+, \neg P^2_+ \vdash}\neg_l}{P^2_+, P^1 \lor \neg P^2_+ \vdash P^1}\lor_l}{P^1 \lor P^2_+, P^1 \lor \neg P^2_+ \vdash P^1, P^1}cut}{P^1 \lor P^2_+, P^1 \lor \neg P^2_+ \vdash P^1}c_r \qquad \dfrac{\dfrac{\dfrac{\dfrac{P^1 \vdash P^1}{P^1, \neg P^1 \vdash}\neg_l \quad P^2_- \vdash P^2_-}{P^1, \neg P^1 \lor P^2_- \vdash P^2_-}\lor_l \qquad \dfrac{\dfrac{P^1 \vdash P^1}{P^1, \neg P^1 \vdash}\neg_l \quad \dfrac{P^2_- \vdash P^2_-}{P^2_-, \neg P^2_- \vdash}\neg_l}{P^1, P^2_-, \neg P^1 \lor \neg P^2_- \vdash}\lor_l}{P^1, P^1, \neg P^1 \lor P^2_-, \neg P^1 \lor \neg P^2_- \vdash}cut}{P^1, \neg P^1 \lor P^2_-, \neg P^1 \lor \neg P^2_- \vdash}c_r}{P^1 \lor P^2_+, P^1 \lor \neg P^2_+, \neg P^1 \lor P^2_-, \neg P^1 \lor \neg P^2_- \vdash}cut$$

φ (length 17, atomic size 97, symbolic size 169):

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P^1 \vdash P^1}{P^1, \neg P^1 \vdash}\neg_l \qquad \dfrac{\dfrac{P^1 \vdash P^1}{P^1, \neg P^1 \vdash}\neg_l \quad \dfrac{P^2_- \vdash P^2_-}{P^2_-, \neg P^2_- \vdash}\neg_l}{P^2_-, P^1, \neg P^1 \lor \neg P^2_- \vdash}\lor_l}{P^1, P^1, \neg P^1 \lor P^2_-, \neg P^1 \lor \neg P^2_- \vdash}\lor_l}{P^1, \neg P^1 \lor P^2_-, \neg P^1 \lor \neg P^2_- \vdash}c_l \qquad \dfrac{\dfrac{\dfrac{\dfrac{P^1 \vdash P^1}{P^1, \neg P^1 \vdash}\neg_l \qquad \dfrac{\dfrac{P^1 \vdash P^1}{P^1, \neg P^1 \vdash}\neg_l \quad \dfrac{P^2_- \vdash P^2_-}{P^2_-, \neg P^2_- \vdash}\neg_l}{P^2_-, P^1, \neg P^1 \lor \neg P^2_- \vdash}\lor_l}{P^1, P^1, \neg P^1 \lor P^2_-, \neg P^1 \lor \neg P^2_- \vdash}\lor_l}{P^1, \neg P^1 \lor P^2_-, \neg P^1 \lor \neg P^2_- \vdash}c_l \qquad \dfrac{P^2_+ \vdash P^2_+}{P^2_+, \neg P^2_+ \vdash}\neg_l}{P^2_+, P^1 \lor \neg P^2_+, \neg P^1 \lor P^2_-, \neg P^1 \lor \neg P^2_- \vdash}\lor_l}{P^1 \lor P^2_+, \neg P^1 \lor P^2_-, \neg P^1 \lor \neg P^2_-, P^1 \lor \neg P^2_+, \neg P^1 \lor P^2_-, \neg P^1 \lor \neg P^2_- \vdash}\lor_l}{P^1 \lor P^2_+, \neg P^1 \lor P^2_-, \neg P^1 \lor \neg P^2_-, P^1 \lor \neg P^2_+, \neg P^1 \lor \neg P^2_- \vdash}c_l}{P^1 \lor P^2_+, \neg P^1 \lor P^2_-, P^1 \lor \neg P^2_+, \neg P^1 \lor \neg P^2_- \vdash}c_l$$

Future Work

- Enrich the clause set with more information from the cut-free proof, and use the additional information to constrain the search for refutations of the clause set.
- Try to extend the method so that *complex* propositional cuts could also be introduced. (This might be possible with a mixed structural clause form transformation of the cut-pertinent struct).
- Modify the concept of projection and modify how they are combined with the refutation, so that atomic cuts do not necessarily appear in the bottom of the proof.

The End

- Thanks!
- An announcement: 24th of July, 18:00 (at ESSLLI SD09, in Bordeaux): “Resolution Refinements for Cut-Elimination based on Reductive Methods”
- Questions? Comments? Suggestions?