Computational Representations of Models of First-Order Formulas

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– *Position Statement* –

I am currently writing my doctoral thesis on "computational representation of models of first-order formulas". My main motivation for investigating this topic comes from the field of automated model building (see e.g. [1, 2, 4, 5, 10, 12, 13]), a subfield of automated theorem proving that attempts to design algorithms for finding models of satisfiable first-order formulas. Note that a model of a formula $\neg A$ is nothing else than a counterexample of A, which may be of substantial help for finding the reason why A failed to be valid. Clearly, the ability to represent models of first-order formulas in a computationally feasible way is a necessary prerequisite for building them. However, representation mechanisms for particular models of first-order formulas play an important role in many other fields too, e.g. semantic resolution, model checking, deductive databases, etc. This makes it worthwhile to investigate mechanisms for representing interpretations of first-order formulas symbolically by their own, aiming to accompany work in the field of automated model building.

By not restricting ourselves to finite models, we come to the necessity of coping with symbolic representations. But even for finite models a concise symbolic representation may be a lot easier to deal with than the explicit table-notation of a large finite model. To call a symbolic representation of a (Herbrand) model a *model representation*, we require[1] to have (good) algorithms for the **ground atom test** (i.e. to decide whether a given ground atom is true in the given model), the **equivalence check** (i.e. to decide whether or not two representations represent the same model) and for **clause evaluation** (i.e. to decide whether a given clause is true in the given model).

From a theoretical point of view it makes sense to focus on Herbrand models of skolemized formulas (or clause sets), as Herbrand models exist for all satisfiable formulas of this kind. But this is also justified from a practical viewpoint to utilize the intuitive requirement of **understandability**, because in Herbrand models the domain and the interpretation of the function symbols are clear, fixed and intuitive. However a Herbrand model over a fixed signature is fully specified by a description of its potentially infinite set of (true) ground atoms, i.e. a set of terms (or strings, depending on how we want to look at them). This point of view reveals the (to our opinion) most interesting aspect of our approach: We are lead to investigating model-theoretic properties in terms of syntactical properties of the corresponding true-ground-atom set.

Various representation mechanisms, evolving from automated model building or having been developed independently (e.g. in formal language theory, automata theory, rewriting, etc.) appear as candidates for model representations. Still algorithms for using those

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[1] These requirements were first raised in [4] to characterize the term *model representation*.

formalisms as model representations are not so widely investigated.

I am interested particularly in **characterizing** the models that are representable by the different mechanisms and in **comparing** their expressiveness and their computational feasibility, which includes developing (good) algorithms and analyzing the complexity of the three basic evaluation problems mentioned before.

In my recent work I concentrated on classical term-set representations, in particular on tree automata and grammars. They were compared with other well known representations like atom representations (see [3, 4, 11]), congruences of equational systems, etc. and algorithms and complexity issues were investigated; see [6–9]. For the future it is planned to work towards a more accurate analysis of other representations like repetitive terms, constrained grammars, atoms with equational constraints etc. and to investigate the incorporation of equality. There is also hope that from a deeper insight into model representations, it will be possible to develop automated model building procedures for interesting classes of first-order formulas.

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