The Method CERES

M. Baaz, A. Leitsch, S. Hetzl, C. Richter, H. Spohr

January 15, 2007

1 LK

Basically we use Gentzen's version of **LK** (see [1]), but extend it by equality and definition rules. The axioms may be arbitrary *atomic* sequents. There are two groups of rules, the logical and the structural ones. All rules with the exception of cut have left and right versions; left versions are denoted by $\xi : l$, right versions by $\xi : r$. Every logical rule introduces a logical operator on the left or on the right side of a sequent. Structural rules serve the purpose of making logical inferences possible (e.g. permutation) or to put proofs together (cut). A and B denote formulas, $\Gamma, \Delta, \Pi, \Lambda$ sequences of formulas. In the rules there are introducing or *auxiliary formulas* (in the premises) and introduced or *principal formulas* in the conclusion. We indicate these formulas for all rules. In particular we mark the auxiliary formulas by $+$ and the principal ones by $\star$.

The logical rules:

- $\wedge$-introduction:

$$\frac{A^+, \Gamma \vdash \Delta}{(A \wedge B)^\star, \Gamma \vdash \Delta}\ \wedge\!: l^1 \qquad \frac{B^+, \Gamma \vdash \Delta}{(A \wedge B)^\star, \Gamma \vdash \Delta}\ \wedge\!: l^2$$

$$\frac{\Gamma \vdash \Delta, A^+ \quad \Pi \vdash \Lambda, B^+}{\Gamma, \Pi \vdash \Delta, \Lambda, (A \wedge B)^\star}\ \wedge : r$$

- $\vee$-introduction:

$$\frac{A^+, \Gamma \vdash \Delta \quad B^+, \Pi \vdash \Lambda}{(A \vee B)^\star, \Gamma, \Pi \vdash \Delta, \Lambda}\ \vee\!: l \quad \frac{\Gamma \vdash \Delta, A^+}{\Gamma \vdash \Delta, (A \vee B)^\star}\ \vee\!: r^1 \quad \frac{\Gamma \vdash \Delta, B^+}{\Gamma \vdash \Delta, (A \vee B)^\star}\ \vee\!: r^2$$

- $\rightarrow$-introduction:

$$\frac{\Gamma \vdash \Delta, A^+ \quad B^+, \Pi \vdash \Lambda}{(A \rightarrow B)^\star, \Gamma, \Pi \vdash \Delta, \Lambda}\ \rightarrow\!: l \qquad \frac{A^+, \Gamma \vdash \Delta, B^+}{\Gamma \vdash \Delta, (A \rightarrow B)^\star}\ \rightarrow\!: r$$

- $\neg$-introduction:

$$\frac{\Gamma \vdash \Delta, A^{+}}{\neg A^{\star}, \Gamma \vdash \Delta}\ \neg\colon l \qquad \frac{A^{+}, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, \neg A^{\star}}\ \neg\colon r$$

- $\forall$-introduction:

$$\frac{A\{x \leftarrow t\}^{+}, \Gamma \vdash \Delta}{(\forall x)A^{\star}, \Gamma \vdash \Delta}\ \forall\colon l$$

where t is an arbitrary *term* containing only free variables.

$$\frac{\Gamma \vdash \Delta, A\{x \leftarrow \alpha\}^{+}}{\Gamma \vdash \Delta, (\forall x)A^{\star}}\ \forall\colon r$$

where α is a free variable which may not occur in Γ, Δ, A. α is called an *eigenvariable.*

- The logical rules for $\exists$-introduction (the variable conditions for $\exists : l$ are these for $\forall\colon r$, and similarly for $\exists : r$ and $\forall\colon l$):

$$\frac{A\{x \leftarrow \alpha\}^{+}, \Gamma \vdash \Delta}{(\exists x)A^{\star}, \Gamma \vdash \Delta}\ \exists\colon l \qquad \frac{\Gamma \vdash \Delta, A\{x \leftarrow t\}^{+}}{\Gamma \vdash \Delta, (\exists x)A^{\star}}\ \exists\colon r$$

The structural rules:

- *permutation*

$$\frac{A_1, \ldots, A_n \vdash \Delta}{A_{\sigma(1)}, \ldots, A_{\sigma(n)} \vdash \Delta}\ \pi : l(\sigma) \qquad \frac{\Gamma \vdash A_1, \ldots, A_n}{\Gamma \vdash A_{\sigma(1)}, \ldots, A_{\sigma(n)}}\ \pi : r(\sigma)$$

where σ is a permutation. In $\pi : l$ all formulas on the left side of the conclusion sequent are principal formulas and all formulas on the left side of the premise sequent are auxiliary formulas; similarly for $\pi : r$.

- *weakening*:

$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, A^{\star}}\ w\colon r \qquad \frac{\Gamma \vdash \Delta}{A^{\star}, \Gamma \vdash \Delta}\ w\colon l$$

- *contraction:*

$$\frac{A^{+}, A^{+}, \Gamma \vdash \Delta}{A^{\star}, \Gamma \vdash \Delta}\ c\colon l \qquad \frac{\Gamma \vdash \Delta, A^{+}, A^{+}}{\Gamma \vdash \Delta, A^{\star}}\ c\colon r$$

- The *cut rule:*

$$\frac{\Gamma \vdash \Delta, A^{+} \quad A^{+}, \Pi \vdash \Lambda}{\Gamma, \Pi \vdash \Delta, \Lambda}\ cut$$

As unary structural rules are not important in classical logic we frequently indicate their use by double lines, e.g.

$$\frac{\Gamma \vdash \Delta \quad \Pi \vdash \Lambda}{\Gamma', \Pi' \vdash \Delta', \Lambda'}\ \xi$$

indicates the application of the rule ξ plus a sequence of unary structural rules before and/or after the application of ξ.

2 LKDe

LKDe is an exstension of **LK**. Besides the rules of **LK**, **LKDe** contains definition- and equality rules.

- The *definition rules* directly correspond to the *extension principle* (see [2]) in predicate logic. It simply consists in introducing new predicate- and function symbols as abbreviations for formulas and terms. Mathematically this corresponds to the introduction of new concepts in theories. Let A be a first-order formula with the free variables $x_1, \ldots, x_k$ (denoted by $A(x_1, \ldots, x_k)$) and P be a *unique* k-ary predicate symbol corresponding to the formula A. Then the rules are:

  $$\frac{A(t_1, \ldots, t_k)^\star, \Gamma \vdash \Delta}{P(t_1, \ldots, t_k)^+, \Gamma \vdash \Delta}\ \mathrm{def}_P\colon l \qquad \frac{\Gamma \vdash \Delta, A(t_1, \ldots, t_k)^\star}{\Gamma \vdash \Delta, P(t_1, \ldots, t_k)^+}\ \mathrm{def}_P\colon r$$

  for arbitrary sequences of terms $t_1, \ldots, t_k$. There are also definition introduction rules for new function symbols which are of similar type.

- The *equality rules* are:

  $$\frac{\Gamma \vdash \Delta, s = t^+ \quad A[s]^+_\Sigma, \Pi \vdash \Lambda}{A[t]^\star_\Sigma, \Gamma, \Pi \vdash \Delta, \Lambda}\ =\colon l1 \qquad \frac{\Gamma \vdash \Delta, t = s^+ \quad A[s]^+_\Sigma, \Pi \vdash \Lambda}{A[t]^\star_\Sigma, \Gamma, \Pi \vdash \Delta, \Lambda}\ =\colon l2$$

  for inference on the left and

  $$\frac{\Gamma \vdash \Delta, s = t^+ \quad \Pi \vdash \Lambda, A[s]^+_\Sigma}{\Gamma, \Pi \vdash \Delta, \Lambda, A[t]^\star_\Sigma}\ =\colon r1 \qquad \frac{\Gamma \vdash \Delta, t = s^+ \quad \Pi \vdash \Lambda, A[s]^+_\Sigma}{\Gamma, \Pi \vdash \Delta, \Lambda, A[t]^\star_\Sigma}\ =\colon r2$$

  on the right, where Σ denotes a set of positions of subterms where replacement of s by t has to be performed. We call $s = t$ (or $t = s$) the *active equation* of the rules. Note that, on atomic sequents, the rules coincide with *paramodulation* — under previous application of the most general unifier.

3 Skolemization of Proofs

Skolemization is a transformation on closed formulas which removes all strong quantifiers. There are different types of skolemizations which may strongly differ in the proof complexity of the transformed formula (see [3]).

Definition 1
Let B be a formula. If $(\forall x)$ occurs positively (negatively) in B then $(\forall x)$ is called a strong (weak) quantifier. If $(\exists x)$ occurs positively (negatively) in B then $(\exists x)$ is called a weak (strong) quantifier.

Below we define the structural skolemization operator *sk* defined in [4].

Definition 2 *sk is a function which maps closed formulas into closed formulas; it is defined in the following way: $sk(F) = F$ if F does not contain strong quantifiers, and*

$$sk(F) = sk(F_{(Qy)}\{y \leftarrow f(x_1, \ldots, x_n)\})$$

if (Qy) is in the scope of the weak quantifiers $(Q_1 x_1), \ldots, (Q_n x_n)$ (appearing in this order) and $F_{(Qy)}$ denotes F after omission of (Qy) and f is a function symbol which does not occur in F (if $n = 0$ then f is a constant symbol).

In model theory and automated deduction the definition of skolemization mostly is dual to Definition 2, i.e. in case of prenex forms the existential quantifiers are eliminated instead of the universal ones. The skolemization of sequents, defined below, represents a more general framework covering both concepts.

Definition 3
Let S be the sequent $A_1, \ldots, A_n \vdash B_1, \ldots, B_m$ consisting of closed formulas only and $(A'_1 \wedge \ldots \wedge A'_n) \rightarrow (B'_1 \vee \ldots \vee B'_m)$ be the structural skolemization of $(A_1 \wedge \ldots \wedge A_n) \rightarrow (B_1 \vee \ldots \vee B_m)$. Then the sequent $S' : A'_1, \ldots, A'_n \vdash B'_1, \ldots, B'_m$ is called the skolemization *of S.*

Example 1 *Let S be the sequent $(\forall x)(\exists y)P(x,y) \vdash (\forall x)(\exists y)P(x,y)$. Then the skolemization of S is $S' :\ (\forall x)P(x, f(x)) \vdash (\exists y)P(c,y)$ for a one-place function symbol f and a constant symbol c. Note that the skolemization of the left-hand-side of the sequent corresponds to the other skolemization concept for formulas mentioned above.*

Not only formulas and sequents, but also whole proofs can be skolemized. By a skolemized proof we mean a proof of a skolemized end-sequent. Also proofs with cuts can be skolemized, but the cut formulas themselves cannot. Only the strong quantifiers which "go" into the end sequent are eliminated. Skolemization does not increase the length of proofs.
We do not give a formal definition of proof skolemization *sk* but refer to [4]. We merely give an example illustrating the transformation.

Example 2 *Let* $\varphi =$

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{P(c,\alpha) \vdash P(c,\alpha) \quad Q(\alpha) \vdash Q(\alpha)}{P(c,\alpha), P(c,\alpha) \to Q(\alpha) \vdash Q(\alpha)}\; \to: l}{P(c,\alpha) \to Q(\alpha), (\forall x)P(c,x) \vdash Q(\alpha)}\; \forall : l}{P(c,\alpha) \to Q(\alpha), (\forall x)P(c,x) \vdash (\exists y)Q(y)}\; \exists : r}{(\exists y)(P(c,y) \to Q(y)), (\forall x)P(c,x) \vdash (\exists y)Q(y)}\; \exists : l}{(\forall x)P(c,x), (\forall x)(\exists y)(P(x,y) \to Q(y)) \vdash (\exists y)Q(y)}\; \forall : l$$

Then $sk(\varphi) =$

$$\cfrac{\cfrac{\cfrac{\cfrac{P(c,f(c)) \vdash P(c,f(c)) \quad Q(f(c)) \vdash Q(f(c))}{P(c,f(c)), P(c,f(c)) \to Q(f(c)) \vdash Q(f(c))}\; \to: l}{P(c,f(c)) \to Q(f(c)), (\forall x)P(c,x) \vdash Q(f(c))}\; \forall : l}{P(c,f(c)) \to Q(f(c)), (\forall x)P(c,x) \vdash (\exists y)Q(y)}\; \exists : r}{(\forall x)P(c,x), (\forall x)(P(x,f(x)) \to Q(f(x))) \vdash (\exists y)Q(y)}\; \forall : l$$

4 The Characteristic Clause Set

The *characteristic clause set* encodes the structure of a proof and the essence of the cut formulas used in the proof. Its construction is based on an analysis, which inferences go into a cut and which are propagated to the end-sequent. The characteristic clause set $\mathrm{CL}(\varphi)$ of a skolemized **LKDe**-proof φ is defined inductively. For every node ν of φ we define:

- If ν is an occurrence of an axiom S and S' is the subsequent of S containing only ancestors of cut formulas then $\mathcal{C}_\nu = \{ S' \}$.
- Let ν_1, ν_2 be the predecessors of ν in a binary inference then we distinguish:
  - The auxiliary formulas of ν_1, ν_2 are ancestors of cut formulas then $\mathcal{C}_\nu = \mathcal{C}_{\nu_1} \cup \mathcal{C}_{\nu_2}$.

– Otherwise $\mathcal{C}_\nu = \mathcal{C}_{\nu_1} \times \mathcal{C}_{\nu_2}$, where $\mathcal{C} \times \mathcal{D} = \{ C \circ D \mid C \in \mathcal{C}, D \in \mathcal{D} \}$ and $C \circ D$ is the merge of the clauses C and D, i.e. $\Gamma \vdash \Delta \circ \Pi \vdash \Lambda = \Gamma, \Pi \vdash \Delta, \Lambda$

- Let ν' be the predecessor of ν in a unary inference then $\mathcal{C}_\nu = \mathcal{C}_{\nu'}$.

The characteristic clause set $\mathrm{CL}(\varphi)$ is defined as $\mathcal{C}_\nu$ where ν is the root node of φ. Note that for a cut-free proof φ we have $\mathrm{CL}(\varphi) = \{\vdash\}$ (which is the set consisting just of falsum).

Example 3 *We define the* **LK***-proof φ (for α, β free variables, a a constant symbol):*

$\varphi :=$

$$\dfrac{\varphi_1 \quad \varphi_2}{(\forall x)(\neg P(x) \lor Q(x)) \vdash (\exists y)Q(y)}\; cut$$

$\varphi_1 :=$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(\alpha)^\star \vdash P(\alpha)}{\vdash P(\alpha), \neg P(\alpha)^\star}\;\neg : r}{\neg P(\alpha) \vdash \neg P(\alpha)^\star}\;\neg : l}{\neg P(\alpha) \vdash \neg P(\alpha) \lor Q(\alpha)^\star}\;\lor : r1 \quad \dfrac{Q(\alpha) \vdash Q(\alpha)^\star}{Q(\alpha) \vdash \neg P(\alpha) \lor Q(\alpha)^\star}\;\lor : r2}{\neg P(\alpha) \lor Q(\alpha) \vdash \neg P(\alpha) \lor Q(\alpha)^\star, \neg P(\alpha) \lor Q(\alpha)^\star}\;\lor : l}{\neg P(\alpha) \lor Q(\alpha) \vdash \neg P(\alpha) \lor Q(\alpha)^\star}\;c : r}{\neg P(\alpha) \lor Q(\alpha) \vdash (\exists y)(\neg P(\alpha) \lor Q(y))^\star}\;\exists : r}{(\forall x)(\neg P(x) \lor Q(x)) \vdash (\exists y)(\neg P(\alpha) \lor Q(y))^\star}\;\forall : l}{(\forall x)(\neg P(x) \lor Q(x)) \vdash (\forall x)(\exists y)(\neg P(x) \lor Q(y))^\star}\;\forall : r$$

$\varphi_2 :=$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\vdash P(a)^\star}{\neg P(a)^\star \vdash}\;\neg : l \quad Q(\beta)^\star \vdash Q(\beta)}{\neg P(a) \lor Q(\beta)^\star \vdash Q(\beta)}\;\lor : l}{\neg P(a) \lor Q(\beta)^\star \vdash (\exists y)Q(y)}\;\exists : r}{(\exists y)(\neg P(a) \lor Q(y))^\star \vdash (\exists y)Q(y)}\;\exists : l}{(\forall x)(\exists y)(\neg P(x) \lor Q(y))^\star \vdash (\exists y)Q(y)}\;\forall : l$$

Note that φ_2 also contains non-tautological axioms.
Let Ω be the set of the two occurrences of the cut formula in φ. The ancestors of Ω are marked by $\star$. We compute the characteristic clause set $\mathrm{CL}(\varphi)$:

From the $\star$-marks in φ we first get the clause sets corresponding to the initial sequents:

$$X_1 = \{P(u) \vdash\},\ X_2 = \{\vdash Q(u)\},\ X_3 = \{\vdash P(a)\},\ X_4 = \{Q(v) \vdash\}.$$

The leftmost-uppermost inference in φ_1 is unary and thus the clause set X_1 corresponding to this position does not change. The first binary inference in φ_1 (it is $\vee : l$) takes place on non-ancestors of Ω — the auxiliary formulas of the inference are not marked by $\star$. Consequently we obtain the set

$$Y_1 = \{P(u) \vdash\} \times \{\vdash Q(u)\} = \{P(u) \vdash Q(u)\}.$$

The following inferences in φ_1 are all unary and so we obtain

$$\mathcal{C}_{\nu_1} = Y_1$$

for ν_1 being the position of the end-sequent of φ_1 in φ.
Again the uppermost-leftmost inference in φ_2 is unary and thus X_3 does not change. The first binary inference in φ_2 takes place on ancestors of Ω (the auxiliary formulas are $\star$-ed) and we have to apply union to X_3, X_4. So we get

$$Y_2 = \{\vdash P(a);\ Q(v) \vdash\}.$$

All following inferences in φ_2 are unary leaving the clause set unchanged. Let ν_2 be the occurrence of the end-sequent of φ_2 in φ. Then the corresponding clause set is

$$\mathcal{C}_{\nu_2} = Y_2.$$

The last inference (cut) in φ takes place on ancestors of Ω and we have to apply union again. This eventually yields the characteristic clause set

$$\mathrm{CL}(\varphi) = \{P(u) \vdash Q(u);\quad \vdash P(a);\quad Q(v) \vdash\}.$$

It is easily verified that $\mathrm{CL}(\varphi)$ is unsatisfiable. This is not only a coincidence, but a general principle:

Theorem 1 *Let φ be a skolemized proof. Then $\mathrm{CL}(\varphi)$ is unsatisfiable.*

Proof: In [5].

In case of equational clause sets the concept of unsatisfiability has to be replaced by E-unsatisfiability. For details see [6].

5 Projection and Refutation

Every clause C in the characteristic clause set $\mathrm{CL}(\varphi)$ of a skolemized proof φ defines a *proof projection* $\varphi[C]$. $\varphi[C]$ is defined from φ by omitting all inferences on ancestors of cuts, leaving the clause C as a "residue" in the end-sequent. Equality and definition rules appearing within the input proof are propagated to the projections like any other rules. In fact, $\varphi[C]$ is a cut-free proof of the end-sequent augmented by C. For a formal definition of projection we refer to [5] and [7].
The construction of $\varphi[C]$ is illustrated below.

Example 4 *Let φ be the proof of the sequent*

$$S : (\forall x)(\neg P(x) \lor Q(x)) \vdash (\exists y)Q(y)$$

as defined in Example 3. We have shown that

$$\mathrm{CL}(\varphi) = \{P(u) \vdash Q(u);\quad \vdash P(a);\quad Q(v) \vdash\}.$$

We now define $\varphi[C_1]$, the projection of φ to C_1: $P(u) \vdash Q(u)$:
The problem can be reduced to a projection in φ_1 because the last inference in φ is a cut and

$$\mathcal{C}_{\nu_1} \quad = \quad \{P(u) \vdash Q(u)\}.$$

By skipping all inferences in φ_1 leading to the cut formulas we obtain the deduction

$$\dfrac{\dfrac{\dfrac{P(\alpha) \vdash P(\alpha)}{\neg P(\alpha), P(\alpha) \vdash}\ \neg : l \qquad Q(\alpha) \vdash Q(\alpha)}{\neg P(\alpha) \lor Q(\alpha), P(\alpha) \vdash Q(\alpha)}\ \lor : l}{P(\alpha), (\forall x)(\neg P(x) \lor Q(x)) \vdash Q(\alpha)}\ \forall : l$$

In order to obtain the end-sequent we only need an additional weakening and $\varphi[C_1] =$

$$\dfrac{\dfrac{\dfrac{P(\alpha) \vdash P(\alpha)}{\neg P(\alpha), P(\alpha) \vdash}\ \neg : l \qquad Q(\alpha) \vdash Q(\alpha)}{\neg P(\alpha) \lor Q(\alpha), P(\alpha) \vdash Q(\alpha)}\ \lor : l}{P(\alpha), (\forall x)(\neg P(x) \lor Q(x)) \vdash Q(\alpha), (\exists y)Q(y)}\ \forall : l$$

For $C_2 = \vdash P(a)$ we obtain the projection $\varphi[C_2]$:

$$\dfrac{\dfrac{\vdash P(a)}{\vdash P(a), Q(\beta)}\ w : r}{(\forall x)(\neg P(x) \lor Q(x)) \vdash (\exists y)Q(y), P(a)}\ \exists : r$$

Similarly we obtain $\varphi[C_3]$*:*

$$\dfrac{Q(\beta) \vdash Q(\beta)}{\overline{Q(\beta), (\forall x)(\neg P(x) \lor Q(x)) \vdash (\exists y)Q(y)}} \; \exists : r$$

We have seen that, in the projections, only inferences on non-ancestors of cuts are performed. If the auxiliary formulas of a binary rule are ancestors of cuts we have to apply weakening in order to obtain the required formulas from the second premise.
Let φ be a proof of S s.t. φ is skolemized and let γ be a ground resolution refutation of the (unsatisfiable) set of clauses $\mathrm{CL}(\varphi)$. Then γ can be transformed into a proof $\varphi(\gamma)$ of S with at most atomic cuts. $\varphi(\gamma)$ is constructed from γ simply by replacing the clauses by the corresponding proof projections. The construction of $\varphi(\gamma)$ is the essential part of the method CERES (the final elimination of atomic cuts — provided it is possible at all — is inessential). The resolution refutation γ can be considered as the characteristic part of $\varphi(\gamma)$ representing the reduction to atomic cuts. Below we give an example of a construction of $\varphi(\gamma)$.

Example 5 *Let* φ *be the proof of*

$$S\colon (\forall x)(\neg P(x) \lor Q(x)) \vdash (\exists y)Q(y)$$

as defined in Examples 3 and 4. Then

$$\mathrm{CL}(\varphi) = \{C_1 : P(u) \vdash Q(u);\; C_2 : \vdash P(a);\; C_3 : Q(v) \vdash\}.$$

First we define a resolution refutation δ *of* $\mathrm{CL}(\varphi)$*:*

$$\dfrac{\dfrac{\vdash P(a) \quad P(u) \vdash Q(u)}{\vdash Q(a)}\,R \qquad Q(v) \vdash}{\vdash}\,R$$

and a corresponding ground refutation γ*:*

$$\dfrac{\dfrac{\vdash P(a) \quad P(a) \vdash Q(a)}{\vdash Q(a)}\,R \qquad Q(a) \vdash}{\vdash}\,R$$

The ground substitution defining the ground projection is

$$\sigma : \{u \leftarrow a, v \leftarrow a\}.$$

Let

$$\chi_1 = \varphi[C_1]\sigma,\ \chi_2 = \varphi[C_2]\sigma \text{ and } \chi_3 = \varphi[C_3]\sigma.$$

Moreover let us write B *for* $(\forall x)(\neg P(x) \vee Q(x))$ *and* C *for* $(\exists y)Q(y)$. *Then* $\varphi(\gamma)$ *is of the form*

$$\dfrac{\dfrac{\dfrac{(\chi_2)}{B \vdash C, P(a)} \quad \dfrac{(\chi_1)}{P(a), B \vdash C, Q(a)}}{B \vdash C, Q(a)}\,cut \qquad \dfrac{(\chi_3)}{Q(a), B \vdash C}}{B \vdash C}\,cut$$

Finally we give the general definition of CERES (cut-elimination by resolution) as a whole. As an input we take skolemized proofs φ.

- construct $\mathrm{CL}(\varphi)$,
- construct a resolution refutation γ of $\mathrm{CL}(\varphi)$,
- compute the output $\varphi(\gamma')$ for a ground projection γ' of γ.

Theorem 2 *CERES is a cut-elimination method, i.e., for every skolemized proof* φ *of a sequent* S, *CERES produces a proof* ψ *of* S *s.t.* ψ *contains at most atomic cuts.*

Proof: In [5] and [6].

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