

Vagueness at the interface between
logic, philosophy, and linguistics
Pleasures and pitfalls of interdisciplinarity

Chris Fermüller

Vienna University of Technology
Theory and Logic Group

www.logic.at/people/chrisf/

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However:

Even if we narrow the focus to 'correctness' of inferences involving predicates, like *tall* and *nice*, there is wide disagreement on how to proceed: What problems are to be solved? What counts as a solution? Which 'data' are relevant? What is a 'model'? How to assess its adequateness? What is the role of logic?

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Linguistics – ‘predict relevant phenomena/data’:

- ▶ aim: ‘**predication**’ of linguistic data (often ‘quasi-empirical’)
- ▶ robust intuitions about ‘grammaticality’ at the bases
- ▶ often in focus: very **subtle differences and contrasts**

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- ▶ **Linguistics:**

Are 'sorites dialogues' realistic and (linguistically) relevant? (Conversationalist will 'reflective': retraction, refusal etc.) In any case: pragmatics seems as important as semantics here. Moving from single utterances to whole conversations seems also crucial: From standard (i.e., word and sentence level) semantics to 'logics of conversation'.

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NB. Nick Smith defends a truth functional, degree theoretic solution to the *sorites paradox*. In essence:

- ▶ vagueness is identified as *Closeness*:
Clo: If x and y are very close in P -relevant respects, then $P(x)$ and $P(y)$ are very close in respect of truth.
Def. P is vague iff \exists P -connected, P -diverse set satisfying Clo
- ▶ (to make a long store short!) Clo entails degrees of truth
- ▶ *Tolerance* (Tol) is a particular (classical) instance of Clo
- ▶ when the inductive *sorites* premise is read according to Tol then the argument is valid, but leads to contraction
- ▶ but the inductive premise is not true according to Clo
- ▶ the *sorites* argument is compelling since we mistake Clo for Tol
- ▶ \rightarrow_{Luk} is not needed, the 'surface reading' of the inductive premise is replaced by talk of 'near equality' of truth values
- ▶ Luk gets consequence wrong, which should remain classical(!)

Smith vs. Novak/Hajek (ctd.)

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- ▶ BL – the logic of all continuous t -norms – serves as basis
- ▶ axiomatic approach: axioms for connective At ('almost true')
- ▶ in PA over $BL+At$ the vague predicate 'feasible' is studied
- ▶ generalization to 'FL with evaluated syntax': assigned truth values are attached to formulas on the syntactic level by extending Łuk to FLn
- ▶ aim (citation from the abstract of Novak & Hajek:)
 - we ... show that under a very natural assumption we obtain a consistent fuzzy theory. Thus, sorites is not paradoxical at all.
- ▶ (citation from the conclusion:)
 - no attempt has been made to select any particular semantics of 'almost true' and 'feasible' as 'recommended', 'best', etc. On the contrary ...

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no direct arguments for BL & FLn (rather: 'it works')
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Novak & Hajek present an (elegant, sophisticated) formalism based on Peano Arithmetic. They mainly refer to other mathematical literature about 'feasible numbers' and within *t*-norm based FL, but don't discuss alternative theories.

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- ▶ **linguists (semanticists)** seem ready to employ whatever logical formalism provides the right tools to formalize compositional semantics — the aim is to accurately 'predict' many observed grammatical phenomena (of natural language)
- ▶ **logicians** (mostly) are happy to define large families of logics that are characterized by 'good' mathematical properties (e.g., conjunction as t-norm, implication as corresponding residuum)

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(Compare **Smith** vs. **Hajek & Novak**)

However:

All accounts make use of formal models!

Philosophers, linguists, and logicians alike have to **justify** and **argue** for the **adequateness** and **usefulness** of their models
(according to different purposes).

They all (should) **reflect on criteria** for the **evaluation** of models.

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Viz. also different answers to these questions:

- ▶ What is a language?
- ▶ What is the role of a formal derivation/proof?
- ▶ What is semantics?

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Examples: Can intermediary truth values be derived from 'context models'? Can Lorenzen's dialogue theoretic justification of intuitionistic logic be generalized/modified to fit (e.g.) fuzzy logics? Can supervaluationist models be interpreted in context based mode, or as inducing intermediary truth values, etc?

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- ▶ While some caution and respect for difficulties is appropriate, there is no reason for undue pessimism:
Events like this one provide ample opportunity for better recognition of relevant differences, but also of common topics, intersecting interests, and new ideas that may prove useful for different purposes and fields concerned with vagueness!