

Logical Models of Reasoning with Vague Information 2009

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Invited talks

Fuzziness, Uncertainty and Vagueness: Toward a Less Blurry Picture?

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The issue of vagueness has been a matter of controversy for many years. The debate around this notion has become even more lively with the introduction to fuzzy sets, whose alleged relevance to the handling of vagueness has been much criticized. However this critique is closely related to a recurrent misunderstanding of the role of multivalued logics and the meaning of truth values other than true and false. This talk tries to clarify the situation of fuzzy sets with respect to the representation of uncertainty and vagueness. More specifically, we shall recall various scenarios in which a concept partitions a universe of discourse (sometimes implicitly) into three regions rather than two, thus leading to situations that some people may call vagueness:

- Non-Boolean predicates
- Ill-known extensions
- Similarity-based refinement of extensions
- Collective representations of concepts
- Incomplete information
- ill-described (rough) sets

Several points will be raised, that may trigger some discussions

1. Fuzzy sets capture gradualness, not vagueness proper. The vagueness of a gradual concept lies in the difficulty to determine the right membership function, not in the uncertainty about an elusive or fictitious threshold between membership and non-membership.
2. There is an incompatibility between uncertainty and truth-functionality and a difference between degrees of belief and degrees of truth. In fact, truth-functionality is mathematically inconsistent in all above scenarios but one (the non-Boolean case).
3. The distinction between receiving a piece of linguistic information involving a gradual predicate, and uttering the corresponding statement. It is claimed that part of the dispute between fuzzy sets and other approaches such as supervaluations and the like is due to a lack of precision on the context in which a gradual predicate is used:
 - when uttering a gradual statement like “John is tall”, there is an implicit ill-known threshold that underlies the decision, by the utterer, of making such an assertion or not.
 - when receiving it there is no decision involved (but that of accepting it) and the issue is to find a faithful gradual modeling of the information conveyed, which does not presupposes the need for a threshold between membership and non-membership.

The bottom line is that part of the controversy between fuzzy sets and vagueness is unwarranted because the question of whether a predicate is Boolean or not is distinct from the issue of uncertainty about meaning.

References to author's works

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Semantics as Meeting of Minds

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I propose a semantic theory that is not construed as a mapping of language to the world, but a mapping between individual meaning spaces. The meanings of linguistic entities are established via a “meeting of minds”. The concepts in the minds of communicating individuals are modeled as regions in conceptual spaces. If concepts are convex, it will be possible for interactors to agree on a joint meaning even if they start out from different representational spaces. The theory is then applied to an analysis of the composition of concepts.

Probability, Fuzzy Logic and Bets: Foundational Issues and Some Mathematics

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A very deep foundational problem is to give a definition of the probability of an event. Many approaches are known (the frequentist approach is one of them). Although it does not give a mathematical definition, the approach I prefer is De Finetti's approach to probability in terms of bets. There are many generalizations to this approach. One of them is an extension of probability to many-valued events. This approach leads to a coherence criterion which is similar to the classical one. That is, a book is coherent (i.e., there is no winning strategy for the bettor, independently of the outcome) iff it can be extended to a state (the many-valued counterpart of a probability measure).

A second generalization is the interpretation of conditional probability over many-valued events in terms of bets. Our idea is some kind of 'fuzzification' of De Finetti's approach. That is, betting on A given B is like betting on A , with the only difference that the whole payoff will be multiplied by the truth value of B . If B is completely false, the bet is null. According to this rule, it turns out that a book is coherent (i.e., there is no winning strategy for the bettor, independently of the outcome) iff it satisfies Kroupa's axioms for conditional probability over fuzzy logic.

In the previous interpretations, games are reversible, that is, the bettor may reverse the roles bettor-bookmaker. Real bookmakers, however, are not reversible, as everybody knows. This leads to the following question: is there a criterion of rationality for non reversible bookmakers?

We propose the following criterion: a book is rational iff there is no bad bet for the bettor, that is, there are no bet B and system of bets S such that S gives to the bettor a better payoff than B , independently of the outcome. The reason for our choice is the following: if B is a bad bet, then the bookmaker can make his book more attractive for the bettor (by reducing the betting odd for B) without loosing money when the bettor plays his best strategy. [The idea is that if a book is not attractive, then no bettor will bet on it, and the bookmaker will not earn money from it.]

We find the precise laws that a book should satisfy in order to be rational from this point of view. That is, a book is rational (in the sense that it forbids bad bets) iff it extends to an upper probability which is the pointwise maximum of a family of states iff it satisfies some equational properties which will be made precise during my talk.

Linguistic Conditions on Theories of Vagueness

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Natural language is notoriously vague, and linguistic semantics still has a lot of work to do here. There are some general conditions on linguistic accounts of vagueness like compositionality. My main interest in this talk, however, is to argue that for natural language semantics at least two mechanisms underlying vagueness are required. One mechanism operates on categories and has epistemic underpinnings. The other operates on scales, involves granularity and is not epistemic. Finally, I show new evidence for this distinction from a set of linguistic tests using expressions that regulate vagueness.

Fuzzy Logic and Higher-Order Vagueness

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The major reason given in the philosophical literature for dissatisfaction with theories of vagueness based on fuzzy logic is that such theories give rise to a problem of ‘higher-order vagueness’ or ‘artificial precision’. In this paper I first survey the main suggested solutions to this problem: fuzzy epistemicism; a hierarchy of vague metalanguages; truth values with an internal hierarchical structure (blurry sets); and fuzzy plurivaluationism. I then argue that in order to decide upon a solution, we need to understand the true source of the problem. Two possible sources are discussed: the problem stems from the very nature of vagueness (i.e. from the defining features of vague predicates); or the problem stems from the way in which the meanings of predicates are determined (by the usage of speakers together with the facts about their environment). I argue that the latter is the true source of the problem—and on this basis, that fuzzy plurivaluationism is the right solution.

The Vague Expression of Quantity

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Classic accounts of vagueness have focused on vague adjectives (thin, red, expensive) and nouns (heap). But vagueness is also pervasive in the expression of quantity and amount. This talk investigates three types of vague expressions of quantity: i) inherently vague quantifiers (e.g. many, much, few); ii) round numbers used with an approximate interpretation (e.g. 100 used to convey ‘roughly 100’); iii) expressions with range rather than point-based denotations (e.g. most, more than half). Each presents a distinct set of interpretive issues, and requires a distinct approach to semantic analysis. It is shown that expressions of type (i) lend themselves to an extension of current degree-based accounts of gradable adjectives; those of type (ii) require the introduction of notions of scale granularity (Krifka 2009); and those of type (iii) must be analyzed with reference to scalar alternatives, and raise questions regarding possible representations of quantity comparisons. Taken together, findings from this domain contribute to a broader perspective on the semantics of vagueness and related phenomena.

Contributed talks

Free BL-Algebras, Revisited

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About one year ago we presented a representation theorem for free BL-algebras (Aguzzoli-Bova, *The Free n -Generated BL-Algebra*, submitted). Our result was stated in the terms of an inductive definition: the elements of the free n -generated BL-algebra were expressed as a patching combination of elements of the free k -generated BL-algebras, for $k < n$. We have revised our result and we have managed to eliminate the inductive definition, obtaining a new representation theorem. Each element of the free n -generated BL-algebra is represented as a finite collection of elements of free Wajsberg hoops, organised in a combinatorial structure based on the ordered partitions of the set of generators.

Intuitionistic Logic and Reasoning with Vague Information

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Using the concept of choice sequences we provide arguments, that intuitionistic logic is the preferred way of reasoning with vague/incomplete informations.

On Deductive Fuzzy Logics as Logics of Gradual Properties

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Formal systems of deductive fuzzy logic were originally developed with the intent to describe the logical behavior of properties that are *vague*. Since, however, the link between deductive fuzzy logic and vagueness has often been disputed, it may be desirable to formulate a less controversial motivation for these logics that avoids references to vagueness or partial truth. I will present an explanation of deductive fuzzy logics as logics of *gradual* (rather than *vague*) properties, which may be less subject to usual criticisms, and free of the liability to provide a philosophical explanation of the phenomenon of vagueness. It can still be argued that many instances of the sorites paradox arise from an inappropriate application of classical reasoning, which is only suitable to bivalent properties, to attributes that are in fact gradual, and therefore governed by a different logic.

Several related issues will be discussed in the talk, including the difference between the *logic* and *formal semantics* of gradual properties, the justification of the laws of deductive fuzzy logic as logic of gradual properties, the role of truth values (independent of a conception of partial truth), and the importance of internalization of the deductive apparatus of the logic (which is a distinctive feature of deductive fuzzy logics among other fuzzy approaches and a missing ingredient in Smith's account of vagueness in his book *Vagueness and Degrees of Truth*).

A Semantic Theory of the Interpretation of a Vague Language

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I present a semantics for the interpretation of a language that includes vague predicates, based on a refinement and extension of the “supervaluation” approach. The proposed theory provides a formal characterisation of the space of possible precise interpretations (precisifications) of the language, in terms of parameters that specify the applicability of vague concepts by means of thresholds on the values of observable measurements. These observables also determine a set of possible states of the world. Thus the truth of a proposition depends on both the possible world and the precisification with respect to which it is evaluated.

On the basis of this semantics, the acceptability of a proposition to an agent is characterised in terms of the agent’s beliefs about the world and attitude to admissible interpretations of vague predicates. A further extension, in which probability spaces are defined over the sets of possible worlds and precisifications, is used to give a statistical measure of the acceptability of a proposition to an agent. The theory is applied to analysing certain aspects of the cognitive evaluation of vague information—in particular the sorites paradox.

Some Remarks on Fuzzy Modal Logics

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The purpose of this contribution is to explore recent developments on fuzzy modal logics. The first (and main) part of the talk will consist on an overview of this fuzzy modal framework. Here the fuzzy modal logics $\Lambda(K, \mathbf{A})$ (see [1]), where K is a class of Kripke frames and \mathbf{A} is a BL chain, will be introduced, and several properties that they satisfy will be stated.

In the second (and last) part we will focus on some preliminary (and naive) study about practical cases where the previous logics seem suitable to be applied. The motivation behind these practical cases can be found in the properties stated in the first part of the talk.

References

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Models of Higher-Order Vagueness in FTT

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In this contribution, we present two models of higher-order vagueness in the frame of fuzzy type theory (FTT). First, we sketch the formal system of FTT and discuss its role in the modeling of vagueness. Then, we show how higher-order vagueness can be modeled, first, using some special formulae of FTT and second, using fuzzy/linguistic IF-THEN rules.

Higher-order vagueness arises when we deliberate about consequences of the definition of a vague property that should have definitely positive, definitely negative, and borderline cases. The main idea consists of thinking about the fact that vagueness has no “end”, i.e. that there are no sharp boundaries in any respect. According to [1], there is neither a sharp boundary between positive and negative cases, nor between borderline cases and other sharp boundaries. It means, besides others, that if a vague property $A(x)$ of elements is given, then there are values of x for which “ $A(x)$ is borderline” is itself borderline. In other words, the property “to be borderline” is also vague. We propose a fuzzy logic model of higher-order vagueness in FTT using special formulae expressing properties “to be vague” and “to be borderline” and show some results, e.g., that property “to be borderline”, expressed in FTT, is indeed vague.

Another possibility to model higher-order vagueness in fuzzy logic is based on the use of the theory of fuzzy/linguistic IF-THEN rules [3], i.e. the rules of the form

$$\text{IF } X \text{ is } \mathcal{A} \text{ THEN } Y \text{ is } \mathcal{B}$$

where \mathcal{A}, \mathcal{B} are trichotomous evaluative linguistic expressions [2] of natural language, or they can be arbitrary vague properties. Using these rules we can characterize the vagueness of a given predicate of arbitrary order.

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Vagueness and Non-Transitive Entailment

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In this paper we present and discuss an analysis of the sorites paradox based on a notion of tolerant entailment relation, namely entailment that is non-transitive. Our proposal agrees with that of Zardini 2008, in the idea that limiting the transitivity of the relation of logical consequence can account for the naive intuition that the major premise of the sorites can be taken on board without contradiction. Our approach also bears some affinities with subvaluationism (Hyde 1997), but the resulting logic is essentially different, as we proceed to discuss here. As shown in previous work (van Rooij 2009), our approach is furthermore connected to the framework of Williamson 1994, and leads to a version of the principle of tolerance that is equivalent to Williamson's epistemic formulation.

In the first part of the paper, we define a tolerant relation of satisfaction for first-order logic, based on the notion of semi-order (see Luce 1956, van Rooij 2009), for which the induced relation of entailment is non-transitive. Thus, each predicate of the language comes with an associated similarity relation that is reflexive, symmetric, but not necessarily transitive (see Williamson 1994). Our main departure from classical logic concerns atomic sentences: basically, ' $P(a)$ ' is defined as tolerantly true if there is an object similar to a that classically satisfies P . We compare two versions of the semantics and show that on one of them, a conditional of the form 'if $P(a)$ then $P(b)$ ' is tolerantly valid whenever a and b are similar.

In the second part of the paper, we discuss how much of classical logic the notion of tolerant entailment preserves, and focus on the comparison between our semantics and subvaluationism (Hyde 1997). Like subvaluationist semantics, our semantics is paraconsistent, as even a conjunction of the form ' $P(a)$ and not $P(a)$ ' can be tolerantly true for borderline cases in our framework. However, we show that the semantics is better behaved regarding the class of logical validities that it preserves from classical logic.

Truthlikeness, Graded Similarity and Fuzziness: Some Logic-Based Approaches

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The similarity-based notion of truthlikeness was proposed independently by Hilpinen (1976) within possible worlds semantics and by Tichý (1974) within propositional logic. The basic idea of this *similarity approach* is that the degree of truthlikeness of a sentence φ depends on the similarity between the states of affairs that are compatible with φ and the true state of the world. According to Niiniluoto (1987), we can consider the *truthlike* value of a sentence as its degree of “proximity to the truth”, even though it may be false. This degree should be given by the “distance” that separates (or dually, by the similarity between) the models of this sentence and the models of the “reality”.

Thus, this notion of truthlikeness can be regarded as a special case of the more general concept of similarity and its logical counterparts to some form of similarity-based reasoning, this last concept being often associated with reasoning by analogy which is an important form of non-demonstrative inference. Similarity-based reasoning aims at studying which kinds of logical consequence relations make sense when taking into account that some propositions may be closer to be true than others. A typical kind of inference which is in the scope of similarity-based reasoning responds to the form “if φ is true then ψ is *close* to be true”. Notice that the fact of ψ being close to (or approximately) true has nothing to do with a problem of missing information (uncertainty) not allowing us to know whether ψ is true or false.

In the literature one can find qualitative (or comparative) and quantitative approaches to similarity-based reasoning. Comparative approaches aim at formalizing e. g. expressions like p is closer to q than r , like those of Nicod (1970), Lewis (1973), Williamson (1988) and Konikowska (1997). Quantitative approaches, that are based somehow on a numerical definition of degree of truthlikeness or similarity following Niiniluoto (1987) and Weston (1987). This kind of approach, although not always within a formal logical framework, has been further developed by making use of Zadeh’s fuzzy similarity relations as graded modelings of similarity relations, originally to be used in techniques of categorization and clustering. A key contribution in this direction was made by Ruspini (1991) who pushed forward the idea of similarity as one of possible semantics for fuzzy sets, where membership degrees are understood as similarity degrees to some of the prototypes of the given fuzzy set (Dubois and Prade, 1994). In this direction one can establish interesting links between similarity-based reasoning and fuzziness, see the references.

The aim of this paper is to survey a class of logical formalizations of similarity-based reasoning models where similarity is understood as a graded notion of truthlikeness. We basically focus on semantically-oriented approaches based on several notions of approximate entailments, providing different formalisations in terms of suitable systems of modal and conditional logics, including for each class a system of graded operators with classical semantics, as well as a system with many-value operators. Finally, we also explore some nonmonotonic issues of similarity-based reasoning.

Here are the main references the talk will be based on:

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Vagueness at the Interface between Logic, Philosophy, and Linguistics

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In this talk we plan to summarize and assess experience with debates about theories of reasoning under vagueness involving three different research traditions: formal logic, analytic philosophy, and formal semantics (and pragmatics) in contemporary linguistics. These three fields have clearly overlapping interests in understanding and modeling correct reasoning with vague predicates. Moreover, the same phenomena, in particular the sorites paradox, borderline cases, and graduality are discussed in almost all approaches to vagueness. Nevertheless, quite different methodologies and research goals make interdisciplinary research on vagueness often surprisingly difficult and controversial. We attempt to clarify some crucial, often hidden assumptions and methodological principals that may help to better understand the complications involved. Moreover, we will hint at some topics that show that ideas from the three different research traditions involved can be fruitfully combined, if their distinct aims and methods are properly taken into account. If time permits, we will further speculate on whether the lessons drawn also apply to further relevant disciplines, like computer science, psychology, and cognitive science.

Constraints on Definite Descriptions with Vague Adjectives: Towards a Computational Model

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A characteristic feature of definite descriptions involving vague adjectives—such as ‘*the big triangle*’ or ‘*the long book*’—is that they can be used to refer to an object that differs from others in the current context only in the extent to which it possesses the property expressed by the adjective, even if that property would not generally be judged to be true of the object. Taking a Natural Language Generation perspective, we shall investigate the constraints that license the production of a definite description involving a vague adjective and propose an approach that stresses the role of salience (modelled by means of a clustering algorithm) and the influence of feature variability (i.e. the variability of the range of values along a particular dimension such as *size*) within the conceptual kinds that constitute the adjective’s domain.

On the Complexity of de Finetti Coherence of Łukasiewicz Events

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De Finetti foundation of probability theory relies on the *coherence* of betting odds as follows [2, 3, 4]: Let ϕ_1, \dots, ϕ_k be classical events and let $\mathbf{a} : \{\phi_1, \dots, \phi_k\} \rightarrow [0, 1]$ be an assessment of ϕ_1, \dots, ϕ_k . Then \mathbf{a} is said to be coherent if and only if there is no system of reversible bets on the events leading to a win independently on the truth of ϕ_1, \dots, ϕ_k . Precisely, the assessment \mathbf{a} is coherent if and only if, for every $\mathbf{b} : \{\phi_1, \dots, \phi_k\} \rightarrow \mathbb{R}$, there exists a Boolean valuation $\mathbf{v} : \{\phi_1, \dots, \phi_k\} \rightarrow \{0, 1\}$ such that

$$\sum_{i=1}^k \mathbf{b}(\phi_i)(\mathbf{a}(\phi_i) - \mathbf{v}(\phi_i)) \geq 0. \quad (1)$$

The celebrated de Finetti theorem states that an assessment \mathbf{a} is coherent if and only if \mathbf{a} coincides with the restriction to $\{\phi_1, \dots, \phi_k\}$ of a finitely additive and normalized function P from the free Boolean algebra generated by the ϕ_i 's to $[0, 1]$. In this case, we say that P is a probability measure *extending* \mathbf{a} , or that \mathbf{a} *extends* to a probability measure P . The problem of checking whether or not a *rational* assessment $\mathbf{a} : \{\phi_1, \dots, \phi_k\} \rightarrow \mathbb{Q} \cap [0, 1]$ is coherent is NP-complete [11].

A natural generalization of de Finetti coherence criterion is obtained allowing an infinite-valued interpretation of events ϕ_1, \dots, ϕ_k , instead of their classical two-valued interpretation. A first attempt in this direction has been made by Paris, who firstly extended de Finetti theorem to deal with a generalization of the classical Boolean semantics of the events, namely the semantics of $(n + 1)$ -valued Łukasiewicz logic [1]: An assessment $\mathbf{a} : \{\phi_1, \dots, \phi_k\} \rightarrow [0, 1]$ is coherent if and only if \mathbf{a} extends to a *state* on the finite $(n + 1)$ -valued MV-algebra over $\{0, 1/n, \dots, 1\}$ freely generated by the ϕ_i 's, if and only if for every $\mathbf{b} : \{\phi_1, \dots, \phi_k\} \rightarrow \mathbb{R}$, there exists a valuation $\mathbf{v} : \{\phi_1, \dots, \phi_k\} \rightarrow \{0, 1/n, \dots, 1\}$ satisfying (1). As a straightforward consequence of [8, Theorem 1] and [5, Theorem 4.4.1], deciding the coherence of \mathbf{a} above is an NP-complete problem. In light of Paris work, in [10] Mundici approaches the infinite-valued semantics for the events, showing that the coherence of an assessment $\mathbf{a} : \{\phi_1, \dots, \phi_k\} \rightarrow [0, 1]$ with respect to $[0, 1]$ -valued Łukasiewicz valuations is characterized by the existence of a state on the free MV-algebra generated by the ϕ_i 's, extending \mathbf{a} . In recent work [9], Mundici and Kühr further extend this result to every $[0, 1]$ -valued algebraizable logic with continuous connectives.

In [10], Mundici shows that the coherence of rational Łukasiewicz assessments is decidable, and, as regards to the computational complexity of the problem, Hájek shows that the problem is in PSPACE [8]. We settle the computational complexity issue, showing that the problem is NP-complete.

Theorem 1. *The set $\text{COH-LUK-ASS} = \{\langle \mathbf{a} \rangle \mid \mathbf{a} \text{ is a coherent rational Łukasiewicz assessment}\}$ is NP-complete.*

In light of this, we obtain NP-completeness results for the satisfiability problem of several classes of formulas of probabilistic logics introduced in [6, 7], settling a problem raised by [8, 7].

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Vagueness through Definitions

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Vagueness appears when, instead of a simple two-valued function, intermediate states of membership can be attributed to objects. This occurs in particular in the case of concepts that are presented through the help of an auxiliary set of defining features. For all these concepts, a weak membership order can be recursively defined among the objects at hand, in which the relative salience of the defining features is taken into account. This weak order naturally extends to compound concepts, leading to a satisfactory notion of qualitative membership that is fully compositional.

On an Old Approach toward a Graded Identity

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We consider a 1965 approach by Klaua toward a graded identity. This identity is defined simultaneously together with a graded membership predicate. This definition is somehow similar to the Scott/Solovay approach toward Boolean valued models for set theory.

We discuss this relationship, together with some important properties of this approach toward a universe of sets with graded membership and graded identity predicates.

A Fuzzy Logician Reading Books on Vagueness

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Various mainly critical opinions on fuzzy logic and its meaning for vagueness from the literature (Keefe, Fine, Gaifman, Sanford, Shapiro, Smith) will be mentioned, mathematical fuzzy logic as a well developed many-valued logic with a comparative notion of truth will be very briefly presented and the critical opinions will be commented from the point of view of this logic.

Inferences with Imprecise Conditionals

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When probabilities are not known precisely there are two ways to express imprecision, (i) by interval probabilities or by (ii) second order probability density functions. The present contribution uses second order distributions.

Many inference problems involve conditionals (if–then). The most basic examples of inference schemes are the *modus ponens* or the *modus tollens*. While in classical logic the conditionals are conceived as material implications, in probability logic conditionals are conceived as conditional events. Conditional events are not truth-functional. Moreover, in the tradition of de Finetti the truth value of conditional events is undetermined if the antecedent is false.

We combine second order distributions and probability logic. More specifically, we use the family of beta distributions to express imprecise knowledge in the premises and infer the distribution of the consequence. Usually the set of premises is “incomplete” and it is not possible to derive exactly one distribution for the consequence. In such cases in the usual probability logic we obtain lower and upper probabilities for the consequence. In our approach we obtain mixtures (weighted averages) of beta distributions. Monte Carlo techniques (implemented in the BUGS software, for example) are employed when the mathematics of the mixtures are intractable analytically.

When conditionals are involved, and when the undetermined truth value in the case of a false antecedent is expressed by a second order uniform distribution, then the conditional becomes “non-symmetric” with respect to negation: If we condition on P , then $\neg P$ has a uniform distribution, and we condition on $\neg P$, then P has a uniform distribution. This leads to a number of interesting results. The results are discussed, e.g., with respect to counterfactual reasoning. We conclude with some speculations about modeling human reasoning using this kind of soft probability logic.

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Fuzzy Logics and Games

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The aim of this talk is to discuss applications of game theory to the semantics of fuzzy logics. In particular we concentrate on evaluation games which represent the game-theoretical counterpart of the model theory.

Game theoretical semantics has been extensively studied for a long time and proved to be a useful alternative to the standard semantics for various logical systems. The research of games in fuzzy logics started in the work of Christian Fermüller (e.g. [2]), who applied Lorenzen style dialogue games to capture proof-theoretical features of fuzzy logics. Later Cintula and Majer in [1] provided evaluation games for fuzzy logics.

In this paper we concentrate on two topics: the first one is foundational—we give a definition of the connectives of fuzzy logics in the terms logical games, and discuss the question of (in)dependence of the game definition with respect to the standard semantics. The second topic we shall deal with is an application of the game semantics to one of the limitations of the standard interpretation of fuzzy logics—the requirement of safe models (see e.g. Hájek [3]). In the safe models all the suprema and infima required by the standard interpretation of the existential and general quantifiers are supposed to exist. This requirement can be partially avoided by the proposed game semantics. We introduce a generalization of the standard notion of validity, which is based on existence of winning strategies for the player defending the formula in question. This generalised validity (*g-validity*), is satisfied by broader class of models than the safe ones and coincides on the safe models with the standard validity.

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The Model-Theoretic Study of Many-Valued Logics: A Survey

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The model-theoretic approach to the study of many-valued logics has been recently used to obtain several results by exploiting classical techniques. In this talk, we will review the most important results (quantifier elimination, model completion, amalgamation, etc.) and give a glimpse of how to adopt this approach to treat the representation of uncertainty of non-classical events [6].

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Probability as a Measure of Information Added

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Information theory defines information as the logarithm of a statistical probability. Carnap and Bar-Hillel used the same measure but of an inductive probability as a measure of semantic information. I propose to turn this on its head, starting from plausible qualitative assumptions about information added: the information added by a single proposition to a body of propositions. We find that there are at least two viable notions of information-added: one goes by the novelty value of the added information, the other roughly by the consequences adding it allows one to draw—information-added as an additional resource.

Making the sort of plenitude assumptions common in the area of representation theorems we find that any measure of information-added as novelty value can be rescaled as a probability-like function bearing a strong resemblance to a non-standard concept of probability employed by Morgan and Mares in providing a semantics for intuitionist logic. More significantly perhaps, any measure of information-added as additional resource rescales as a unique Popper function.

Connections can be drawn between this conception of probability as a measure of information-added and Bayesian accounts of confirmation, on the one hand, and probabilistic accounts of the semantics of conditionals on the other.

Base Belief Revision for Finitary Monotonic Logics

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We slightly improve on characterization results already in the literature for base revision. We show that in order to axiomatically characterize revision operators in a logic the only conditions this logic is required to satisfy are: finitariness and monotonicity. A characterization of limiting cases of revision operators, full meet and maxichoice, is also offered. Finally, we distinguish two types of bases naturally arising in the context of graded fuzzy logics.

About Vagueness, Typicality and Similarity

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Introduction. Vagueness is related to the informal idea that the contours or limits of the scope of words lack precision or clearness. Vagueness may be equated with the idea that a vague concept partitions the universe of discourse (sometimes implicitly) into more than two parts. As investigated in [3], there are many distinct informational scenarios that may lead to such a situation.

Here, ‘vagueness’ is used in a more restricted way. Vagueness is often associated to the idea of ‘uncertainty in meaning’, e.g. [5]. Still this may be understood in different ways that have to be distinguished. Indeed a statement of the form X is A , where X refers to a single-valued attribute, conveys a piece of information that is *imprecise* if the extension of the set representing A is not a singleton, as in the statements ‘Peter is 25 or 26’, or ‘John is between 20 and 30 years old’. The statement X is A is *fuzzy* if A is a fuzzy set, as in ‘Bob is young’. In both cases, we are *uncertain* about what is the precise age: Peter may be 25 or may be 26; the *possibility* that $age(Bob)$ is 25 can be taken as the degree of membership of 25 to the fuzzy set representing ‘young’ [9], i.e. $\pi_{age(Bob)}(u) = \mu_{young}(u)$, $\forall u$.

But, this does not mean that ‘young’ is vague, or involve any uncertainty in itself, at least as long as there is some agreement on the membership function representing it in the considered context. The gradualness of the predicate ‘young’ is essentially due to the fact that the numerical attribute ‘age’ has an attribute domain which is a continuum, and which is interfaced with a discrete set of linguistic labels (here such as ‘young’, ‘old’, and so on). Zadeh himself considered that vagueness should not be confused with fuzziness. He wrote (in a footnote p. 396 in [9]): “Although the terms fuzzy and vague are frequently used interchangeably in the literature, there is, in fact, a significant difference between them. Specifically, a proposition, p , is fuzzy if it contains words which are labels of fuzzy sets; and p is vague if it is both fuzzy and insufficiently specific for a particular purpose.” We use ‘vague’ here in a slightly different sense. The statement X is A will be regarded as *vague*, whatever the specificity of A , only if the membership function of A is open to variability, or in other words if the precise location of the border of A is not known: A is more like a *flou* set [4] where some elements are certainly in A , and others are only possibly in A .

This agrees with a flexible reading of propositions, based on *similarity*, useful for maintaining consistency as outlined in the following, after briefly discussing the notion of *typicality* also often associated with vagueness.

Introducing typicality in formal concept analysis. Formal concept analysis (FCA) [8] defines concepts as (extension, intension)-pairs (X, Y) , such that $X = \{x \in Obj | R(x) \supseteq Y\}$ and $Y = \{y \in Prop | R^{-1}(y) \supseteq X\}$ from a relation R , called ‘context’, which states for each object $x \in Obj$ the complete set of its properties $y \in Prop$. Gradualness in properties can be taken into account by allowing R to be fuzzy [1]. Typicality can be introduced in FCA by keeping R crisp, and introducing degrees among objects and among properties. The basic idea can be summarized by the two principles [3]: (A) An object x is all the more normal (or typical) w.r.t. a set of properties Y as it has all the properties $y \in Y$ that are sufficiently important; (B) A property y is all the more important w.r.t. a set of objects X as all the objects $x \in X$ that are sufficiently normal have it.

Let us consider, e.g., the sets of birds $X = \{albatross, parrot, penguin, kiwi\}$, and properties $Y = \{\text{‘laying eggs’}, \text{‘having two legs’}, \text{‘flying’}, \text{‘having feathers’}\}$, with typicality degrees: $X^t(albatross) = X^t(parrot) = 1$, $X^t(penguin) = \alpha$, and $X^t(kiwi) = \beta$, with $1 > \alpha > \beta$ (kiwis

do not fly and have no feathers). Let us compute the fuzzy set of important properties according to principle (B) above, i.e., $Y^i(y) = \min_x X^t(x) \rightarrow R(x, y)$, with $a \rightarrow 1 = 1$ and $a \rightarrow 0 = 1 - a$. This choice expresses the idea that a bird not having property y makes a property all the less important in the definition of the concept *bird* as this bird is considered more typical. Let $Y^i(y)$ define the degree of importance of property y , in the definition of *bird*, $\forall y$. Let us compute the fuzzy set of typical objects according to principle (A) above by $\mu(x) = \min_y Y^i(y) \rightarrow R(x, y)$. We get $\mu(\text{albatross}) = Y^i(\text{parrot}) = 1$; $\mu(\text{penguin}) = \alpha$; $\mu(\text{kiwi}) = \beta$ (since $(1 - a) \rightarrow 0 = a$). We have $\forall x \mu(x) = X^t(x)$, and thus a (fuzzy) Galois connection.

Then, the statement ‘Tweety is a bird’ is represented by $\forall y \pi_{\text{Tweety}}(y^c) = 1 - Y^i(y)$, where y^c is the negation of y (to parallel with $\pi_{\text{attribute}(x)}(u) = \mu_A(u)$).

Vague understanding for maintaining consistency. Assume K_1 (resp. K_2) is a reliable source that says tomorrow ‘the sky will be overcast’ (*oc*), (resp. ‘the sky will be open’ (*os*)). Then classical merging operators Δ lead to $\Delta(K_1, K_2) = \{oc \vee os\}$. Let us introduce the literal *pc* = ‘tomorrow the sky will be partially cloudy’ which is *close* both to *oc* and *os*, and have a *vague* reading of sources as $K_1 = \{oc \vee pc\}$ and $K_2 = \{pc \vee os\}$. This enlargement of propositions by proximity enables us to get $\Delta(K_1, K_2) = \{pc\}$. This idea, reminiscent of taking advantage of vagueness in dialogues [7], can be generalized by allowing progressive enlargements of the meaning of propositions using possibilistic logic [6].

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Multidimensional Adjectives versus Nouns

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In this paper I defend the thesis whereby linguistic differences between different predicate-types are naturally explained by assuming that they are associated with different types of degree function. Elsewhere (Salt 18), I successfully support this view by showing that puzzling differences between positive and negative adjectives (Kennedy 1999) directly derive from differences in their degree functions. The present paper focuses on the degree functions of multidimensional predicates (e.g., *healthy* is characterized by dimensions like *blood pressure [bp]*, *pulse*, etc.), arguing that the differences between nominal and adjectival multidimensional predicates boil down to differences in their degree functions, i.e. differences in the way entities' degrees in their dimensions combine so as to determine their status in the given predicate. I support my claims empirically and theoretically.

Cognitive concept-theories assume that an entity is classified under a concept like *bird* iff (roughly) its mean degree in the concept's dimensions (e.g. *small*, *flying*, *perching*, etc.) reaches a certain standard (for an extensive review of the empirical support see Murphy 2002). This analysis has been applied to the semantics of predicates (Lakoff 1987). Many adjectives (*healthy*, *normal*, *typical*, *good*, etc.) have several dimensions. Yet, I propose that in adjectives, unlike nouns, the instances' mean degrees in the dimensions need not reach the standard. The dimensions are not combined via averaging (mean operations). Rather, they combine through quantification (Boolean operations). Moreover, I propose that adjectives split into two different types:

- (i) Adjectives like *sick* and *atypical* are '**disjunctive**': Entities are required to reach the standard in but *one* dimension, e.g., intuitively, entities that satisfy at least one property which functions as a dimension of *sick* (measure of sickness) in a context are considered *sick*.
- (ii) Adjectives like *healthy* and *typical* are '**conjunctive**': Entities are required to reach the standard in *all* the dimensions, even unimportant ones; e.g., when one is healthy in every respect except that one has the flu, strictly speaking, one is *not healthy*.

In sum, I propose that the default interpretation of **conjunctive** adjectives involves (implicit) **universal** quantification on dimensions, while that of **disjunctive** adjectives involves **existential** quantification.

The context dependency of the dimension set of multidimensional adjectives makes it hard to support or refute the present proposal. However, in what follows, I present a new experimental method for studying categorization, designed especially to overcome this difficulty. I show that my proposal is supported by a study of Google search results for "*P except*" with different predicates *P*.

Exception phrases are only compatible with universal quantification, as shown in (1).

- (1) a. **Somebody except for Dan is singing*
- b. *Everybody / nobody except for Dan is singing*

Given my proposal, an *except*-phrase should more easily lend itself to an interpretation in which it operates on the dimension-set, in conjunctive adjectives (as in (2a)), than in disjunctive ones (cf. (2b)). In the latter, this requires the accommodation of a non-default universal quantifier (as in *sick in every respect*).

- (2) a. I am a 64-year-old man, quite **healthy except** for high blood pressure
 b.# ... , quite **sick except** for (normative) blood pressure
 c. $[[\text{Dan is healthy except wrt blood pressure}]]^+_c = 1$ iff
 $\forall Q \in (DIM(\text{healthy}, c) - \{(\text{healthy wrt}) \text{ blood pressure}\})$: $[[\text{Dan is } Q]]^+_c = 1$
 s.t. $DIM(P, c)$ is a set of predicates, P 's dimensions in c (Dan is healthy wrt all dimensions except bp in c)

Yet, on my proposal, a *negated* disjunctive adjective like *not sick* should denote the entities that fall under *no* 'sick' dimension, so I predict that under negation 'except' *should* operate on the dimension-set, as in (3).

- (3) a. They do **not appear to be sick, except** for the diarrhea
 b. $[[\text{Dan is not sick except wrt blood pressure}]]^+_c = 1$ iff
 $\neg \exists Q \in (DIM(\text{sick}, c) - \{(\text{sick wrt}) \text{ blood pressure}\})$: $[[\text{Dan is } Q]]_c = 1$ iff:
 $\forall Q \in (DIM(\text{sick}, c) - \{(\text{sick wrt}) \text{ blood pressure}\})$: $[[\text{Dan is } Q]]_c \neq 1$
 (Dan is sick wrt no dimension except bp in c)

My predictions are borne out by the facts. In one study (ESPP 16), I count different uses of *except* with 8 adjectives, in ~70 Google entries with each. As predicted, **most** (three quarters) of the dimension-set uses with the disjunctive antonyms *sick*, *atypical*, *different* and *abnormal* **are negated**. Conversely, except for one, **none** of the dimension-set uses of *except* with the conjunctive adjectives *healthy*, *typical*, *identical* and *normal* **is negated**.

Note, however, that the absence of negated dimension-set readings with conjunctive adjectives may merely reflect the low frequency of negated forms compared to non-negated forms in natural use, casting doubts on the validity of my conclusions. My second study improves upon its predecessor in terms of the number of predicates and search-results studied (12 adjectives, 3 nouns, ~100 Google entries with each), and in controlling for the low frequency of negated forms (by separately searching for negated forms, like, e.g., *not healthy except*, and evaluating the proportion of dimension-set readings within a set of negated forms, versus their proportion within a set of non-negated forms). The results are as follows.

First, while no dimension-set uses are found with the three nouns examined (e.g., *table*, *bird*, *mother*), hundreds of such uses are found with the examined adjectives. These data are explained iff adjectival dimensions combine via quantifiers, not averaging.

Second, the likelihood of a dimension-set use of *except* with the presumed conjunctive adjectives *healthy*, *typical*, *identical* and *normal* **is (between 2 to 7 times) higher when not negated than when negated, as predicted**. Conversely, the likelihood with the presumed disjunctive adjectives *sick*, *atypical*, *different* and *abnormal* **is (between 3 to 10 times) higher when negated than when not negated, as predicted**. Finally, a third group of adjectives exhibits a mixed pattern.

The distinction between categorization tasks that do/don't involve averaging has neural and developmental correlates (Ashby and Maddox 2005). This motivates my proposal as to its lexical correlates. It also sheds new light on other contrasts between adjectives and nouns. For example, *wrt*-phrases are bad with nouns (cf. *healthy wrt bp* vs. *#a bird wrt flying*), excluding nominalizations (*health*; *typicality*) and nouns with adjectival entries (*an Italian*). Modifying a predicate P with a *wrt*-phrase makes sense iff entities may be regarded as P in one respect, and as 'not P ' in another respect, i.e. iff P 's dimensions are categorization criteria, i.e., iff either P or P 's negation is conjunctive (as is the case in multidimensional adjectives). In one-dimensional adjectives like *tall* we cannot find two respects (hence *#is tall wrt height* is odd), and nouns with several dimensions are mean-based, not

conjunctive (so *#is a bird wrt flying* is odd). Similarly, quantification over respects is ruled out when the domain (the set of categorization criteria) is a singleton (as in *# tall in some/most/every respect*) or is empty (in nouns; cf. *# a bird in some/most/all respects / # generally a bird*).

If the degree functions of multidimensional adjectives are not mean-based, what are they? Time permitting, I'll discuss this question, its implications for the analysis of comparison statements (cf., Bale 2007), and related empirical findings.

On Vagueness and Granularity

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Many accounts of vagueness rely on a rather wide notion of reality. Attempts to specify the deeper reasons for vagueness frequently lead to questions like the following (cf., e.g., [Sha]): Is the source of vagueness of a linguistic nature or to be searched on a metaphysical level? That is, do we encounter vagueness because our language allows vague propositions only, or are the objects in the world already subjected to vagueness? The concern about such a problem relies on a clear separation of the observer on the one hand and observed objects on the other hand; without assuming an observer-independent reality the question cannot even be understood.

To approach the problem how to interpret properly the vagueness of human statements, we argue in favour of a more restrictive notion of reality. What we say about the world depends on perceptions, so the minimum of what must be taken as real, if anything at all, is the perception itself. In contrast, the contents of perceptions need not be considered as something which would exist without the observer, but can be understood as the means to enable us to describe perceptions. Adopting this viewpoint, we conclude that whatever question we pose about the world in general, we can either reformulate it in terms of perceptions and thus endow it with a meaning or we should not expect a valuable answer. The above mentioned question falls presumably into the latter category.

The corresponding characterisation of mathematics stresses the notion of a structure rather than axioms and inference rules. Namely, perceptions are systematised by mutual comparison; and to describe how given objects may differ from other ones with respect to a specific property, this is what mathematical structures are good for. For example, the adjective “large” is applicable to an object which can be appraised with respect to its spacial extent and is used to distinguish it from other objects which have been observed as less large. Disregarding any other property which could be associated to objects, we can construct a mathematical structure modelling the notion of “size”. Constructing, say, the positive rationals \mathbb{Q}^+ , is an iterative process based on the experience that lengths can be added and divided into equal parts.

What comes out is a structure which is closed under what is imaginable as a size relative to other sizes. Needless to say, when considering the infinite model resulting from the repeated application of the idea to concatenate and to divide into equal parts no matter if we operate in huge or tiny dimensions, only the first few steps are in accordance with experience; the rest works by analogy. Furthermore, these first steps reflect statements which must be called vague; and we end up with a structure which by construction does not leave any further possibility with regard to the considered property, that is, we end up with a precise notion of size. In particular, preciseness is secondary: it is built upon what is called vague.

We wonder what these ideas imply for the design of a framework to reason about vague information. We think that the following procedure suggests itself. For a perceptual notion like “size”, we first have to check what we actually have in mind: (i) We want to argue about the notion in general, taking into account all distinctions with regard to it. (ii) We want to argue about perceptions according to which objects are actually distinguishable by means of it. (iii) We aim at both (i) and (ii) in a consistent way.

If we have in mind (i), we are obviously just asked to do usual mathematics. We may take, for the above example, the structure $(\mathbb{Q}^+; +, \cdot, 0, 1)$ which is fine enough to include everything imaginable, formulate valid statements in the corresponding language as axioms, find valid inference rules, and proceed.

If we aim at (ii), we may, as it seems, do just the same, but in an adapted setting. There is no necessity to proceed up to the finest possible level; we may equally well stay at a coarse level. Consider another example, namely one of Aristotle's syllogisms (see, e.g., [Pet]): "If nearly all A are B and all B are C , then some C are A ." We may take the set of all finite subsets of some fixed set; we may use relations $A \subseteq B$ as well as $A \Subset B$, the latter meaning " A is a small subset of B ". We leave the latter notion as vague as it is; we just proceed as usual, specifying the properties which we assume about small subsets.

Finally, if we aim at (iii), the known difficulties arise. In particular the standard example of the apparently paradox consequences of vagueness, the sorites paradox, can be formulated only if we take into account both levels: the fine and the coarse one.

The fine level being required in any case, we might first choose a model like in (i) and then extend the framework to include reasoning on a coarse level. Adequate approaches exist. A possibility is to introduce a metric and to treat situations as distinct only if separated by a fixed minimal distance. One version of this idea is Williamson's Logic of Clarity [Wil]; but note that his motivation is exactly opposite to ours. According to this approach the value for distinguishability is fixed; if we want to have it flexible, we may use Ruspini's approach to similarity-based reasoning [Rus]. Corresponding logics have been developed in a series of papers by Esteva, Godo, and others [EGGR].

We opt for the latter proposal. We modify the formalism in certain points according to the present context. In particular, we allow to use several pseudometrics rather than a single metric, one for each considered property. On this basis we furthermore propose a first-order version of the logic.

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