Reply to David Ripley's Comments on *Comparing context updates in delineation and scale based models of vagueness*

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In his comments to my contribution 'Comparing Context Updates in Delineation and Scale Based Models of Vagueness' David Ripley points out some severe technical flaws. As he shows by a counter-example, the main result of the paper does not hold. The goal of this reply is to explain where exactly the source of the problem lies and to point out a possible way to solve it.

In the paper I take two contextual approaches to vagueness both describing the notion of context update very precisely, namely one by Kyburg and Morreau and one by Barker. Besides analyzing strengths and weaknesses of these approaches I try to prove that they both make the same predictions, once the same situation is modeled twice using each approach. This is done by introducing an 'intermediate representation' of contexts and showing how contexts as defined by each approach can be translated into this representation and also the other way round. Finally, once this connection is established one can observe that a context update (with consistent information) has exactly the same effect on the intermediate representation in either approach. However, as Ripley points out, these translation functions are not perfectly reversible, they rather form antitone and monotone Galois connections between contexts and their intermediate representations. In order to illustrate this I will stick to the same notation as in the paper and in Ripley's comment: the mappings from Kyburg and Morreau's models into the intermediate representation and back will be denoted as T_{km} and T_{km}^{-1}, and T_b as well as T_b^{-1} for Barker's models. For directly translating between Kyburg and Morreau's and Barker's contexts I use the mappings $K = T_{km}^{-1} \circ T_b$ and $B = T_b^{-1} \circ T_{km}$. Assume two corresponding models, one according to Kyburg and Morreau and the other one according to Barker, let the initial contexts be denoted as p_0 and C_0, respectively, and let the resulting contexts after a (successful) update be called p and C. Theorem 8 then states that the intermediate representations of p and C coincide: $T_{km}(p) = T_b(C)$. As Ripley shows by giving a counter-example this claim does not hold due to the non-reversibility of these mappings. However, it is still true—at least for his example—that $K(C) = p$ and that $B(p) = C$. One might be tempted to reformulate Theorem 8 in such a way that it does not directly refer to the intermediate representation but makes use of these connections instead and thus try to save Theorem 9. As it turns out, this does not suffice; one has to go deeper and rethink the notion of *corresponding models*. Intuitively, two models of the same situation, one as defined by Kyburg and Morreau and the other as

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defined by Barker, are called *corresponding* if they are constructed 'by making the same assumptions'. This entails that penumbral connections—explicit restrictions on the context space—in the first kind of model correspond to the (scalar) data in Barker's models. As the definition is formulated in the paper, it relies on the presented mappings between contexts and their intermediate representations, but it fails to capture this intuitive notion of correspondence. Ripley points to this deficiency in his counter-example to the Theorem 8: whereas in the context C_0 the proposition $\neg(Pb \wedge Pd)$ is true, it does not hold at context p_0, but they both fulfill the technical definition of corresponding contexts. This objection however is only partially valid: Kyburg and Morreau define *truth* at an incomplete precification point in a supervaluationist manner and in this case the proposition holds at all complete precifications of p_0. Nevertheless his observation is right that the definition is not strong enough to capture the intended notion.

In the following I will sketch how to change the relevant definitions of the paper to remedy this situation. First, let us recapitulate the mapping T_b from Barker's contexts to sets of classical worlds. It is beneficial to think of T_b as the partition of a context induced by $\mathscr{A}$.[1] The translation can be viewed as an *abstraction* of the context away from concrete degree values. This point of view makes it immediate that there cannot—in general—exist an inverse mapping T_b^{-1} and enables us to concentrate on the relationship between Kyburg and Morreau's notion of context and on this abstraction. There is one subtle problem with the mapping T_{km} as defined in the paper, namely its ignorance of penumbral connections. As already seen by Ripley's counter-example to Theorem 8 this inclusion of precification points violating penumbral connection leads to the failure of the theorem. Therefore we suggest another definition for T_{km}:

DEFINITION 1 *Let $\mathscr{P}$ be a precification space and $p \in \mathscr{P}$ a partial interpretation. Then the translation from p to a set of classical worlds, denoted as $T_{km}p$, is defined as the set of all complete precifications of p.*

$$T_{km}p =_{\scriptscriptstyle DEF} \{S_q \mid q \in \mathscr{P}, p \leq q, \text{ and } q \text{ is complete}\},$$

where S_q is defined as the smallest set such that $R(u) \in S_q$ iff $u \in l^+(R,q)$ and $\neg R(u) \in S_q$ otherwise for all predicates $R \in \mathscr{R}$ and all objects $u \in \mathscr{U}$.

In the paper, I gave an example why this definition of T_{km} is not fruitful: this way one cannot distinguish between different precification points which share the same set of complete precifications. However, since Kyburg and Morreau define truth at an incomplete point in a supervaluationist style, and not as local truth at a point, this loss of information does not seem to do any harm here. Incidentally, this revised version of T_{km} also sheds new light on the use of Shapiro's contextual approach to vagueness instead of supervaluation for Kyburg and Morreau's models as advocated in the paper. As T_{km} now heavily relies on complete precification points and in Shapiro's approach such precification points are not required to exist, this already gives hints to what respect Shapiro's approach may be more expressive than Kyburg and Morreau's as well as Barker's.

[1] As defined by Ripley, $c_1 \mathscr{A} c_2$ if and only if c_1 and c_2 agree on all atomic propositions.

Finally, the definition of *corresponding* contexts should be reformulated:

DEFINITION 2 *Let C_0 be a context as defined by Barker and $\mathscr{P}$ be a precification space in the sense of Kyburg and Morreau. C_0 and $\mathscr{P}$ are called* corresponding models *if the following conditions are met:*

- *for each vague predicate p in C_0 there is predicate P in $\mathscr{R}$ and vice versa,*
- *for each individual* **a** *in C_0 there is an object a in $\mathscr{U}$ and vice versa,*
- *for each $m \in \mathscr{P}$ there exists $C \subseteq C_0$ such that $T_{km}m = T_bC$, and*
- *for each non-empty $C \subseteq C_0$ there exists $m \in \mathscr{P}$ such that $T_bC = T_{km}m$.*

Note that now there is no reference to the inverse mappings in the definition of corresponding models and also note that this definition implies $T_{km}p_0 = T_bC_0$ for corresponding models. On the one hand this definition strictly ensures that all penumbral connections present in Kyburg and Morreau's model are enforced in each possible context in Barker's model and on the other hand the third clause forces Barker's model to be large enough to account for all possible complete precifications. Ensuring that penumbral connections are always in force nicely demonstrates why explicit penumbral connections in Kyburg and Morreau's approach are more expressive than the implicit ones in Barker's model. Consider, e.g., one predicate P, three objects a, b, and c, and the set $\mathscr{P}$ of precifications:

$$\{\{Pa\},\{Pa,\neg Pb,Pc\},\{Pa,Pb,\neg Pc\},\{\neg Pa\},\{\neg Pa,Pb,Pc\},\{\neg Pa,\neg Pb,\neg Pc\},\{\}\}.$$

Such a situation cannot be expressed in Barker's approach: there have to be possible Barker-style worlds corresponding to all of the four complete precifications. An update with Pb singles out the third and the fifth world, but there is no precification point (except the root) which has just these two as its complete precifications.[2]

Equipped with these new definitions of corresponding models and translations let us again take a look at Theorem 8. Let $\mathscr{P}$ and C_0 be two corresponding models and s a (consistent) proposition; initially, we have $T_bC_0 = T_{km}p_0$. It is now easy to see that in Barker's model all possible worlds not fulfilling s are filtered out, which amounts to all classical worlds $c \in T_bC_0$ which do not satisfy s being filtered out; let the resulting context be denoted as C. On the other hand, also in Kyburg and Morreau's model surely—as s is consistent with the current context—s will hold in all complete precifications of the new precification point p reached. However we still need to check that indeed $T_{km}p = T_bC$. We first show $T_{km}p \supseteq T_bC$: assume there exists a classical world $w \in T_bC$ such that $w \notin T_{km}p$. But then, by the definition of corresponding models, there exists a precification point $p_1 \geq p_0$ such that $w \in T_{km}p_1$ and s true at p_1. As $T_{km}p$ has been obtained just by updating with s and nothing else, $p_1 \geq p$ holds, but due to the monotony constraint on partial precifications this yields a contradiction to $w \notin T_{km}p$. The other direction, $T_{km}p \subseteq T_bC$ is analogous.

[2] Also, for Kyburg and Morreau's approach this situation is not entirely unproblematic: the update with Pb results in one of the respective two precifications, but it is not well-defined which one.

Summing up, I hope I have convincingly shown how to straighten out the errors made in my paper. In any case, there is a strong correspondence between these two approaches that seem to be so fundamentally different at the first glance.

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