Comments on *Vagueness in Quantity* by Stephanie Solt

PAUL ÉGRÉ[1]

Overview

There is a very active trend of research in linguistics at the moment about the right way to interpret vague expressions of quantification such as "many", "few" and "most". In this note I would like to focus my discussion on the second part of Solt's exciting paper, regarding the meaning difference between the two determiner expressions "most" and "more than half". The problem of the relation between those expressions has been addressed in a number of recent works, in particular by M. Ariel (2003, 2004), M. Hackl (2009) and by P. Pietroski et al. (2009). Like Solt, these authors give empirical evidence for the view that "most" and "more than half" make different contributions to the meaning of the sentences in which they occur. Importantly, however, they tend to agree that the truth conditions for complete sentences of the form "most *A*s are *B*s" and "more than half of *A*s are *B*s" are logically equivalent.[2] Solt's main emphasis in her paper is slightly different. It rests on the idea that "more than half" and "most" manipulate different scales of measurement, corresponding to different levels of informativeness. Because I find myself in agreement with most of Solt's account in her paper (not to say more than half of it), in this commentary I would like first to adduce further evidence in favor of her semantic analysis of "most", in particular concerning the need for measure functions. I then discuss the status of the entailment relations between "most" and "more than half" and some tentative counterexamples to either direction of implication between the two determiners.

"Most" and measure functions

At least two sets of facts have been presented in the literature to suggest that "most" and "more than half" might have different meanings. The first, investigated in particular by Ariel (2003, 2004), is that "most" typically receives "larger majority interpretations than 'more than half' " (Ariel 2003). This is illustrated by Solt's example "most Americans are female", a sentence which seems less appropriate to use than "more than half of Americans are female" to describe the proportion of 50.7% of female among Americans. The second set of observations concerns the fact that the determiner "most", which rests on superlative morphology (see Hackl 2009), does not specify an explicit comparison

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[2]This is not exactly so for Ariel, who considers that "most" semantically excludes "all", unlike "more than half". For her, "most students attended the party" must therefore be false when "all students attended the party" is true, whereas "more than half of the students attended the party" still counts as true. Unlike Ariel, I consider this difference as pragmatic rather than semantic. I shall not say more about it here though.

point for the minimum proportion of As that is needed to say that "most *A*s are *B*s". In contrast to that, "more than half", which uses the comparative "more", fixes a precise lower threshold by means of the proportional expression "half". This morphological difference plays a central part in Hackl's decompositional theory of the determiner "most" in particular. Besides those two sets of observations, Solt in her paper puts particular emphasis on a third contrast, which concerns the fact that "most" can quantify over domains that are not enumerable, or otherwise for which no obvious measure is available, whereas "more than half" appears to require either enumerability, or a scale of measurement with clear measurement units. An illustration of this phenomenon is given by Solt concerning the combination of "most" with mass nouns, as in the pair "most racism is not so blatant" vs. "(?) more than half of racism is not so blatant".

The first aspect of Solt's paper on which I wish to dwell concerns precisely the idea that the semantics of "most" must involve a measure function in order to take account of quantification over domains that are not enumerable. On her account, "most$(A)(B)$" means that the measure of things that are A and B is greater than the measure of things that are A and not B. Interestingly, the idea that the semantics of "most" requires a measure function has been put forward independently by Schlenker (2006) and by Chierchia (2010: p. 138, fn. 44), each time in the context of broader considerations about mass nouns. Schlenker, for instance, points out that a sentence like "most of the water in the room is poisoned" can only be judged true or false depending on how water is measured (for instance it will depend on whether it is the overall volume, the number of water containers in the room, and so on). Similarly, Chierchia points out that we can meaningfully judge the following sentence about numbers to be true:

(1) Most numbers are not prime.

despite the fact that the set of prime numbers and the set of composite numbers have equal cardinality.

It is worth lingering on that particular example, because prima facie, it gives a direct counterexample to the standard cardinality analysis of "most" given in generalized quantifier theory, and moreover because it lends further support to Solt's analysis. Indeed, on the standard analysis, "most *A*s are *B*s" should mean that the cardinality of the set AB is greater than the cardinality of the set $A\overline{B}$. However, since there are as many prime numbers as there are natural numbers, this predicts that (1) should be false in that case. There is a legitimate sense in which we can utter (1) truly, however. Interestingly, it seems odd to say:

(2) (?) More than half of the natural numbers are not prime.

In this case the oddness does not appear to originate from the fact that the sentence would be false. For it would be equally odd to say that "(?) exactly half of the natural numbers are not prime", in order to mean that "there are as many primes as there are nonprimes". Moreover, the oddness cannot result from the fact that the natural numbers are not enumerable, since they are. Rather, the problem seems to be exactly related to the fact that the scale of cardinal numbers is no longer a ratio scale once we reach infinite cardinals, in agreement with Solt's remarks. Indeed, although there is an absolute zero point on the scale of cardinal numbers, this scale is no longer a ratio scale for infinite cardinalities, since on a ratio scale, when $\mu(A) = a$ and $\mu(B) = 2 \cdot a$, one can meaningfully say that "there are twice as many *B*s as *A*s", and infer "there are more *B*s than *A*s". At the very least, infinite cardinalities do involve a departure from the idea that the concatenation of an interval of a given size with an interval of the same size will yield a larger interval, as happens for measures defined over the real numbers.

The situation is different with (1). As suggested by Chierchia, we likely accept this sentence in relation to a measure of the frequency of prime numbers among natural numbers. Let us elaborate on this: we know from the prime number theorem that the number of prime numbers less or equal to n tends to $n/\log(n)$ as n tends to infinity (see e.g. Jameson 2003). This entails that the probability for a number less or equal to n drawn at random to be prime can be approximated by $1/\log(n)$. Hence, the larger the initial segment of natural numbers we consider, the lower the probability will be that a given number in that segment is prime. When n tends to infinity, this probability tends to 0, and the probability for a number not to be prime tends to 1. This can be seen as one way to associate a measure to the set of prime numbers. Whether we actually understand a sentence such as (1) in relation to that measure is open to discussion. Maybe what we have in mind when we utter (1) is something related, but simpler, like the following: for any n sufficiently large, we can observe that the number of prime numbers less or equal to n is strictly less than the number of nonprime numbers less or equal to n. That is, any initial segment of the natural numbers that is sufficiently large contains more composite numbers than prime numbers. Possibly, therefore, we may be using a standard cardinality measure to check the meaning of (1), but restricting cardinality comparison to arbitrarily large segments.

Could "more than half" fail to entail "most"?

Besides the emphasis on measure functions, another aspect of Solt's discussion concerns the idea that "most" is used "when the number or measure of *A*s that are *B* is *significantly greater* than the number/measure of As that are not B" (emphasis mine). By contrast, "more than half" will be true even if the number or measure of *A*s that are *B* is just above a half on the relevant scale. One aspect Solt does not make entirely explicit is whether the notion "significantly greater" should then be part of the truth conditions of sentences with "most", or whether it is only pragmatically relevant. Supposing it were part of the truth conditions for "most", this would directly explain that "most" gets larger majority interpretations than "more than half". On the other hand, it would predict that a sentence like "most Americans are female" should be judged false for majority proportions such as 50.7%, that are only slightly above 1/2.

Solt leaves open whether this should be so, or whether we only have a pragmatic preference for using "more than half" in that case. As I understand Solt's account, however, the notion of significant difference should likely *not* be part of the truth conditions of "most". It appears to be relevant primarily in relation to our analogical representation of magnitudes in order to check that the measure of AB is greater than the measure of $A\overline{B}$. The reason I see for this is that we can still use "most" in cases in which we can rely on precise measurement for the comparison of the set AB with the set $A\overline{B}$. For instance, Pietroski et al. in their study point out that even though subjects use "most" in accordance with the laws of psychophysics to estimate whether there are more yellow dots than non-yellow dots in a visual display, "one extra yellow dot suffices (up to the stochastic limits of the ANS to detect this difference) for judging that most dots are yellow". This means that whenever "more than half$(A)(B)$" can be judged true, "most$(A)(B)$" should also be true, at least if the same dimension of comparison is used each time.

On the other hand, if dimensions of comparison can vary for "most" and "more than half", judgments may come apart, but only as a result of pragmatic ambiguity. For instance, imagine a jar filled with 100 very small black marbles at the bottom, and 20 big red marbles on top of them, and suppose that the 100 black marbles fill a fifth of the volume of the jar, while the 20 red left occupy the rest of the jar's volume. In the cardinality sense, "more than half of the marbles in the jar are black" is true. However, I could easily imagine people to judge that "most of the marbles in the jar are red" (and therefore not black), making use of some form of coercion, to mean "most marbles are red *in proportion to the jar's volume*" (consistently with Solt's remarks about "many").[3] It would be interesting to see if one can say in the same sense: "[in proportion to the jar's volume] more than half of the marbles in the jar are red".

As I see Solt's theory, the entailment from "more than half" to "most" is semantically robust, and the fact that larger majority interpretations are assigned for "most" calls for a pragmatic explanation. Regarding the converse entailment, from "most$(A)(B)$" to "more than half of$(A)(B)$", I wonder if an appropriate way to describe Solt's view may not be in terms of Strawson-entailment (von Fintel 1999). That is, "most$(A)(B)$" will entail "more than half$(A)(B)$" only if the presuppositions of "more than half" are satisfied, that is, only when a common ratio scale can be used for both expressions. Solt's example "most racism is not so blatant" is an example of that sort: the entailment to "more than half of racism" only fails because no obvious ratio scale is available for the latter to be used felicitously.

Relative majorities

Could there be, however, reasons to think that "most" and "more than half" need not entail each other either way, even when the same scale can be used for both? One tentative example I can think of concerns the case of relative or qualified majorities. Consider an election in which the rule is that a proposal will be accepted only if it gets a majority of at least 60% of the votes, and consider a case in which the proposal gets only 54% of the votes. Plausibly, one should be willing to say that "more than half of the people voted for the proposal". But it might be judged false that "most people voted for the proposal", if "most" in that case is evaluated against the minimum of 60%. This would be a case where in order for the set of AB to be counted as greater than the set of $A\overline{B}$, the proportion of AB simply must exceed 60%. Conversely, even when the majority rule is set at 50%, one may find occurrences of "most people voted for candidate A" even as the proportion of people who voted for candidate A is less than 50%, provided A is the candidate with the largest number of votes, and with a significant relative majority in comparison to other candidates (viz. 40% for A vs. less than 4% for each of 15 competitors).

Such examples are not conclusive against the entailment relations between "most" and "more than half", however. For the first example, my intuitions easily change if

[3] I believe actual experiments have been made on examples of that sort, which I remember G. Chierchia to mention in a lecture. Unfortunately, I can no longer track the references for this example.

the majority rule is set at 75%, and exactly 70% voted for the proposal. In that case, it seems fine to me to say that "although most people voted for the proposal, the majority was not reached". For the second example, a more natural determiner expression precisely seems to be "a majority" rather than "most". Ariel (2004: pp. 696–97) considers actual election cases with reports such as "Yosifov was elected by a majority of 41.6%". While use of "a majority" is perfectly appropriate, it is unclear to me whether one could say "most people voted for Yosifov" in that case, except maybe for a situation in which Yosifov would have gotten an overwhelming proportion of votes in comparison to each alternative candidate. If so, it may be that "most" is used for comparison to subsets of the whole domain, namely to mean "most people voted for Yosifov *in comparison to any other candidate they could vote for*". That is, the sentence may be judged true by comparing the proportion of people who voted for Yosifov to the distribution of votes among other candidates (that is, to the respective proportion of each other candidate), assuming the resulting ratio to be high each time. If such uses of "most" exist, consequently, I believe that they remain entirely compatible with the measure-theoretic analysis of "most" outlined in Solt's paper.

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Paul Égré
Institut Jean-Nicod
Département d'Etudes Cognitives
Ecole Normale Supérieure
29, rue d'Ulm, 75005 Paris
E-mail: paul.egre@ens.fr