Comments on *Vagueness in Language: The Case Against Fuzzy Logic Revisited* by Uli Sauerland

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Uli Sauerland's contribution to this volume is very important indeed, since it addresses a question of central relevance to anyone interested in logical approaches to reasoning in face of vagueness: why has fuzzy logic been so thoroughly discarded, and consequently neglected, by linguists as a possible tool for modeling vague language? The main "culprit" is quickly identified to be Hans Kamp's seemingly very influential paper "Two theories of adjectives" from 1975. (We refer to Sauerland's paper for precise references.) Clearly it is time for revisiting Kamp's arguments, variations of which, as Sauerland points out, can actually be found in a number of other places as well, including Rescher's classic monograph on many-valued logic. Of course one may and should also ask why engineers, mathematicians, and logicians who propagate fuzzy logic as a tool for handling vague propositions and predicates have largely ignored the work of semanticists of natural language. I think that an analysis of Kamp's arguments is a good starting point for answering that latter question too. However, my own assessment is somewhat different from Sauerland's. In particular I think that a number of implicit, but essential methodological assumptions, that separate natural language semantics from mathematical fuzzy logic, are largely to be blamed for the mutual disinterest if not outright dismissal. This would hardly be of much interest, if it were not the case that both disciplines claim that "models of vague language" are important items on their respective research agenda.[1]

Sauerland recapitulates Kamp's argument in a clear and succinct manner, that nicely assists its analysis from a contemporary point of view. I do not want to go into all components of the argument here, but rather want to focus on three aspects that may serve to illustrate what I mean by implicit underlying methodological principles that I think are relevant here: (1) logical contradictions, (2) ambiguity of logical connectives, and (3) truth-functionality.

1. Kamp claims that it is "absurd" to assign the value $\frac{1}{2}$, or in fact any other value but 0, to a sentence of the form $\phi \wedge \neg\phi$, asking rhetorically "how could a logical contradiction be true to *any* degree?" I find it interesting that seemingly only quite recently linguistic research has been conducted that confirms an intuition that not only in fuzzy logics,

[1]The situation is considerably complicated by the fact that the term "fuzzy logic" is used to label quite distinct areas of engineering, mathematics, and more recently mathematical logic, as pointed out at many places of this volume. However for the purpose of this short comment we may ignore these complications and focus on the question whether the family t-norm based fuzzy logics is of potential relevance in the context of formal models of reasoning with vague propositions and concepts.

but also, e.g., in paraconsistent logics is taken for granted. Namely that competent speakers are quite ready to accept—at least to some degree—sentences of the indicated contradictory form if vague predicates are involved. One can of course maintain that an acceptance of an utterance like "He is nice and not nice" indicates that it has to be read as "He is nice in one respect but not so in another respect". But the experiments by Alxatib/Pelletier and by Ripley, that Sauerland comments on, indicate considerable agreement to sentences where the involved gradable predicates make it very difficult to argue that they are not of the logical form $\phi \wedge \neg\phi$.

Of course, fuzzy logicians are quick to point out that acceptability is routinely a matter of degree, anyway. According to their approach the acceptability of ϕ and $\neg\phi$ to *equal degree*, rather than spelling logical disaster, only indicates the presence of a clear borderline case. There is no need to advertise here the benefits of such an approach, e.g., in dealing with sorites series. However fuzzy logicians often show little understanding of a methodological principle that seems to be implicit in almost all relevant linguistic literature: treat acceptability as a bivalent category; i.e., truth-conditions should be formulated in a manner that allows one to decide whether they apply or not, once the context is fixed and all relevant ambiguities are sorted out. I presume—and even think to understand myself well enough—that there good reasons for adhering to this principle in linguistics. In fact I submit that fuzzy logicians mostly respect an analogous principle themselves: treat validity and logical consequence as bivalent categories. Of course, there is also work on graded consequence and other forms of "fuzzification" of the meta-level. However even there, on pain of violating well established scientific standards, one better remains grounded in good old classical (bivalent) mathematics at the appropriate level of investigation. But of course working in classical mathematics (on the meta-level) does not prevent one to study non-classical models.

2. A standard reply of fuzzy logicians to the charge that a sentence of the form "*A* and not *A*" should receive the value 0 is to point out that in Łukasiewicz logic (and in many other logics for that matter) one can choose between two different forms of conjunction: strong conjunction (t-norm conjunction), with $\phi \& \neg\phi$ always yielding 0, and weak conjunction (lattice conjunction), where $\phi \wedge \neg\phi$ may receive any value between 0 and $\frac{1}{2}$. This of course triggers the question whether "and" could be ambiguous. To a logician this question may seem naive, since from the viewpoint of modern logic the answer is obvious: of course we can and should distinguish different forms of conjunction, i.e., different meanings of "and"! One of the hallmarks of the important area of substructural logics is the distinction between "additive" and "multiplicative" versions of connectives like conjunction. But also other logics, like some that are successfully applied in software verification, demonstrate the need to distinguish between different forms of conjunction in particular contexts. T-norm based fuzzy logics are certainly in good company here: in fact there are strong analogies between the weak/strong distinction in (e.g.) Łukasiewicz logic and the additive/multiplicative distinction in (e.g.) Girard's linear logic.

The fact that contemporary logic, for the most part, wholeheartedly embraces pluralism not only at the level of different logical systems but frequently also differentiates forms of conjunction within a single system, does not answer the question whether the word "and" is ambiguous in standard English (disregarding its extensions to specific mathematical and technical vocabularies). I presume that many linguists will disagree with my conviction that the latter question is unanswerable empirically. After all well accepted methods, in particular the ellipsis test, seem to be available for testing for ambiguity. But I have never seen any of these tests applied successfully to words that correspond to logical connectives and I strongly doubt that it can be done at all. Sauerland dismisses the idea that "and" might be lexically ambiguous as "clearly *ad hoc*" without presenting further arguments. However this only confirms my conviction that not clear empirical facts, but rather certain methodological presuppositions are at play here. From conversations with linguists I have learned that to support the idea that "and" might be ambiguous one should at least be able to point to languages where the allegedly different forms of conjunctions are indeed expressed by different words. It so happens that I currently struggle to learn a bit of Khmer (Cambodian). I was quite surprised to be exposed already in the first two lessons of my textbook[2] to three different Khmer expressions that are all translated simply as "and" there, but that seemingly are not used fully interchangeably `http://www.english-khmer.com/` returns two distinct Khmer words for "and"). Note that I do not claim to have found a language that witnesses the ambiguity of conjunction. In fact, I can think of various alternative explanations for having different words in a language that all roughly correspond to the English "and". I only want to emphasize that if, on the one hand, logicians and computer scientists have good reasons to distinguish between distinct forms of conjunctions and, on the other hand, linguists find good reasons to insist that there is only one core meaning of the word "and", then this is not because one of the two disciplines got it all wrong. It rather indicates that the question what the word "and" means and, in particular, whether it is ambiguous cannot be answered satisfactorily without subscribing to a whole package of theoretical and methodological assumptions that reflect different research goals and traditions.

3. As recognized by Sauerland (see his footnote 3 on a corresponding observation by Libor Běhounek) the main problem with a "naive" attempt to use fuzzy logic to model the meaning of vague language, as derived from observable behavior of competent speakers, is its insistence on truth-functionality.[3] But again, I think that a quite fundamental methodological confusion is sometimes impeding discussions about "models of reasoning with vague language" in logics and linguistics, respectively. The particular confusion that I have in mind, however, seems to be easily resolvable by pointing out the ambiguity of the word "model" here. Whereas linguists aim for a systematic *description* of observable linguistic behavior—or so the story goes—logicians prefer their formal models to be interpreted *prescriptively*, as tools for successful reasoning, in particular in scenarios where the degree of nesting of logical operators transcends levels commonly observed in natural language use. Truth-functionality is a good case in point. In fact we need not focus on fuzzy logic to understand the issue. Already for classical logic it is clear that the principle of truth-functionality is not well reflected in the actual behavior of human agents in face of logically complex reasoning tasks. Nor is the actual use

[2] Richard K. Gilbert, Sovandy Hang: "Cambodian for Beginners", Paiboon Publishing, 2004.

[3] In this short comment, I have to remain silent on the various fuzzy logic based models (in a wider sense) that are decidedly not truth-functional. Didier Dubois's contribution to this volume discusses some of them. But he also emphasizes that fuzziness and vagueness are to be treated as two distinct features anyway.

of simple natural language phrases of the form “If *A* then *B*”, “*A* and *B*”, etc., always directly conforming to the corresponding classical truth tables. Yet it is hard to deny that imposing truth-functionality is a very useful methodological principle for (a certain type of) “models of reasoning”. A familiar analogous case is probabilistic reasoning: while psychologists point out that human reasoners hardly adhere to the mathematical laws of probability theory, one can also demonstrate in which sense and under which circumstances they actually *should* do so.

The issue about using truth-functional logics for modeling observable linguistic behavior is often taken to be fully settled by the above remarks: logics (including fuzzy logics) are prescriptive tools, but linguists strive for formal descriptions of natural language features as reflected in empirical data. However I think that this plea for non-interference is definitely too quick. First of all, the heavy use of logical machinery (largely of higher order classical and intensional logics) witnesses the fact that formal logic is not just a prescriptive tool, but undoubtedly useful also in defining complex descriptive models, if not handled too naively. The latter qualification is meant to emphasize the well known fact that, whatever we mean by the “logical form” of a sentence in natural language, we can hardly expect it to be obvious or always easily and uniquely determined. Secondly, I maintain that not only logicians, but also semanticists of natural language, abstract away considerably from “naked” empirical data when constructing their models. As we know well from philosophy and history of science: observations are always theory-laden; and that is not a defect, but rather a constructive hallmark of scientific reasoning. On the other hand, fuzzy logicians cannot credibly claim that they do not care at all about natural language use, but only aim at purely prescriptive models that provide examples of interesting mathematical artifacts. Such claims clearly contradict the manner in which the considerable amount of research on “fuzzy linguistic variables”, “linguistic hedges”, “fuzzy quantifiers”, etc., is motivated and explained.

It is probably still too early to judge whether an interaction between mathematical fuzzy logic and natural language semantics that is profitable for both sides is worthwhile and feasible. I think that the work of Uli Sauerland and a number of his colleagues in linguistics justifies cautious optimism: it witnesses a level of interest in contemporary fuzzy logics that, with hindsight, renders the all too brief encounter between the two fields in the 1970s a rather unhappy historical fact. Needless to say that I eagerly hope for more awareness of contemporary linguistic research on vagueness on the side of fuzzy logic as well. In any case, I am convinced that a better mutual understanding of the often quite different aims, methods, and theoretical as well as cultural background of the two research fields will prove to be profitable well beyond attempts to simply combine techniques originating in different corners of expertise about the multi-faceted phenomenon of vagueness.

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