

Proof Analysis with HLK, CERES and ProofTool

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Motivation

Proof Analysis and Transformations with CERES, HLK and ProofTool

- What are the main goals of CERES (and HLK and ProofTool)?
 - The automated analysis and transformation of mathematical proofs.
 - The elimination of lemmas from mathematical proofs, in order to obtain more “direct”, “elementary”, “analytic” proofs, i.e. that do not contain notions that are not already in theorem.
- Two facts:
 - lemmas in informal mathematical proofs correspond to cuts in formal proofs in sequent calculi.
 - Cuts can be eliminated (Gentzen's Hauptsatz).
- Approach to achieve the goals:
 - ① Formalize mathematical proofs in sequent calculi.
 - ② Eliminate the cuts (automatically, by computer).
 - ③ Interpret the resulting cut-free proofs, to obtain “direct” mathematical proofs of the given theorems.

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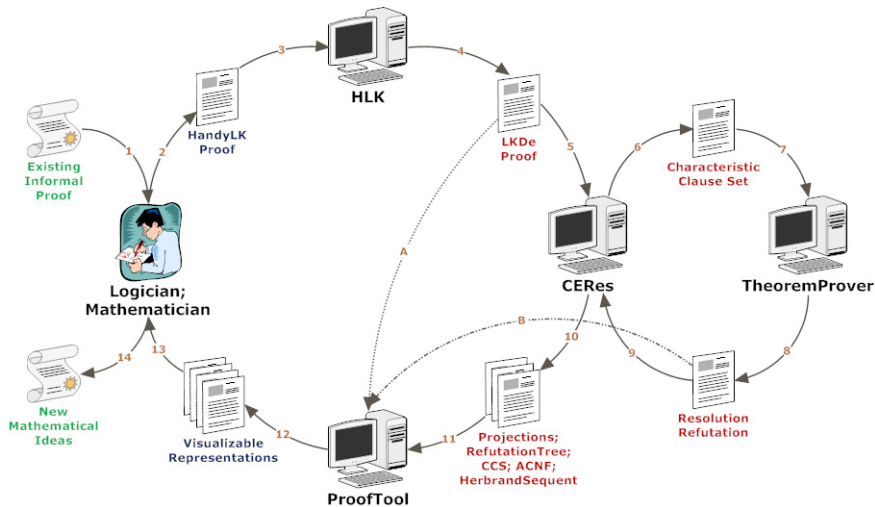
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System Demonstration

- The Tape Proof
 - From proofs with (a version of) the Infinite Pigeon-Hole Principle as a lemma to different proofs without it.
- The (much more significant) Proof of the Infinity of Primes
 - From Fürstenberg's Proof to Euclid's Proof.

Examples of Successful Cases

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- The (much more significant) Proof of the Infinity of Primes
 - From Fürstenberg's Proof to Euclid's Proof.

- Extension to Second-order Logic
 - Motivation: proofs with induction.
 - Difficulties: higher-order resolution and higher-order skolemizability of proofs.
- Elimination of Single Cuts
 - Motivation: it might be interesting to eliminate only some cuts/lemmas.
- Resolution Refinements for Cut-Elimination
 - Motivation: sometimes current theorem provers are not able to refute the unsatisfiable characteristic clause sets produced by CERES.

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- Improvements for ProofTool
- Translation of proofs written in languages other than HLK: COQ, Isabelle, Mizar, ForTheL.
 - Motivation: to be able to analyze a large number of proofs already formalized in these languages.
 - Potential Difficulties: incompatibilities in the theoretical side (e.g. different logical calculi used) and in the implementation (e.g. absence of output of an easily convertible formal proof object).

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Many Thanks and One Announcement

- Thank you!
- On Wednesday, 30th of July, 11:30-12:00 (MKM Conference):
 - Talk **Herbrand Sequent Extraction**.
 - A very relevant complement for this talk:
the theoretical methods used in CERES will be explained.