A Further and Effective Liberalization of the δ-Rule in Free Variable Semantic Tableaux*

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Abstract

In this paper, we present a further liberalization of the δ-rule in free variable semantic tableaux. It is effective in that (1) it is both a natural and intuitive liberalization, and (2) can reduce the proof size exponentially as compared to previous versions of the δ-rule.

1 Introduction

Proof procedures for first order predicate logic such as semantic tableaux need means to deal with existential quantifiers. In general there are two different ways to do this. One way is to Skolemize the formula to be proven in a preprocessing step, obtaining a purely universal formula at the expense of a richer signature. The other approach is not to use a preliminary Skolemization but to add a tableau expansion rule for treating the essentially existential formulae, so that Skolemization is performed during the proof construction when existential formulae are encountered on tableau branches. In substance, there is no difference in applying either of the two methods, but we believe that adding a rule for the existential formulae to the tableau expansion rules and eliminating the preliminary Skolemization phase makes the proof procedure more compact and is generally preferable.

In this paper we follow the second approach, presenting an expansion rule for existential formulae based on the global Skolemization technique described in [4] and [3]. The central idea of our method is to perform – during the proof – a "delayed" global Skolemization of the formula to be proved. This approach differs from the widespread "local" Skolemization in that the (infinitely many) Skolem function symbols for eliminating all existential quantifiers are introduced in a single shot.[1]

We will define a δ-rule going beyond existing δ-rules in the literature in that sense, which is able to reflect structural similarities in a natural way. This reduces the number of Skolem functors and of variables dependencies in the proofs.

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[1] In [4] and [3] the possibility is contemplated to get rid of the universal quantifiers as well, returning a formula devoided of quantifiers.

2 Preliminaries

2.1 Uniform Notation

Before going into details, we have to introduce some notation. For the sake of simplicity we use Smullyan's uniform notation, which has the advantage of being compact, cutting down on the number of cases that must be considered. Smullyan divides the logical operators and consequently the formulae of the language into four categories: conjunctive formulae called α-formulae, disjunctive formulae called β-formulae, universally quantified formulae called γ-formulae, and existentially quantified formulae called δ-formulae. According to this notation, our interest is clearly devoted to the δ-formulae.

2.2 Baaz and Fermüller's Rule

We assume the reader to be familiar with the standard expansion rules of semantic tableaux (the previous versions of the δ-rule are described in [7], [5], [6], [2]). Here, we only give the formulation of the δ-rule developed by Baaz and Fermüller [1]; their rule, called δ^*-rule, can be seen as the starting point for developing the rule we present in this paper. Schematically, the δ^*-rule can be described as follows:

$$\frac{\delta}{\delta_0(f_{[\delta]}(x_1, \ldots, x_n))}$$

where $x_1 \ldots, x_n$ are the relevant variables (in the sense defined in [1]) occurring in δ, and $f_{[\delta]}$ is the function symbol assigned to δ. This function symbol has not necessarily to be new, but it is allowed to use the same function symbol more than once when the δ-rule is applied to existential formulae that are identical up to variable renaming (including renaming of the bound variables in the formula).

Hence, along with the initial signature Σ, a new enriched signature Σ^* is introduced, containing all the Skolem function symbols that may occur during the construction of a tableau for a formula over Σ. Let $\mathcal{L}^{\Sigma^*}$ be the set of well-formed formulae over Σ^* (for a detailed explanation see [1] or [2]); and let Δ be the set of all δ-formulae in $\mathcal{L}^{\Sigma^*}$. Further, let the equivalence relation R_* over Δ be defined as follows: $\delta_1 R_* \delta_2$ iff δ_1 is identical to δ_2 up to variable renaming (for all $\delta_1, \delta_2 \in \Delta$). Clearly, the set of all the δ-formulae of the language is divided into equivalence class. Each of these classes is assigned a Skolem function symbol.

Our further liberalization of the δ-rule allows a different construction of the extended signature Σ^{*^*}, minimizing the number of redundant Skolem symbols. We use an equivalence relation R_{*^*} over set Δ such that two δ-formulae δ_1 and δ_2 are equivalent iff they can be (uniquely) related to the same relevant extracted key formula as defined in the following section.

2.3 Key formulae

In this section, we introduce the notion of *key formulae*, which is of great importance to the technique of global Skolemization because it characterizes the formulae in the language that are assigned their own Skolem function symbol. Each formula of the language corresponds to a unique key formula, of which it inherits the related Skolem function symbol.

In the following, we adapt the notion of key formulae to the context of semantic tableaux. Before giving a formal definition, we intuitively clarify the concept with some examples.

EXAMPLE 2.1 *Suppose we have a δ^*-tableau proof in which, at a certain point, we have to apply the δ^*-rule to the δ-formula $(\exists x)r(x,y)$. Clearly, the expansion results in the formula $r(g(y),y)$.*

Now let us suppose that later in the proof, we have to expand the formula $(\exists w)r(w,z)$. It is identical with the former formula up to variable renaming, so we can use the same Skolem function symbol g and the expansion results in the formula $r(g(z),z)$.

However, if we encounter the δ-formula $(\exists x)r(x,k(z))$, we must introduce a new Skolem function symbol f obtaining the result $r(f(z),k(z))$. On the other hand, if we employ our new δ^{*^*}-rule and come across $(\exists x)r(x,k(z))$, we can use the same Skolem function symbol g obtaining $r(g(k(z)),k(z))$. Applying δ^{*^*} yields this result, because the three δ-formulae mentioned above match with the same key formula $r(x_1,x_2)$.

EXAMPLE 2.2 *Let us suppose that the following formulae occur in a tableau proof:*

$$(\exists x)p(x,y)$$
$$(\exists w)p(w,f(f(z))$$
$$(\exists x)p(x,h(h(h(z)))$$

If we apply any of the previous versions of the δ-rule (see [5], [6], [2] and [1]), then we have to assign a different Skolem function symbol to each of them. On the other hand, applying the δ^{*^*}-rule, it is possible to relate all these formulae to the key formula $p(x_1,x_2)$ and, thus, to assign them the same function symbol.

We now proceed to formally define the notion of key formulae. This definition is slightly different from the one given in [4] and [3], because it is adapted to the tableaux, allowing key formulae to contain quantifiers.

Let us assume that we have infinitely many variables $\ldots, x_{-2}, x_{-1}, x_0, x_1, x_2, \ldots$. In particular we can single out the sequences:

1. $x_{-1}, x_{-2}, \ldots$
2. $x_1, x_2, \ldots$

and the special variable x_0.

DEFINITION 2.1 *A formula φ is called* canonical *with respect to the variable x_0 if:*

- *There is a $k \geq 0$ such that the bound variables of φ are $\{x_{-1},\ldots,x_{-k}\}$, appearing in φ in the order $x_{-1},\ldots,x_{-k}$ from left to right, and each of these variables is quantified only once (but may occur multiply).*
- *There is an $n \geq 0$ such that $Free(\varphi) \setminus \{x_0\} = \{x_1,\ldots,x_n\}$ (where $Free(\varphi)$ is the set of free variables in φ), these variables appear in φ in the order $x_1,\ldots,x_n$ from left to right, and each of them appears only once in φ.*

Every formula of the language has a corresponding canonical formula with respect to a variable (in general the existentially quantified one).

EXAMPLE 2.3 *The canonical formula φ_1 corresponding to the formula*

$$\varphi = (\exists y)(\exists z)(R(x, f(y), z, h(w, w)) \wedge Q(u, v))$$

with respect to x is

$$\varphi_1 = (\exists x_{-1})(\exists x_{-2})(R(x_0, f(x_{-1}), x_{-2}, h(x_1, x_2)) \wedge Q(x_3, x_4)) \ .$$

We define key formulae to be canonical formulae that are most general with respect to substitution.

DEFINITION 2.2 *A formula φ is called a* key formula *if*

- *it is canonical with respect to x_0,*
- *for all ψ that are canonical with respect to x_0, if there is a substitution σ that is free for $(\exists x_0)\psi$ such that $\varphi = \psi\sigma$, then $\psi = \varphi$.*

EXAMPLE 2.4 *We continue from Example 2.3. The key formula with respect to x corresponding to*

$$\varphi = (\exists y)(\exists z)(R(x, f(y), z, h(w, w)) \wedge Q(u, v))$$

is

$$\varphi_2 = (\exists x_{-1})(\exists x_{-2})(R(x_0, f(x_{-1}), x_{-2}, x_1) \wedge Q(x_2, x_3)) \ .$$

Only key formulae deserve their own Skolem function symbol, so the following bi-unique correspondence exists:

$$\varphi \to h_\varphi \ ,$$

where φ is a key formula according to Definition 2.2 and h_φ is the corresponding Skolem function symbol.

Any δ-formula of the language (uniquely) corresponds to a key formula as the following theorem states:

THEOREM 2.1 *Let ψ be a δ-formula not containing occurrences of any of the variables x_{-1}, $x_{-2}, \ldots$, and let x_i be a variable ($i \geq 0$). Then there is a unique key formula φ and a free substitution σ such that:*

- *ψ and $\varphi\sigma$ are identical up to renaming of the bounded variables.*
- *$x_0\sigma = x_i$ and x_i does not occur in $x\sigma$ for any $x \neq x_0$.*

Proof. We give an effective algorithm to construct the key formula φ from the formula ψ:

1. In ψ, rename all the bound variables (from left to right) by $x_{-1}, x_{-2}, \ldots$ calling the obtained formula ψ_1.
2. Locate in ψ_1 the leftmost term t_1 not containing x_i or any bound variable and continue the process until a t_n is found such that there is no term not containing x_i or any of the bound variables occurring after it. In that way, the tuple $[t_1, \ldots, t_n]$ is obtained.

Now, let φ be the formula obtained by the simultaneous substitution of $[x, t_1, \ldots, t_n]$ by $[x_0, x_1, \ldots, x_n]$.

Then, φ is a key formula because

- it is canonical with respect to x_0;
- for all χ canonical with respect to x_0, if there is a σ that is free for $(\exists x_0)\chi$ and $\varphi = \chi\sigma$, then $\chi = \varphi$.

By construction, φ is canonical with respect to x_0, so the first point is trivially proved.

We proceed to prove the second point. Let χ be a formula which is canonical with respect to x_0. We have to show that, if there is a σ that is a free substitution for $(\exists x_0)\psi$ such that $\varphi = \chi\sigma$, then $\chi = \varphi$.

Let us suppose that $\sigma = \{x_1/t_1, \ldots, x_n/t_n\}$ is a substitution such that $\varphi = \chi\sigma$. As σ is free for $(\exists x_0)\chi$ and does not instantiate x_0, the terms $t_1, \ldots, t_n$ do not contain x_0, but (by construction) these terms are the variables $x_1, \ldots, x_n$ and so $\sigma = \epsilon$ and $\varphi = \chi$, which concludes the proof. $\square$

We go on to show how the notions we have introduced will be used to optimize the tableau calculus. For that purpose, we make use of the fact that the length of proofs in first-order tableaux is closely related to the number of applications of the γ-rule. In substance the problem of shortening the length of tableau proofs is reduced to the one of finding a way to close all the tableau branches as soon as possible. Reasons why it may not be possible to close a tableau are:

Variable Dependencies. To close the tableau requires to unify a term with a variable occurring in it.

Ground Terms. There are too many ground terms and not enough free variables (and since all the free variables in the tableau come from γ-rule applications, obviously the more free variables are needed the more γ-rule applications are required).

3 The δ^{*^*}-Rule

The δ^{*^*}-rule addresses both problems mentioned at the end of the previous section, leading to a potentially exponential speedup over the δ^*-rule.

Baaz and Fermüller's definition of the δ^*-rule reduces the number of variable dependencies; in this way a non-elementary speedup is obtained, compared to the δ^{+^+}-rule of Beckert, Hähnle, and Schmitt described in [2]. For that purpose, Baaz and Fermüller introduce the notion of *relevant variables* of a formula φ with respect to a free variable x. One can go further by using a recursive definition of *relevant variables.* Instead, however, we define the notion of *relevant extracted formulae*, that not only allows to reduce the number of variable dependencies (and thus the number of arguments in the Skolem terms) but also to use the same Skolem symbols for existentially quantified formulae that differ only in *irrelevant* subformulae such as, for example, the formulae $(\exists x)p(x) \wedge q$ and $(\exists x)r \wedge p(x)$. In the definition of the δ^{*^*}-rule, this idea will be combined with the concept of key formulae explained in the previous section.

DEFINITION 3.1 *Let φ be a formula, and let S be a set of variables. We define the* relevant extracted formula *for φ w.r.t. S, denoted by $RelF(\varphi, S)$, as follows, where we indicate with Λ the "empty formula":*[2]

1. *If $Free(\varphi) \cap S = \emptyset$, then $RelF(\varphi, S) = \Lambda$,*
2. *otherwise:*
   (a) *$RelF(\varphi, S) = \varphi$ if φ is atomic,*
   (b) *$RelF(\varphi, S) = \neg RelF(\psi, S)$ if $\varphi = \neg\psi$,*
   (c) *$RelF(\varphi, S) = RelF(\psi, S) * RelF(\chi, S)$ if $\varphi = \psi * \chi$, where $*$ is a binary connective,*
   (d) *$RelF(\varphi, S) = (Qy)RelF(\psi, S \cup \{y\})$ if $\varphi = (Qy)\psi$, where Q is a quantifier.*

The above definition of *relevant extracted formulae* subsumes the definition of *relevant variables $Rel(\varphi, x)$* as given in [1] in the sense that

$$Free(RelF(\varphi, \{x\})) \setminus \{x\} \subseteq Rel(\varphi, x) \ .$$

Now we have everything at hand to give the formal definition of our δ^{*^*}-rule:

DEFINITION 3.2 *Let $\delta = (\exists x)\varphi$, and let φ_1 be the corresponding key formula w.r.t. x, such that $\varphi_1\sigma \equiv \varphi$. Further let $\varphi_2 = RelF(\varphi_1(x_0), \{x_0\})$ be the relevant extracted key formula, let $S = Free(\varphi_2) \setminus \{x_0\}$, and let $h_{\varphi_2} \in \mathcal{F}^{(|S|)}$ be the corresponding Skolem function symbol.*

The δ^{*^*}-rule can be schematically described as follows:

$$\frac{\delta}{\delta_0(h_{\varphi_2}(\vec{S})\sigma)} \quad \text{if } \varphi_2 \neq \Lambda. \qquad\qquad \frac{\delta}{\delta_0(x)} \quad \text{if } \varphi_2 = \Lambda.$$

where $\vec{S}$ are the terms substituted for the free variables of the relevant extracted key formula φ_2 in the substitution σ defined above.

4 The Extended Signature Σ^*

As we have already pointed out, in case the formula to be proved is not yet Skolemized, the initial signature is enriched. In previous versions of the δ-rule, namely the δ^{+^+}-rule and the δ^*-rule, the signature structure is of interest, because sometimes it is possible to use the same symbol more than once.

Now, analogously, we can analyze how the augmented signature for the δ^{*^*}-rule is constructed starting from an initial signature Σ. For that purpose we define the operator *sk* that, given a signature, computes an extended signature enriched by new function symbols produced by the Skolemization of the δ-formulae over Σ. It is defined as follows:

$$(\Sigma)_{sk} = \{P_\Sigma, F_\Sigma \cup \{f \mid f \text{ is a Skolem function symbol corresponding to a relevant extracted key formula } \varphi \in \mathcal{L}^\Sigma\}\}$$

[2] Note, that Λ does not have any semantics, and that $\neg\Lambda$ syntactically reduces to Λ, $\Lambda * \psi$ to ψ ($*$ is any propositional connective), $\psi * \Lambda$ to ψ and $(Qy)\Lambda$ to Λ.

The extended signature Σ^{*^*} is recursively defined as follows:

$$\begin{array}{rcll} \Sigma^0 & = & \Sigma & \\ \Sigma^{n+1} & = & (\Sigma^n)_{sk} & \quad \text{for } n \geq 0 \\ \Sigma^{*^*} & = & \bigcup\limits_{n \geq 0} \Sigma^n & \end{array}$$

Finally, we define the notion of the rank of a Skolem function symbol.

DEFINITION 4.1 *The* rank *of a Skolem functional symbol f is the smallest $n \geq 0$ such that $f \in \Sigma^n$.*

When we prove a formula in which the maximal nesting degree of existential quantifications is n, then no Skolem function symbols of a rank greater than n can occur in a tableau proof. Consequently, the number of recursive steps needed to construct a signature for such a tableau proof starting from the initial signature is exactly n.

5 Exponential Speedup

In this section, we show that using the δ^{*^*}-rule instead of the δ^*-rule can shorten proofs exponentially.

THEOREM 5.1 *There is a class of formulae $(\phi_n)_{(n \geq 1)}$ such that, if $b^*(n)$ (resp. $b^{*^*}(n)$) is the number of branches of the shortest closed tableau for ϕ_n using the δ^*-rule (resp. δ^{*^*}-rule), then the shortest closed tableau for ϕ_n using the δ^*-rule has*

$$b^*(n) = O(2^{b^{*^*}(n)})$$

branches.

Proof. We recursively define the following class of formulae:

$$\begin{array}{rcl} \phi_1 & = & \mathit{false} \\ \phi_n & = & (\forall x)(\forall y)(\phi_{n-1} \ \lor \ [p_n(x,y) \land \\ & & \qquad ((\forall v)(\exists z)(\neg p_n(z, f(v))) \lor (\forall w)(\exists z)(\neg p_n(z, f(f(w)))))]) \end{array}$$

for $n \geq 2$

The theorem is then proven by showing that

1. the number $b^{*^*}(n)$ of branches of the smallest closed δ^{*^*}-tableau for ϕ_n is linear in n,
2. the number $b^*(n)$ of branches of the smallest closed δ^*-tableau for ϕ_n is exponential in n.

Intuitively, the reason for the different behavior of the δ^{*^*}- and the δ^*-rule on the above formula class is that the δ^{*^*}-rule uses *the same* Skolem function symbol h to Skolemize the two existential formulae in the second part of ϕ_n; therefore, a single copy of the literal $p_n(x_1, y_1)$ is sufficient to close the two branches that contain these existential formulae, and the closed tableau $T_n^{*^*}$ for ϕ_n contains only *one* copy of $T_{n-1}^{*^*}$. The δ^*-rule, on the other hand, introduces two *different* Skolem function symbols h and g. As a result, two

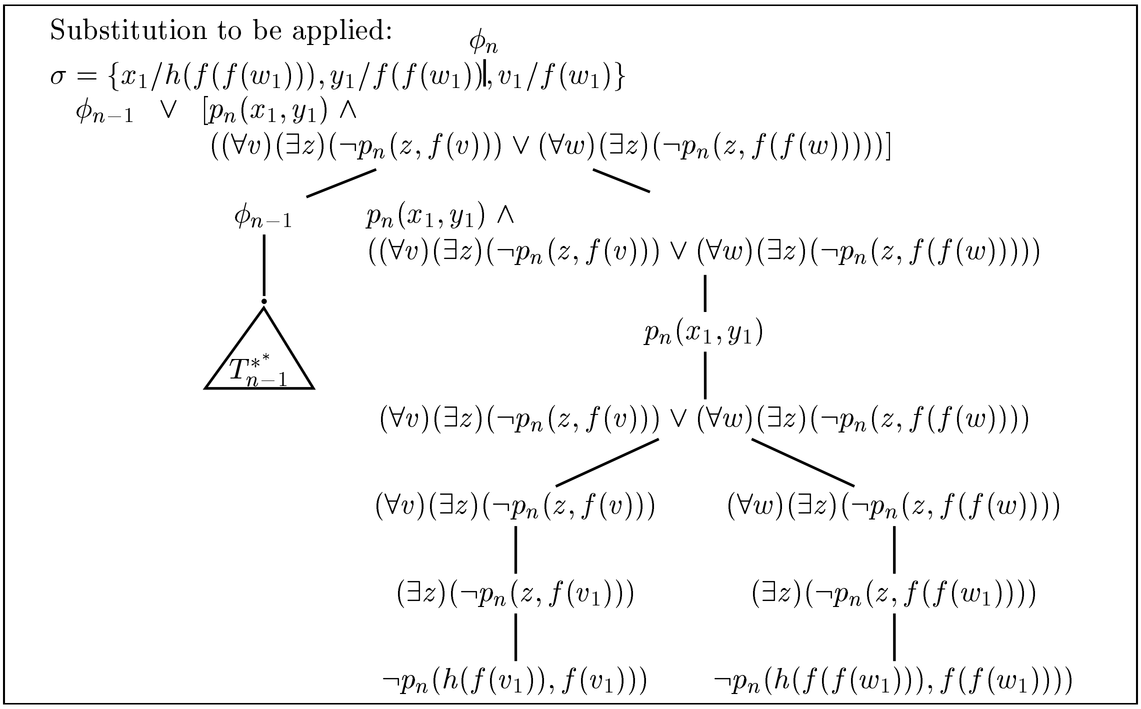

Figure 1: A minimal δ^{*^*}-tableau for ϕ_n that is closed after application of the substitution σ shown at the top.

instances $p_n(x_1, y_1)$ and $p_n(x_2, y_2)$ have to be generated; this, however, means that the closed tableau T_n^* for ϕ_n must contain *two* copies of T_{n-1}^*.

$b^{*^*}(n)$ *is linear in* n. It is easy to see that the tableau $T_n^{*^*}$ shown in Figure 1 is a smallest closed δ^{*^*}-tableau for ϕ_n, The number $b^{*^*}(n)$ of branches of $T^{*^*}(n)$ is

$$b^{*^*}(n) = b^{*^*}(n-1) + 2$$

for $n \geq 1$, which implies that $b^{*^*}(n)$ is linear in n.

$b^*(n)$ *is exponential in* n. Similar to the previous case, it is easy to see that the tableau T_n^* shown in Figure 2 is a smallest closed δ^*-tableau for ϕ_n, The number $b^*(n)$ of branches of $T^*(n)$ is

$$b^*(n) = 2b^*(n-1) + 2$$

for $n \geq 2$, which implies that $b^{*^*}(n)$ is exponential in n. ■

Note that the above proof uses only one of the two main features of the δ^{*^*}-rule, namely the fact that it uses the concept of key formulae for assigning Skolem function symbols to δ-formulae. The same result can be proved solely based on the second main feature, which is to ignore non-relevant sub-formulae.

REMARK 5.1 *Notice that it is possible to adapt to our case Baaz and Fermüller proof given in [1], thus proving a non-elementary speedup of the δ^{*^*}-rule over the δ^*-rule. Without going into details, it is sufficient to find a suitable "justifying formula" in the sense of [1] to obtain the desired result.* □

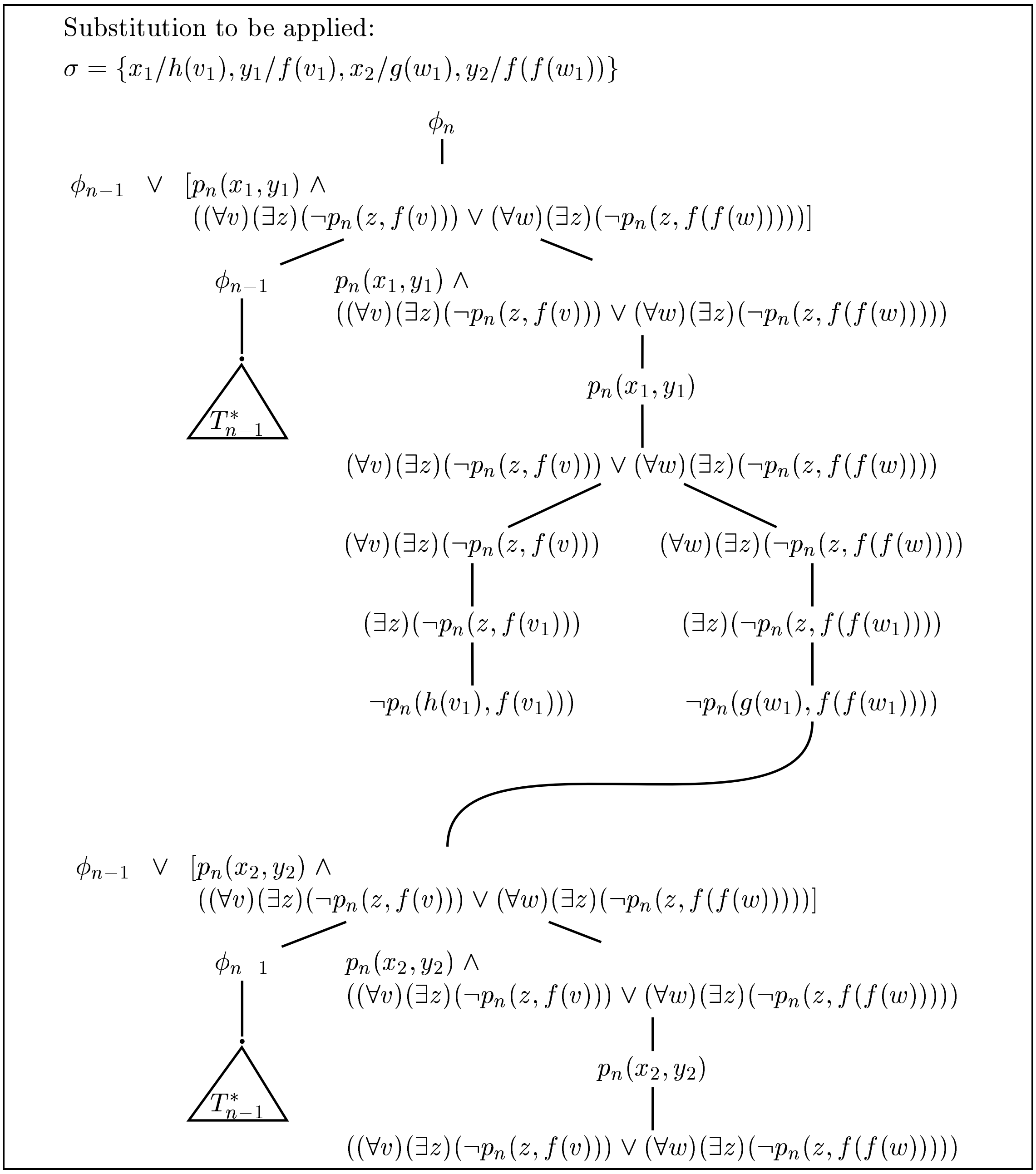

Figure 2: A minimal δ^*-tableau for ϕ_n that is closed after application of the substitution σ shown at the top.

6 Proving Soundness of δ^{*^*}

The fact that the δ^{*^*}-rule is a liberalization of the δ^*-rule implies that completeness is trivially preserved and does not have to be proved.

The soundness proof, which because of space restrictions we can only sketch, follows the same lines as that for the δ^*-rule. First, satisfiability of tableaux is defined, and then it is proved that satisfiability is preserved when a tableau is expanded or a (closing) substitution is applied to a tableau.

The main idea for proving that satisfiability of tableau is preserved is to inductively define a sequence $(\mathcal{M}_n)_{n\geq 0}$ of structures that all have the domain $\mathcal{D}$. This sequence is an approximation of a structure $\mathcal{M}^* = \langle \mathcal{D}, \mathcal{I}^* \rangle$ for the signature Σ^{*^*}. The interpretation $\mathcal{I}^*$ is constructed in such a way that the Skolem function symbols are interpreted in the right way.

The initial structure $\mathcal{M}_0 = \langle \mathcal{D}, \mathcal{I}^0 \rangle$ is an arbitrary structure for the signature $\Sigma = \Sigma^0$ that satisfies the formula to be proved; $\mathcal{M}_{n+1} = \langle \mathcal{D}, \mathcal{I}^{n+1} \rangle$ is a structure for the signature Σ^{n+1}; $\mathcal{I}^{n+1}$ coincides with $\mathcal{I}^n$ on all symbols in Σ^n. The function symbols of rank $r \leq n$ are already been interpreted in $\mathcal{M}_n$.

To define the interpretation of the function symbols of rank $n+1$, let us consider the formula $\delta = (\exists x)\delta_0$ and let φ be the key formula corresponding to δ_0 and $\sigma = \{y_1/x, y_2/t_1, \ldots, y_{n+1}/t_n\}$ the substitution such that $\varphi\sigma \equiv \delta_0$. Let $\varphi_1 = \mathit{RelV}(\varphi, \{x_0\})$ be the corresponding relevant extracted key formula and let $f_{[\delta]_{R_{**}}}$ the related Skolem functional symbol introduced to Skolemize the formula $\delta = (\exists x)\delta_0$. The Skolem term substituted to the variable x, $f_{[\delta]_{R_{**}}}(t_1, \ldots, t_i)$, where $t_1, \ldots, t_i$ are the terms in δ_0 corresponding to the free variables of φ_1 is interpreted as follows:

1. If there is a variable assignment A such that $(\mathcal{M}^n, A) \models \varphi$, according to the semantics of the existential quantifier then we choose an element c of the domain such that:

   $$(\mathcal{M}^n, A[x \leftarrow c]) \models \varphi_0$$

   and define

   $$[f_{[\delta]_{R_{**}}}(t_1, \ldots, t_i)]^{\mathcal{I}^{n+1}} = c$$

2. otherwise we define

   $$[f_{[\delta]_{R_{**}}}(t_1, \ldots, t_i)]^{\mathcal{I}^{n+1}} = c$$

   where c is an arbitrarily chosen element of the domain.

7 Conclusion

We have introduced a new version of the δ-rule in semantic tableau, based on the δ^*-rule of Baaz and Fermüller.

The new rule carries mainly two features:

1. For Skolemization, we identify formulae that are identical up to irrelevant subformulae, and assign them the same Skolem function symbol.

2. We abstract from the terms in an existential formula before assigning it a Skolem symbol; the terms are used as arguments of the Skolem term that is introduced.

As we already pointed out, both these features independently enable an exponential reduction in proof complexity.

Key formulae and extracted key formulae are not expensive to calculate. Global Skolemization has already been implemented in $SETL$. As a next step, we plan to implement a semantic tableau system employing the δ^{*^*}-rule and several additional optimizations. We believe that our version of the δ-rule offers many advantages over previous versions and a good ratio between reduction in proof length and costs of execution.

What we want to emphasize as the main point of our work is not the fact that it is possible at all to gain an exponential speedup, but that our rule (in particular due to the second feature mentioned above) triggers Skolemization in a quite natural way. If a proof introduces functions in a generalizing manner, we keep closer to the usual *intuition* of what the meaning of a function is, i.e., an abstraction that is applicable to *different* elements.

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