Muddy children

There are \( n \) children playing together. During their play some of the children, say \( k \) of them, get mud on their foreheads. Each can see the mud on others but not on his own forehead. Along comes a father, who says, “At least one of you has mud on your head”. He then asks the following question, over and over: “Can any of you prove that you have mud on your head?” Assuming that all the children are perceptive, intelligent, truthful, and that they answer simultaneously, what will happen?

There is a proof that the first \( k-1 \) times the father asks the question, the children will all say “no” but that the \( k \)-th time the children that are dirty will answer “yes”.
Kripke model with 3 children:

After father's announcement:
After all children say 'No':

Possibilities:
Let $\mathcal{A}$, a set of agents, and $\mathcal{P}$, a set of propositional variables, be given.

- A possibility $w$ is a function that assigns to each propositional variable $p \in \mathcal{P}$ a truth value $w(p) \in \{0, 1\}$, and to each agent $a \in \mathcal{A}$ an information state $w(a)$.

- An information state $\sigma$ is a set of possibilities.