Comments on *Comparing Context Updates Delineation and Scale Based Models of Vagueness* by Christoph Roschger

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In ‘Comparing Context Updates in Delineation and Scale Based Models of Vagueness’, Christoph Roschger argues that two frameworks, due respectively to Kyburg & Morreau and Barker, bear important similarities to each other in certain respects. Roschger begins by summarizing the respective frameworks, which I will not repeat, and proceeds to define four mappings: for each framework, he provides a mapping both to and from subsets of the space of classical possible worlds. By composing these mappings, Roschger uses these subsets as intermediaries, to move from one framework to the other and back again. This note will be spent reviewing these mappings. My goal is to clarify how they do their respective jobs, and to correct some errors in Roschger's work.

1 Reviewing the mappings

This section steps through the mappings Roschger defines between three spaces: the space of partial interpretations, used by Kyburg & Morreau, the space of Barker-contexts, as I will call the contexts used in Barker's framework, and the space of sets of classical possible worlds, used by Roschger as an intermediate between these other spaces. Note that the worlds appearing in Barker-contexts are more richly structured than classical possible worlds; they include thresholds on scales for various predicates. I'll call these worlds “Barker-worlds” to avoid confusion with the classical possible worlds Roschger uses.

Notation: I use $\mathfrak{P}$ for the set of all partial interpretations; any precisification space $\mathscr{P}$ is thus a subset of $\mathfrak{P}$. Similarly, I use $\mathfrak{S}$ for the set of all possible worlds; any set S of worlds is a subset of $\mathfrak{S}$. Roschger takes possible worlds to be consistent and complete sets of literals; I'll extend a similar approach to partial interpretations, taking them to be consistent sets of literals (let $\mathscr{L}_{\text{lit}}$ be the set of literals). I write $w \Vdash A$ to indicate that the possible world or the Barker-world w satisfies the sentence A.

For the mappings T_b and T_b^{-1}, a crucial notion will be that of a Barker-world *agreeing atomically* with a classical possible world. This happens when they satisfy all the same atomic sentences as each other. For classical worlds, atomic satisfaction is handled via membership, and for Barker-worlds, it is handled via the scale structure; nonetheless, these two approaches may produce the very same results. I write $c\mathscr{A}s$ to mean that the Barker-world c agrees atomically with the classical possible world s.[1]

[1]Note that if there is some c such that $c\mathscr{A}s$ and $c\mathscr{A}s'$, then $s = s'$ (possible worlds are individuated by the atomic sentences that hold there), while there might be distinct c, c' such that for some s, $c\mathscr{A}s$ and $c'\mathscr{A}s$ (distinct Barker worlds can agree on all atomic sentences).

The rest of the notation will follow Roschger. To save space I skip the proofs below, except for counterexamples to some of Roschger's claims; none of the omitted proofs is anything but routine.

1.1 T_{km} and T_{km}^{-1}

$T_{km} \colon \mathfrak{P} \mapsto \wp(\mathfrak{S})$ takes a partial interpretation as input and returns a set of possible worlds. In particular, $T_{km}(p) = \{s \in \mathfrak{S} \mid \forall A \in p \colon s \Vdash A\}$; T_{km} returns the set of possible worlds that satisfy every literal in p. Conversely, $T_{km}^{-1} \colon \wp(\mathfrak{S}) \mapsto \mathfrak{P}$ takes a set of possible worlds and yields a partial interpretation: $T_{km}^{-1}(S) = \{A \in \mathscr{L}_{\mathrm{lit}} \mid \forall s \in S \colon s \Vdash A\}$, including all and only the literals that every member of S agree in satisfying. While partial interpretations may occur in precisification spaces, nothing in either of these definitions pays attention to any precisification space that p occurs in. All that matters is p itself.

The spaces $\mathfrak{P}$ and $\wp(\mathfrak{S})$ these maps move between are both ordered by $\subseteq$, and both maps are antitone: if $p \subseteq p'$, then $T_{km}(p') \subseteq T_{km}(p)$, and if $S \subseteq S'$, then $T_{km}^{-1}(S') \subseteq T_{km}^{-1}(S)$. A more complete partial interpretation imposes more requirements, and so corresponds to a more restricted set of worlds.

T_{km}^{-1} really does reverse T_{km}; that is, for any p, $p = T_{km}^{-1}(T_{km}(p))$. On the other hand, it is not the case that $S = T_{km}(T_{km}^{-1}(S))$ for every S, despite Roschger's claim in his Proposition 4.[2] For a counterexample, consider any case in which $T_{km}^{-1}(S) = T_{km}^{-1}(S')$ while $S \neq S'$ (Roschger provides one such case immediately after his Proposition 4). Since $T_{km}^{-1}(S) = T_{km}^{-1}(S')$, $T_{km}(T_{km}^{-1}(S)) = T_{km}(T_{km}^{-1}(S'))$. This cannot be identical to both S and S', since $S \neq S'$, so either S or S' must provide a counterexample (in fact, both might). One direction of the claim does hold, however: $S \subseteq T_{km}^{-1}(T_{km}(S))$.

It follows from the above that T_{km} and T_{km}^{-1} form a(n antitone) Galois connection between $\mathfrak{P}$ and $\wp(\mathfrak{S})$: $p \subseteq T_{km}^{-1}(S)$ iff $S \subseteq T_{km}(p)$. These maps thus preserve a fair amount of the structure of the two spaces. Moreover, T_{km} is perfectly reversible: one can recover p from $T_{km}(p)$.

1.2 T_b and T_b^{-1}

T_b and T_b^{-1} tie their respective spaces together slightly less tightly. Throughout this section, everything should be considered relative to some fixed Barker-context C_0; every Barker-context C will be taken to be a subset of C_0.

$T_b \colon \wp(C_0) \mapsto \wp(\mathfrak{S})$ takes a Barker context C and maps it to a set of possible worlds as follows: $T_b(C) = \{s \in \mathfrak{S} \mid \exists c \in C \colon c \mathscr{A} s\}$. Conversely, $T_b^{-1} \colon \wp(\mathfrak{S}) \mapsto \wp(C_0)$ takes a set of classical possible worlds and maps it to a subset of C_0 in a similar way: $T_b^{-1}(S) = \{c \in C_0 \mid \exists s \in S \colon c \mathscr{A} s\}$. Again, the spaces involved in these mappings are naturally ordered by $\subseteq$; this time, the mappings are both monotone: if $C \subseteq C'$, then $T_b(C) \subseteq T_b(C')$, and if $S \subseteq S'$, then $T_b^{-1}(S) \subseteq T_b^{-1}(S')$.

Unlike the Kyburg & Morreau case, here T_b^{-1} is not a true inverse of T_b; we can have C such that $C \neq T_b^{-1}(T_b(C))$. This is because T_b and T_b^{-1} pay attention only to which atomic sentences a given Barker-world satisfies, but there may well be a Barker-

[2] The claim does hold for sets S that are themselves $T_{km}(p)$ for some partial interpretation p; in these cases, it follows from the fact that $p = T_{km}^{-1}(T_{km}(p))$, by applying T_{km} to both sides.

context C that differentiates between Barker-worlds that agree on all atomics. However, we do have one direction: $C \subseteq T_b^{-1}(T_b(C))$. We also do not have $S = T_b(T_b^{-1}(S))$; for a particular $s \in S$, there might be no $c \in C_0$ such that $c\mathscr{A}s$; when this happens, $s \notin T_b(T_b^{-1}(S))$. Again, we have one direction: $T_b(T_b^{-1}(S)) \subseteq S$.

T_b and T_b^{-1} form a Galois connection (this time monotone) between $\wp(\mathfrak{S})$ and $\wp(C_0)$: the above facts guarantee that $C \subseteq T_b^{-1}(S)$ iff $T_b(C) \subseteq S$. Again, a considerable amount of structure is preserved by the mappings. This time, though, neither mapping is perfectly reversible.

2 How the mappings interact

The goal of Roschger's paper, though, is not just to map Kyburg & Morreau's machinery and Barker's machinery into a common space. It is to map each of them into the other, using $\wp(\mathfrak{S})$ as an intermediary. For a given C_0, we have $T_b^{-1} \circ T_{km} \colon \mathfrak{P} \mapsto \wp(C_0)$ and $T_{km}^{-1} \circ T_b \colon \wp(C_0) \mapsto \mathfrak{P}$ connecting the two approaches. For brevity, I'll write K for $T_{km}^{-1} \circ T_b$ and B for $T_b^{-1} \circ T_{km}$. From the above, it is quick to see that B and K themselves form a(n antitone) Galois connection between $\mathfrak{P}$ and $\wp(C_0)$; that is, $p \subseteq K(C)$ iff $C \subseteq B(p)$.

Roschger defines *correspondence*: a precisification space $\mathscr{P}$ *corresponds* to a Barker-context C_0 iff: 1) they have the same domain, and interpret the same atomic predicates; 2) for every $p \in \mathscr{P}$, there is a nonempty $C \subseteq C_0$ such that $C = B(p)$ (relative to C_0); and 3) for every $C \subseteq C_0$, there is a $p \in \mathscr{P}$ such that $p = K(C)$.

One immediate problem with this definition is that condition 3 is not meetable with only consistent partial valuations: $K(\emptyset)$ must be the absolutely inconsistent interpretation (the interpretation that satisfies *every* literal), but $\emptyset \subseteq C_0$ for any C_0. Since Roschger gives no suggestion of allowing inconsistent partial valuations, I assume this is a problem. Corresponding to the restriction to nonempty C in condition 2, then, I'll assume a weakened condition 3, allowing that there may be no $p \in \mathscr{P}$ such that $p = K(\emptyset)$. If this modification is not made, then there are no corresponding models.

Note as well that condition 2 is very weak. $B(p)$ is the set of Barker-worlds in C_0 that are compatible with p, and there is always some such set, although it may be empty. But for there to be a nonempty such set, all it takes is a single Barker-world in C_0 compatible with p. This world, though, might decide a huge variety of propositions that p remains silent on.

One might hope that in corresponding models $p = K(B(p))$ and $C = B(K(C))$, for every p and nonempty C. But one would be disappointed; the connection between corresponding models can be less tight than this. There is no guarantee in general that $p = K(B(p))$, only that $p \subseteq K(B(p))$; the move from a partial interpretation to a Barker-context and back again can result in a *gain* in information, even in corresponding models. Similarly, there is no guarantee that $C = B(K(C))$, only that $C \subseteq B(K(C))$; the move from a Barker-context to a partial interpretation and back again can result in a *loss* of information.

Whether these connections between corresponding models are tight enough or not depends on the goal in play. The key goal here, I take it, is embodied in Roschger's

Theorem 9, which depends on Proposition 8: that for any corresponding $\mathscr{P}$ and C_0 and any update of a certain form consistent with C_0, there is a unique most general $m \in \mathscr{P}$ such that the update is true at m and, where C is the result of applying the update to C_0, $T_{km}(m) = T_b(C)$.

What to make of this proposition, and the theorem that depends on it, is an interesting question; I won't explore it here, since unfortunately they are both false. Begin with Proposition 8: it can be that there is no such m. A simple example of this involves a precisification space $\mathscr{P}$ with three partial interpretations and a Barker-context C_0 with two Barker-worlds. Assume a language with a single predicate P and three names referring to three distinct things in the domain; let Pa, Pb, and Pd be the three resulting atomic sentences. Now let $\mathscr{P} = \{p_0, p_1, p_2\}$, where $p_0 = \{Pa\}$, $p_1 = \{Pa, Pb, \neg Pd\}$, and $p_2 = \{Pa, \neg Pb, Pd\}$. Let $C_0 = \{c_1, c_2\}$, where $c_1 \Vdash Pa, Pb$, and $\neg Pd$, and $c_2 \Vdash Pa, \neg Pb$, and Pd. (The scales can be set up any which way, so long as these satisfaction relations result.) These are corresponding models, by the above (modified) definition: every $p \in \mathscr{P}$ has some nonempty $B(p) \in C_0$ and every nonempty $C \subseteq C_0$ has some $K(C) \in \mathscr{P}$.[3]

Now, update these models with an assertion of Pa. C_0 updated in such a way is just C_0 itself, but there is no $m \in \mathscr{P}$ such that $T_{km}(m) = T_b(C_0)$. The best candidate for such an m would be p_0, but $T_{km}(p_0)$ will include the worlds $\{Pa, Pb, Pd\}$ and $\{Pa, \neg Pb, \neg Pd\}$; neither of these worlds is in $T_b(C_0)$. The argument Roschger gives for Proposition 8 assumes that $S = T_{km}(T_{km}^{-1}(S))$, but this is the mistaken Proposition 4.[4]

Theorem 9 is supposed to establish that corresponding models make equivalent predictions when given matching inputs. The above counterexample to Proposition 8 is also a counterexample to Theorem 9, as the proposition $\neg(Pb \wedge Pd)$ holds at C_0 but not at p_0.[5] I think the best way to understand the failure of Theorem 9 is as undermining the interest of Roschger's notion of *corresponding models*. It may yet be the case that there is a relation that can hold between a Kyburg & Morreau model and a Barker model such that when it holds the models make equivalent predictions. That would be an interesting and valuable discovery. Roschger's relation of correspondence, despite his claims, is not such a relation.

[3]In particular, $B(p_0) = C_0, B(p_1) = \{c_1\}$, and $B(p_2) = \{c_2\}$, while $K(C_0) = p_0$, $K(\{c_1\}) = p_1$, and $K(\{c_2\}) = p_2$. In this pair of models, then, we have $K(B(p)) = p$ and $B(K(C)) = C$; they are even more intimately related than corresponding models are required to be.

[4]Although this is a counterexample to Proposition 8 in its present form, it might be thought that Proposition 8 is meant to hold only for assertions that *add* to the current context, rather than simply reiterating something—Pa in the above example—already in the context. But there are counterexamples to this weakened claim as well. For example, start from the C_0 in the counterexample above, and add one more Barker-world c_3, such that $c_3 \Vdash \neg Pa, Pb$, and Pd, to yield C_0'. Then let $\mathscr{P}' = \{K(C) \mid C \subseteq C_0' \text{ and } C \neq \emptyset\}$. $\mathscr{P}'$ and C_0' correspond to each other, and again satisfy the extra conditions that $p = K(B(p))$ and $C = B(K(C))$. However, an assertion of *any* of Pa, Pb, or Pd in this case will both add information and yield a counterexample even to the weakened Proposition 8. (An assertion of Pa, in particular, results in narrowing C_0' down to C_0, and moving to a subtree of $\mathscr{P}'$ that is just like $\mathscr{P}$; then the counterexample given in the main text works as it does there. Assertions of Pb or Pd work similarly.)

[5]If Theorem 9 is weakened to a claim only about atomic propositions, it still fails, although the counterexample is a bit more complex: add to the main-text counterexample an additional proposition Pe, and let Pe hold at every partial interpretation in $\mathscr{P}$ as well as every Barker-world in C_0. Now add one additional partial interpretation p_3 to $\mathscr{P}$, at which only Pa holds. It can be verified that the resulting models are still corresponding, although they no longer satisfy $K(B(p)) = p$, as $K(B(p_3)) = p_0$. An assertion of Pa is now sufficient to guarantee Pe in the Barker model, but not in the Kyburg & Morreau model.

In sum, I think Roschger has made a compelling case that there are important similarities between the frameworks of Kyburg & Morreau and Barker, but that he has not succeeded in describing just what those connections are. More work will be necessary to gain an understanding of the relations between these frameworks. The questions driving Roschger's paper are about the similarity or possible equivalence of predictions made by these two frameworks; but the formal and structural situation needs to be clarified before these questions can be properly addressed. I hope this note contributes something towards that clarification, as a step towards the important questions that remain.

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